Homework problems

Group II

Rasmus
Mattias
Torbjörn

Problem No 1.9

Ultra-high energy neutrinos in the Universe

If there exist neutrinos of ultra-high energies in Nature, they may scatter off relic neutrinos. The neutrino-neutrino cross section in the Standard Model of particle physics is very small. Its maximum value, $\sigma = 0.15\,\mu b = 1.5{\cdot}10^{31}\,\mathrm{cm}^2$, is reached at center-of-mass energy $\sqrt{s} \approx M_Z \approx 90$ GeV when neutrinos pair-annihilate through the resonant Z-boson production. The observation of photons produced in Z-decay chains would be indirect evidence for the existence of relic neutrinos. Find the mean free path of ultra-high energy neutrino in the present Universe with respect to the above process. Does one expect a cutoff in the spectrum of ultra-high energy neutrinos, similar to the GZK cutoff in the spectrum of ultra-high energy protons (see Problem 1.8)?

Problem A5

useful GR math

Check the following properties of the Christoffel symbols and covariant derivative:

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\begin{aligned}
&\Gamma^{\mu}_{\nu\mu} = \partial_\nu \ln\sqrt{-g}\,,\\
&g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = -\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\lambda\mu})\,,\\
&\nabla_\mu A^\mu = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}A^\mu)\,,\\
&\nabla_\mu A^{\mu\nu} = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}A^{\mu\nu})\,, \quad \text{for} \quad A^{\mu\nu} = -A^{\nu\mu}\,,\\
&\nabla_\mu\nabla^\mu\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi)\,,
\end{aligned}
$$

Problem No 2.6

Klein-Gordon equation for a scalar field in expanding Universe

Show that for a slowly varying scale factor, solutions to equation

$$\frac{1}{a^2}\partial_\eta(a^2\partial_\eta\phi) - \partial_i\partial_i + m^2a^2\phi = 0$$

are superpositions of

$$\phi(\eta, \vec{x}) = \frac{1}{a(\eta)\sqrt{\Omega(\eta)}}\, e^{i\int^\eta \Omega(\eta)d\eta}\, e^{-\vec{k}\vec{x}} \cdot (1 + \mathcal{O}(\partial_\eta a))\,,$$

where $\Omega(\eta) = \sqrt{k^2 + m^2a^2(\eta)}$. Thus, coordinate frequency (derivative of the exponent with respect to conformal time η) equal $\Omega(\eta)$, while the physical frequency is

$$\omega(\eta) = \frac{\Omega(\eta)}{a(\eta)}$$

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