

**Written exam, FYTB13/FYTA12, Electromagnetism,
May 30, 2018, 14.00–19.00.**

Allowed material: (a) one A4-sheet with notes; (b) pens, erasers, rulers and similar tools for drawing; (c) something to drink, eat, snacks; pillows, towels, and similar necessities.

Total of 30 points, 15 will be required to pass.

Note! The problems are not ordered in difficulty. Read the text of each problem carefully before attempting to solve it, and make sure to motivate all steps and assumptions carefully.

Use *separate* sheets for each problem, mark them with your ID number, and write on only *one* side. The **results** will be displayed in the theoretical physics corridor as soon as they are ready and at the latest on Friday June 15.

1. Coaxial charges [9p]

Consider an infinitely long cylindrical rod with radius a carrying a uniform charge density ρ . The rod is surrounded by a co-axial cylindrical metal-sheet with radius b that is connected to ground. The volume between the rod and the sheet is filled with a linear, isotropic, and homogeneous dielectric with dielectric constant ε .

- a)** [3p] Starting from Maxwell's equations on differential form, calculate the electric field in the regions $s < a$ and $a < s < b$.
- b)** [2p] Use the results from **(a)** to calculate the potential V in the regions $s < a$ and $a < s < b$. (If you have not solved **(a)** you can use $\mathbf{E}_{s < a} = C_1 \hat{\mathbf{s}}$ and $\mathbf{E}_{a < s < b} = C_2 \hat{\mathbf{s}}/s$.)
- c)** [2p] Calculate the free surface charge on the sheet at $s = b$. (*Hint:* what is V (and thereby \mathbf{E}) for $s > b$?)
- d)** [2p] Calculate the bound surface charges at $s = a$ and $s = b$.

2. Spinning ring [8p]

A thin circular metallic ring with radius a , conductivity σ , and cross-sectional area A will have the resistance $R = 2\pi a/(\sigma A)$. The ring is rotating with angular frequency ω around an axis along a diameter through the ring, and it is placed in a constant, homogeneous magnetic field \mathbf{B} , orthogonal to the axis of rotation.

- a)** [2p] Calculate the magnetic flux through the ring, Φ_m , as a function of time. (Assume for simplicity that the normal of the surface spanned by the ring is parallel to the magnetic field at $t = 0$.)
- b)** [2p] What is the induced EMF, \mathcal{E} , in the ring (neglecting its self-inductance), as a function of time? (If you have not solved **(a)** you can use $\Phi_m = C_1 \cos(\omega t)$)
- c)** [2p] What is the instantaneous induced current in the ring, expressed in terms of the resistance R ? (If you have not solved **(b)** you can use $\mathcal{E} = C_2 \sin(\omega t)$)
- d)** [2p] Calculate the (time-dependent) magnetic dipole moment that the current in the ring gives rise to.

3. Standing wave [6p] A standing electromagnetic wave in vacuum has the \mathbf{E} -field

$$\mathbf{E}(\mathbf{r}, t) = E_0 \hat{\mathbf{e}} \cos(\mathbf{k} \cdot \mathbf{r}) \cos(\omega t),$$

where \mathbf{k} is a wave vector, ω is an angular frequency, $\hat{\mathbf{e}}$ is a unit vector $\perp \mathbf{k}$, and E_0 is the amplitude of the electric field.

a) [2p] Express $\mathbf{E}(\mathbf{r}, t)$ as a superposition of two electromagnetic plane waves moving in opposite directions.

b) [2p] Use Maxwell's equations to determine the magnetic field $\mathbf{B}(\mathbf{r}, t)$ of the standing wave. Express your answer as a product of two factors: one depending only on \mathbf{r} , the other only on t .

c) [2p] Determine the Poynting vector of this wave, as a function of space and time. What is its time-averaged value, and what does that mean?

4. Surfing on a wave? [7p]

Consider a pointlike particle with charge q and mass m , moving in the electromagnetic fields of a plane wave propagating along the z -axis. Assume that the fields are determined by Maxwell's equations in vacuum, and determined by a vector potential (complex notation!) given by

$$\tilde{\mathbf{A}}(\mathbf{r}, t) = \hat{\mathbf{x}} A_0 e^{ikz - i\omega t}$$

where A_0 is a (real) amplitude. Assume so called *axial gauge*, where the scalar potential V is zero.

a) [3p] Determine the *physical* fields $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{B}(\mathbf{r}, t)$.

b) [2p] Determine the Lorentz force on the particle as a function of position, velocity and time.

c) [2p] Now specialize to the case of the particle moving in the z direction: assume $z(t) = z_0 + vt$, with z_0 an initial position. How large then is the force as a function of time, and in what direction does it point? Can this setup be used to accelerate the particle, and if so, under what conditions?

— Good Luck! —