Phase Transitions (PT)

Universe was hot and may have experienced rearrangement of the ground state \( \Rightarrow \) phase transitions.

No direct experimental indications at \( T > \text{a few MeV} \).

Examples of PT: \( T > 280 \text{ MeV} \) \( \Rightarrow \) no hadrons.

- QCD phase transition:
  - Quark-gluon plasma \( \leftrightarrow \) colorless pseudoscalar mesons (hard to check \( \rightarrow \) no traces).
  - Chiral phase transition: \( \Rightarrow \) formation of \( \langle \eta \eta \rangle \) condensate.

- EW phase transition:
  - \( T_\text{EW} \approx 100 \text{ GeV} \)
  - EW symmetry breaking \( \Rightarrow \) Higgs boson.

- Grand Unified PT:
  - \( T_\text{GUT} \approx 10^{16} \text{ GeV} \)
  - \( \langle H \rangle = 0 \).

It is possible to have many PTs in 100 GeV \( < T < M_\text{EW} \)
\( \Rightarrow \) shed some light on mysteries of Cosmology.

- Possible formation of baryon asymmetry
- Topological defects
- Dark Matter
- Dark Energy

We are limited only to theories with small couplings (like deconfined phase in QCD PT).

In what follows, we consider the EW PT (small coupling).

Equilibrium state \( \rightarrow \) minimum of the Grand Thermal effective potential.

Let small chem. pot. \( \Rightarrow \) this potential corresponds to the free energy of the field system:

\[
F = \frac{\partial V_{\text{eff}}(T, \phi)}{\partial \text{volume}} \text{ effective potential}
\]

We assume thermal equilibrium, \( \langle H \rangle = \langle \phi \rangle_T \) at \( T \).

\( \text{at } T = 0 \) \( V_{\text{eff}}(T = 0, \phi) = V(\phi) \) absolute minimum of \( V \) in the field theory breaks:

\[
3U(2) \times U(1)_Y \rightarrow U(1)
\]
At $T > 0$ symmetry may be restored to $T < T_{\text{c}}$ temperature of the phase transition. Two different types of PT

1st order
- (jump in heat capacity)
- $\langle \phi \rangle_{T}$ instantaneous
- no thermal eq. m

2nd order
- (continuous heat capacity)
- $\langle \phi \rangle_{T}$ thermal eq. m

Two different types of effective potential dependence on $\phi$

Phase I $\rightarrow$ Phase II
(Symmetry I $\rightarrow$ Symmetry II)
\[ U(\phi) = U_{0}(\phi) + \frac{1}{2} m^{2} \phi^{2} \]

1st order EW transition is of particular importance for Cosmology!

It cannot occur homogeneously in all space $\rightarrow$ spontaneous nucleation of bubbles in new phase.

\[ V_{c} = V_{\text{eff}}(T_{c}, \phi = 0) \]
\[ V_{t} = V_{\text{eff}}(T, \phi = 2 \langle \phi \rangle_{T}) \]
latent heat $AV$ is released during the PT.

\[ \frac{1}{2} R^{2} (V_{t} - V_{c}) < 0 \] decreasing
We also need to take into account the surface tension.

\[ F(R) = 4\pi R^2 \mu - \frac{4\pi}{3} R^3 \Delta V, \quad \Delta V = V - V_t > 0 \]

At small \( R \), tension collapse, the bubble expands! (as \( R \to 0 \), \( F(R) < 0 \))

Minimum size \( \frac{\partial F}{\partial R} = 0 \) \( \Rightarrow \) critical bubble, 

\[ R_c = \frac{2\mu}{\Delta V} \]

Re, \( Fe \to \infty \) for \( \Delta V \to 0 \).

Probability for \( \text{"thermal jump" per unit time per unit volume} \)

\[ F = 4\pi R^2 \mu - \frac{4\pi}{3} R^3 \Delta V = \frac{16\pi}{3} \Delta V \]

\[ n = A T^4 e^{-Fe/T} \text{ valid for } Fe > T \]

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by dimension, Boltzmann factor.

\[ n \sim \text{supercritical phase,} \]

\[ \text{cooling rate,} \]

\[ \text{the particles are trapped in old } \phi > 0 \text{ state due to barrier (colder particles!)} \]

In Cosmology, effective nucleation of bubble begins

\[ \Gamma' = A T^4 e^{-Fe/T} \sim H^4(T) \left( \frac{M^2_{pl}}{H^2} \right)^4 \]

\[ t_{\text{nucleation}} = \frac{1}{n} \Rightarrow \text{at } T \sim 100 \text{ GeV} \]

\[ H^4 = \frac{M^2_{pl}}{T^2} \sim 10^6 \text{ GeV} \]

\[ \text{bubble size } \sim T^{-1} \Rightarrow R_c \sim 10^{-16} \text{ cm} \]

\[ \Rightarrow \text{very few bubbles are formed!} \]

Let us calculate the surface tension (subnuclear size)

Neglecting bubble curvature:

\[ \phi(x) \to 0 \quad x \to \infty \]

\[ \phi(x) \to <\phi> \quad x \to 0 \text{ (center)} \]
\[ F/\phi I = \int_0^\infty \frac{1}{4\pi r^2} d\sigma \int \left[ \frac{1}{2} \left( \frac{d\phi}{dr} \right)^2 + V_{\text{eff}}(\phi) - V_\infty \right] d\sigma \]

\( \varpi = r - R \) in the thin wall approximation

\[ F/\phi I = 4\pi R^2 \int_{-\infty}^{\infty} d\varpi \left[ \frac{1}{2} \left( \frac{d\phi}{d\varpi} \right)^2 + V_{\text{eff}}(\phi) - V_\infty \right] \]

formally extended

\( \phi \) is a solution of Euler-Lagrange eqn:

\[ \frac{d^2 \phi}{d\varpi^2} = \frac{\partial V_{\text{eff}}(\phi)}{\partial \phi} \]

\( \phi(\varpi) \)

\( \phi_r < \phi < \phi_f \)

Boundary conditions:

\[ \phi_r \to < \phi_f \quad r - R \to -\infty \]

\[ \phi_r \to 0 \quad r - R \to +\infty \]

Inside the bubble wall

\[ V_r = -V_\infty, \quad \Delta V \text{ is small} \]

\( \phi(\varpi) \leftarrow U(\phi_f) = U(0) \rightarrow \text{solution of e.o.m.} \]

\[ \phi_r \leftarrow \text{classical plc.} \]

\[ \frac{d\phi}{d\varpi} \left( \frac{d\phi}{d\varpi} - V_{\text{eff}} - v \right) = -(R-r) \]

Free energy of the wall

\[ F_w = 4\pi R^2 \left( \phi_f - \phi_r \right) \]

\[ \phi_r = \int_0^\infty V_\infty (\phi_r(\varpi) - \phi_f) d\varpi \]

H/W check these formulae