Homework problems (not exam!)  

Group III  

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Problem No 2.5  
Redshift in open/closed universe  
Consider photons and massless scalar particles with actions  
\[ S = -\frac{1}{4} \int d^4 x \sqrt{-g} g^{\mu\nu} g^{\lambda\rho} F_{\mu\lambda} F_{\nu\rho} \]  
and  
\[ S = \frac{1}{2} \int d^4 x \sqrt{-g} g^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi, \]  
respectively. Show that the relation  
\[ z(t) = \frac{a_0}{a(t)} - 1, \]  
between redshift and scale factor remains valid in open and closed Universes, provided that the wavelength \( \lambda(t) \) is small compared to the radius of spatial curvature \( a(t) \) at all times between emission and absorption.  

Problem 3.7  
Speed of sound in a universe with barotropic e.o.s.  
Is it possible in expanding Universe with equation of state \( p = p(\rho) \) to evolve from the regime \( (p + \rho) > 0 \) to the regime \( (p + \rho) < 0 \) without violating the real-valuedness of the sound speed \( u_s^2 = \frac{\partial p}{\partial \rho} \)?  

Problem 3.8  
Total size/lifetime of the closed universe  
Find the relation between \( a_m \) and the total mass in the closed Universe whose Friedmann equation is  
\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{a_m}{a^3} - \frac{1}{a^2} \]  
What would be the maximum size of the universe having 1 kg of non-relativistic matter? What is total lifetime for such a universe?