Homework problems 2

Group III

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Problem No 4.3

Redshift of the deceleration-to-acceleration transition epoch with the barotropic e.o.s

At what z does the transition from deceleration to acceleration occur for dark energy with e.o.s. $p = w\rho$, $w = \text{const}$? For what value of the parameter w this transition would occur now?

Problem No 4.17

Future of the Universe filled with quintessence

Assuming that today $\phi = \phi_0 \gg M_{Pl}$, study the future of the Universe in a model with potential $V(\phi) = \frac{1}{2}m^2\phi^2 + \epsilon\phi$, where $|\epsilon| \ll m^2 M_{Pl}$ is a small parameter, and m is such that $V(\phi_0) \sim \rho_c$. Hint: Use the results of Sec. 4.8.1.

Problem 5.5

Charge particles abundances in early Universe

Estimate the contribution of electrons and protons into total energy at T = 50 keV and T = 1 eV. Neglect the presence of light nuclei in the plasma.

Problem 6.8

Photon last scattering epoch

Show that at $T > 2500$ K the probability that 2s-atom gets ionized in the reaction $2s + \gamma^{th} \to e + p$ is higher than the probability that it becomes 1s-state due to the process $2s \to 1s + 2\gamma$. Here, γ^{th} denotes thermal photon. Hint: Use ionization cross section near threshold

$$\sigma_{2s} = \frac{2^{16}\pi^2}{3e^8}\frac{1}{\alpha m_e^2} = \frac{2^7}{e^4}\sigma_{1s} \approx 2.34\,\sigma_{1s}\,,$$

We note that the temperature 2500 K is lower than $T_r = 0.26$ eV $= 3000$ K, so the approximation of partial equilibrium is indeed valid at photon last scattering epoch. We note also that the typical time scales $t \lesssim \Gamma_{2s}^{-1}$ are much smaller than the time scale of the cosmological expansion, so the expansion does not spoil the equilibrium for processes $e + p \leftrightarrow 2s + \gamma$.

Problem 6.17

Sound velocity in the cosmic medium

Find sound velocity in the baryon-electron-photon plasma before recombination as function of temperature assuming that the baryon-to-photon ratio is known.