Lecture 18: Photon last scattering. Horizon problem

We are interested in the moment of time when photons scattered for the last time, and after which relic photons propagate freely.

Compton scattering: $\gamma + e \rightarrow \gamma + e$

mean free path (or time)

$\tau = \frac{1}{\xi \cdot n_e(T)}$

density of free electrons

$n_p = n_e \approx n_H \Rightarrow n_e \approx \frac{n_B}{2} = \frac{5}{\pi^2} \frac{E}{k_B T_{re}}$

for $T_{re} \approx 0.3$ eV

when $T = 0.26$ eV $\Rightarrow \approx 2500$ K

At these temperatures $\rightarrow$ kinetic equilibrium (when rate of energy exchange between different 3 components of cosmological plasma is faster than cosmological expansion)

$\frac{n_e}{n_B} (T_{re}) \ll \frac{n_B}{n_e} (T_{re})$, at $\omega = \Delta_H$ hydrogen binding energy $\Delta_H = 13.6$ eV

$\frac{n_e}{n_B} (T, \omega) \sim e^{-\Delta_H/k_B T} \ll 1$, $\omega \gg T$

We are interested in production of ground state hydrogen

$\rightarrow$ 1s-state: typical reactions

$e + p \leftrightarrow 1s + \gamma$ (1)

$2p \leftrightarrow 1s + 2\gamma$ (2)

$e + p \leftrightarrow 1s + 1\gamma$ (3)

ionization cross section

$\sigma_{ion} \gg \sigma_T$

for $n_{1s} \gg n_e$

(1) does not effectively change the number of free electrons and 1s-atoms.

(2) is more favorable for $\gamma$ to be absorbed back by $\gamma$-atom than by $e$-atom.

$\sigma_T = \frac{2}{3} \frac{m_e^2}{4 \pi} = 6.3 \cdot 10^{-24}$ cm$^2$
Consider transition \((2)\, 2s \rightarrow 1s + 2p\), width \(\Gamma_{2s} \approx 8.2 \text{ s}^{-1}\) (slow decay).  


More likely to get ionized by a thermal photon with \(\Delta \nu \gg \Delta\nu_t\).

\[2s + e^- \rightarrow e^- + p\]

In thermal equilibrium (expansion does not spoil it).

Saha equation for \(2s, e^-p\) in thermal equilibrium:

\[
\frac{n_e n_p}{n_e^2} = \frac{N_{2s}}{(meT)^{3/2} \cdot e^{-\Delta_{2s}/T}}
\]

\(\Delta_{2s} = \Delta H/4\)

Some of them going into \(1s\)-state (binding energy of \(2s\)-state)

\[
\left(\frac{dne}{dt}\right)_{2s \rightarrow 1s} = -\Gamma_{2s} N_{2s}
\]

Finally, \((3)\): \(1s + X \rightarrow 2p\) get redshifted away, so \(2p \rightarrow 1s + X\) effectively increases the number of \(1s\)-states with the same rate:

\[
\left(\frac{dne}{dt}\right)_{2p \rightarrow 1s} = -\Gamma_{2s} N_{2s}
\]

In total, \[
\frac{dne}{dt} = -\Gamma_{\text{eff}}(T) N_{2s}
\]

\(\Gamma_{\text{eff}} \approx 2 \Gamma_{2s}\)

\[
\Rightarrow \frac{dne}{dt} = -\Gamma_{\text{eff}} \left(\frac{2\pi}{T \cdot me}\right)^{3/2} e^{-\Delta_{2s}/T} \cdot n_e^2
\]

For relativistic matter, \(\frac{T}{T} \sim -H = -\frac{T^2}{M_{PL}^2}\) photon temperature:

\[
\frac{dT}{T} = -\frac{dt}{t} \Rightarrow \ln T = -\ln t + \text{const}
\]

\[
\Rightarrow t + dt = -\frac{dT}{T} \cdot H
\]

\[
\frac{1}{n_e} = \Gamma_{\text{eff}} \cdot \frac{T}{\Delta_{2s} \cdot H} \left(\frac{2\pi}{T \cdot me}\right)^{3/2} e^{-\Delta_{2s}/T} + \text{const}
\]

Small at \(t \approx t_r\).
\[ n_e(t) = \frac{\Delta n}{4 \Gamma_{\text{eff}} T} \left( \frac{T \text{Me}}{2\pi} \right)^{3/2} e^{-\Delta n / 4T} \]

\[ \Delta_{23} = \Delta n / 4 \]

\[ n_e^{eq}(t) = \frac{\Delta n}{4 \Gamma_{\text{eff}} T} \left( \frac{T \text{Me}}{2\pi} \right)^{3/2} e^{-\Delta n / 4T} \]

\[ \Rightarrow \text{different from equilibrium formula } n_e^{eq}(t) ! \]

Average number of collisions of a photon with free electrons from \( t \) until present time \( (t + \infty) \)

\[ N(t) = \int \left( \frac{\Gamma_{\text{eff}} n_e}{T} \right) dt' \]

\[ \Rightarrow \text{av. number of collisions per unit time} \exp \left( -\frac{T}{T_r} \right) \]

\[ dT = \frac{dT'}{T_r H} \]

\[ \Rightarrow 1 = \int_{0}^{\infty} \Gamma_{\text{eff}} n_e \frac{dT'}{H(T_r) \Delta n} \]

\[ \Rightarrow \text{equation for } T_r ! \]

\[ \left( \frac{T_r \text{Me}}{2\pi} \right)^{3/2} e^{-\Delta n / 4T_r} = \frac{\Gamma_{\text{eff}}}{\Gamma_T} \]

\[ \Rightarrow T_r = 0.26 \text{ eV} \Rightarrow H_r \approx 4.5 \text{ km/s/Mpc} \]

\[ H = \frac{2}{3L} \Rightarrow \Delta t = \left( \frac{2}{3} \right) H^{-1} = 480 \text{ thousand years} \]

\[ \text{CMB photons give us a photographic picture of the Universe at } T_r = 0.26 \text{ eV} (z_r = 1000) ! \]

\[ \Rightarrow \text{imprinted the typical length scale of the Universe} \]

\[ \Rightarrow \text{horizon scale } \ell_{H_r} \]

Recombination occurs at MD epoch \( (T_{RD} \rightarrow MD \approx 1 \text{ eV}) \)

\[ \ell_{H_r} = \frac{2}{H_r} \]

\[ t = t_r \quad t = \infty \]

\[ \ell_{H_r} = \frac{2}{H_0 \sqrt{\Delta M} (1 + z_r)^{3/2}} \]

\[ \ell_{H,r} = \frac{2}{H_0 \sqrt{\Delta M} (1 + z_r)^{3/2}} \]

\[ \ell_{H,r} (t_0) = \frac{2}{H_0 \sqrt{\Delta M}} \cdot \frac{1}{(1 + z_r)^{3/2}} \approx \frac{1}{10} \]

\[ \text{(present horizon)} \]
\[ \frac{L_H^0}{L_H} \sim (1 + 2 \pi)^{3/2} \sim 3 \times 10^4 \ \text{regions which were causally disconnected by recombination, but these regions are exactly the same from CMB observations.} \]

How did it happen? [Horizon problem] → solved in inflationary theory.