Written exam problems, part 3

Problem No 8.2

Find the lowest possible freeze-out temperature of hypothetical massless particles, assuming 50 % uncertainty in $N_{\nu,eff}$. Do the same for uncertainty of 10 %.

Problem No 9.2

Let us extend the Standard Model by adding new real scalar field X which interacts with the Higgs field H only. Let us add the following term to the Standard Model Lagrangian,

$$\Delta\mathcal{L} = \frac{1}{2}\partial_\mu X \partial^\mu X - \frac{\kappa}{2} H^\dagger H X^2 - \frac{m^2}{2} X^2 \, .$$

The discrete symmetry ($X \to X$) ensures the stability of the scalar particle X, so it is a dark matter candidate.

Let the vacuum expectation value of the field X be equal to zero and $m_h \simeq 125$ GeV. Find the range of parameters m, κ in which X-particles make all of dark matter.

Problem No 9.3

Let us assume that there exist stable, electrically charged particles $X^\pm$ much heavier than proton. Let us recall that baryons in the early Universe are predominantly either protons or α-particles (^{4}He nuclei). Mass fraction of α-particles is 25%.
(1) Find the binding energy of an atom consisting of X^--particle and proton. The same for X-α atom.
(2) Assuming that the number density of X-particles is small compared to that of baryons, find out in which state they predominantly exist today: bound state with proton, bound state with α-particle or free state.
(3) Assuming that $X^\pm$-particles were in thermal equilibrium in the early Universe, and that the asymmetry between X and X^+ equals η_X, find the present mass density of relic X-particles as function of their mass M_X and the asymmetry η_X.
(4) Place the bounds on the asymmetry η_X making use of the fact that searches for anomalous heavy isotopes ("wild hydrogen" and "wild helium") set upper bounds on the present mass density of these isotopes at the level [78, 79] $\Omega_X < 10^{10} \cdot (10 \text{ TeV} / M_X)$ at $M_X < 10$ TeV (see also [1, 79] for other mass intervals).

Problem No 10.2

Let the effective potential be

$$V_{eff}(\phi) = \frac{\lambda}{4}\phi^2(\phi - v)^2 - \epsilon\phi^2 \, ,$$

where λ, v, and ϵ are positive parameters. Find the conditions at which the thin-wall approximation is valid. Find the surface tension and wall thickness in the thin-wall approximation, as well as the size of the critical bubble R_c; estimate the probability of nucleation of a bubble of the new phase inside the phase with $\phi = 0$ in the thin-wall approximation at temperature T.

Problem No 11.3

Find the relic positron abundance at the present epoch.