Homework problems

Group I

Problem No 1.8

Greisen-Zatsepin-Kuzmin (GZK) effect

Interaction of photon with proton at sufficiently high energies may lead to the absorption of photon and creation of π-meson. Let the cross section of the latter process in the center-of-mass frame be

$$\sigma = 0.5 \text{ mb} \quad \text{at} \quad \sqrt{s} > m_\Delta$$

otherwise zero, where s is the total energy of photon and proton, $m_\Delta = 1200$ MeV (Δ-resonance mass), 1 mb $= 10^{27}$ cm^2. Find mean free path of a proton in the present Universe with respect to this process as a function of proton energy. At what distance from the source does proton lose 2/3 of its energy? Ignore all photons (e.g., emitted by stars), except for CMB.

Refs:
K. Greisen, Phys. Rev. Lett. 16, 748 (1966);
G. T. Zatsepin and V. A. Kuzmin, JETP Lett. 4, 78 (1966).

Problem No 2.3

Geodesic lines

If one chooses a time coordinate which does not coincide with proper time of particles at rest, FLRW metric takes the form

$$ds^2 = N^2(t)dt^2 - a^2(t)\gamma_{ij}dx^i dx^j \, .$$

Show by direct calculation that in this metric too, world lines of particles at rest are geodesic (this is of course obvious, since these lines are the same lines as in the text). (Hint: similar calculation, but for proper time, has been done in lectures, see also the course book).

Problem No 2.5

Redshift in open/closed universe

Consider photons and massless scalar particles with actions

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}\, g^{\mu\nu}\, g^{\lambda\rho}\, F_{\mu\lambda}F_{\nu\rho}$$

and

$$S = \frac{1}{2}\int d^4x\, \sqrt{-g}\, g^{\mu\nu}\, \partial_\mu\phi\partial_\nu\phi \, ,$$

respectively. Show that the relation

$$z(t) = \frac{a_0}{a(t)} - 1\,,$$

between redshift and scale factor remains valid in open and closed Universes, provided that the wavelength $\lambda(t)$ is small compared to the radius of spatial curvature $a(t)$ at all times between emission and absorption.

Problem 3.8

Total size/lifetime of the closed universe

Find the relation between a_m and the total mass in the closed Universe whose Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{a_m}{a^3} - \frac{1}{a^2}$$

What would be the maximum size of the universe having 1 kg of non-relativistic matter? What is total lifetime for such a universe?