Homework problems

Group III

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Problem No 2.5

Redshift in open/closed universe

Consider photons and massless scalar particles with actions

$$S = -\frac{1}{4}\int d^4x\sqrt{-g}\, g^{\mu\nu}\, g^{\lambda\rho}\, F_{\mu\lambda}F_{\nu\rho}$$

and

$$S = \frac{1}{2}\int d^4x\, \sqrt{-g}\, g^{\mu\nu}\, \partial_\mu\phi\partial_\nu\phi\,,$$

respectively. Show that the relation

$$z(t) = \frac{a_0}{a(t)} - 1\,,$$

between redshift and scale factor remains valid in open and closed Universes, provided that the wavelength $\lambda(t)$ is small compared to the radius of spatial curvature $a(t)$ at all times between emission and absorption.

Problem 3.7

Speed of sound in a universe with barotropic e.o.s.

Is it possible in expanding Universe with equation of state $p = p(\rho)$ to evolve from the regime $(p + \rho) > 0$ to the regime $(p + \rho) < 0$ without violating the real-valuedness of the sound speed us defined as $u_s^2 = \partial p/\partial\rho$?

Problem 3.8

Total size/lifetime of the closed universe

Find the relation between a_m and the total mass in the closed Universe whose Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{a_m}{a^3} - \frac{1}{a^2}$$

What would be the maximum size of the universe having 1 kg of non-relativistic matter? What is total lifetime for such a universe?