Problems Chapter 5, QFT 2013

1. Show that the trace of a product of an odd nr of gamma-matrices is zero
2. Show that $\mathrm{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] = 4[g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}]$
3. Show that $\gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu$
4. Show that $\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho}$
5. Show that for $m = 0$
$$\sum_{\text{spins}} \left|\bar{v}(p')\gamma^\mu \frac{1+\gamma^5}{2} u(p)\right|^2 = 2(p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu} p\cdot p' - i\varepsilon^{\alpha\mu\beta\nu} p'_\alpha p_\beta)$$
6. a) Starting from the proper contractions show that the scattering amplitude for $e^-(p_1)\mu^-(p_2) \to e^-(p'_1)\mu^-(p'_2)$ scattering in QED is given by $i\mathcal{M} = \dfrac{ie^2}{(p_1-p'_1)^2}\bar{u}(p'_1)\gamma^\mu u(p_1)\bar{u}(p'_2)\gamma_\mu u(p_2)$

   b) Use the results in a) to show that, neglecting the electron mass, the spin-averaged matrix-element squared for $e^-(p_1)\mu^-(p_2) \to e^-(p'_1)\mu^-(p'_2)$ scattering is given by
$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{8e^4}{(p_1-p'_1)^4}\left[(p_1\cdot p'_2)(p'_1\cdot p_2) + (p_1\cdot p_2)(p'_1\cdot p'_2) - m_\mu^2(p_1\cdot p'_1)\right] = \frac{2e^4}{t^2}\left[u^2+s^2\right]$$
   where in the last step it has been assumed that $m_\mu = 0$ and the Mandelstam variables are $s = (p_1+p_2)^2$, $t = (p_1-p'_1)^2$, and $u = (p_1-p'_2)^2$.

   c) Show that in the center-of-mass frame this gives (keeping m_μ)
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{CM}} = \frac{\alpha^2}{2|\vec{k}|^2(E+|\vec{k}|)^2(1-\cos\theta)^2}\left[(E+|\vec{k}|)^2 + (E+|\vec{k}|\cos\theta)^2 - m_\mu^2(1-\cos\theta)\right]$$
   where the incoming muon has momentum $p_2 = (E, -|\vec{k}|\hat{z})$, the outgoing one $p'_2 = (E, -\vec{k})$ and θ is the angle between the two such that $\vec{k}\cdot\hat{z} = |\vec{k}|\cos\theta$. What is the origin of the $1/\theta^4$ behaviour for small θ?
7. Show that in the center-of-mass system the Mandelstam variables (in the case when all particles have the same mass m and energy E) are given by
$$\begin{aligned} s &= E_{cm}^2 \\ t &= -2|\vec{p}|^2(1-\cos\theta) = [\text{m=0}] = -\frac{s}{2}(1-\cos\theta) \\ u &= -2|\vec{p}|^2(1+\cos\theta) = [\text{m=0}] = -\frac{s}{2}(1+\cos\theta) \end{aligned}$$
   where $E_{cm} = 2E$. Note that $s+t+u = 4m^2$. In general $s+t+u = \sum m_i^2$
8. Show that $(\not{p}+m)\gamma^\mu u(p) = 2p^\mu u(p)$ and that $\bar{u}(p)\gamma^\mu(\not{p}+m) = 2p^\mu\bar{u}(p)$
9. Show that the Compton amplitude vanishes if we replace $\varepsilon_\nu(k)$ with k_ν. Hint: rewrite the second term in $p'-k$ instead of $p-k'$ and make use of problem 8.

Please send an email to Johan.Rathsman@thep.lu.se at least one hour before the problem solving session stating the problems that you are willing to solve on the blackboard and the problems that you would like to see the solution to. (Click on the email address to get a preformated mail.)