1. Interactions in Nature

3. Gravity: (a) classical \rightarrow General Relativity (see previous lecture)
   well describes gravitational interactions of massive objects at energy scales
   \[ E \ll M_{\text{Pl}} \approx 1.22 \times 10^{19} \text{ GeV} \] (Planck scale)
   at \[ E \approx \frac{1}{2} \] Planck time \[ 5.38 \times 10^{-44} \text{ s} \]
   at \[ E \approx \frac{1}{2} \] Planck length \[ 1.6 \times 10^{-35} \text{ m} \]
   quantum \[ \rightarrow \] not exists yet!

   \[ \downarrow \]
   models:
   - quasi-classical
   - loop gravity
   - string theory etc.

   \[ \rightarrow \] can be well neglected in all phenomena we deal with in laboratory

2. Electromagnetic Interactions: (a) classical (based on Special Relativity)
   works well for not extremely strong e.m. fields or extremely short distances
   Maxwell electrodynamics
   \[ \rightarrow \] quantum \[ \Rightarrow \] QED

   \[ \Downarrow \]
   field theory
   \[ \leftarrow \]
   Special Relativity
   Quantum Mechanics

Free Fermion Lagrangian
\[ L_{\text{free}} = \bar{\psi}(x) \left( i \gamma^\mu D_\mu \psi(x) - m \psi(x) \right) \psi(x) \]

\[ \rightarrow \] invariant under (global) gauge transformation
\[ \psi(x) \rightarrow \psi'(x) = e^{i \alpha} \psi(x), \quad \alpha = \text{const} \]

if we make it (local) \[ \alpha \rightarrow \alpha(x) \] \[ L \] loses invariance!

in order to restore it, one introduces new gauge field
which transforms as
\[ A'_\mu(x) = A_\mu(x) + \frac{1}{2} \epsilon_{\mu \nu \rho} \partial_\nu \alpha(x) \Rightarrow \]

\[ L_{\text{free}} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} \quad L_{\text{int}} = e \bar{\psi}(x) \gamma_\mu A_\mu \psi(x) \]
All interactions in Nature are gauge interactions!

- e.m. gauge transfers are classified according to the Abelian gauge group (U(1)_{em})
  - no photon self-interactions
  - e.m. charge is screened by vacuum polarization
  - "running" e.m. charge:
    \[ e \to e(r) = e_0 \left( 1 - \frac{2}{3}\ln \frac{r}{r_0} + O(\alpha^2) \right) \]
  - long-range force! (exists at any distance from the charge)
  - light can propagate billions of light-years away!

- Strong interactions \( \rightarrow \) non-Abelian interaction
  - no classical analog
  - gluons are color-charged
  - they interact with each other
  - three types of color charge!
  - strong interactions = "relations" in the "color space"

\[ U(1) \rightarrow SU(3) \]

\[ U(a_1) \otimes U(a_2) = \sum_i U(a_i) \]

\[ \text{local substructure} \rightarrow 8 \text{ gluons} \]

\[ u_1 \otimes u_2 \rightarrow \left( \begin{array}{c} u_\uparrow \cr u_\downarrow \end{array} \right) \]

\[ \left( \begin{array}{c} a_1 \cr a_2 \end{array} \right) \otimes \left( \begin{array}{c} a_1' \cr a_2' \end{array} \right) = |A \rangle \]

- in particular, explains the baryon puzzle:

Due to gluon self-interactions, defines the properties of strong interactions:

\[ a_3 = \frac{\alpha}{2\pi} \]

\[ m_\text{gluon} \frac{d^2 \sigma}{d^3 p} = \beta (x, s), \beta (s) = \left( \frac{11 - 2n_f}{3} \right) \]

\[ QED \text{ vacuum polarization:} \]

- \( \approx \frac{e}{R^2} \)
**Strong coupling is running in opposite way:**

Confinement: Color charge cannot travel macroscopic distances (exists only inside colorless objects - hadrons, \(<1\text{ fm size}\)).

\[
V_{\text{QCD}} = \frac{g^2}{4} \alpha_s(r) + a_\pi
\]

QED potential: \(V_{\text{QED}} \sim \frac{1}{r}\). Linear string potential: \(V_{\text{string}} \sim -\frac{1}{r}\).

At ENeV description in terms of quarks/gluons loses any sense! New gg pair produced from vacuum!

**Weak interactions**

Responsible for e.g. neutron \(\beta\)-decay \(n \rightarrow p + e^- + \bar{\nu}_e\) non-Abelian gauge group \(SU(2)_W\) \(\Rightarrow\) massive gauge bosons \(Z^0, W^\pm\)

\(\Rightarrow\) short-range (but not confined!)

\(\Rightarrow\) Dual to the Higgs mechanism! Explains masses and provides longitudinal pol. for \(Z^0, W^\pm\)

**The Standard Model and unification of forces**

- **Particle content**
  - Quark sector: \((u, d, s, c, t, b)\) generations
  - Lepton sector: \((e, \mu, \tau, \nu_e, \nu_\mu, \nu_\tau)\)
  - Force carriers: \(X^0, W^\pm\)

- **Matter fields**
  - Spin: \(\frac{1}{2}\) (fermions!)

- **The Higgs boson**
  - Spin: 0

Except for neutrino oscillations, the SM is in excellent agreement with all existing experimental data so far!

**SM gauge group:** \(SU(3)_C \times SU(2)_W \times U(1)_Y = G_{SM}\)

By spontaneous symmetry breaking, SM predictions agree with EW predictions too.
Correspondingly, $W^\pm$, $Z^0$, and $\nu_e, \nu_\mu, \nu_\tau$ in the Higgs phase (after SSB).

SM particles form complete multiplets w.r.t. $G_{SM}$ group.

- $SU(3)_c$: 8 gluon field $G^a_i$, $a = 1, 8$, $\Delta_t = 2\frac{1}{2}$, of the adjoint $SU(3)_c$ representation.
- $SU(2)_L$: 3 gauge (massless) fields $V^a_i$, $i = 1\ldots 3$, of the adjoint of $SU(2)_L$.
- $U(1)_Y$: 1 gauge (massless) field $B$, of the adjoint of $U(1)_Y$.
- $\tau^i \in$ Pauli matrices.

Fermion fields are conveniently described in terms of the 4-component Weyl spinors:

\[ \psi_L, \psi_R \]

They form fundamental and anti-fundamental representations of $SU(2,\mathbb{C})$ group, which is the universal (2-fold) covering group of $SU(2)$.

Proper transformations:

\[ \psi \rightarrow \Gamma \psi \]

Parity transformation $(P) \rightarrow$ inversion of space and rotations.

- Inversion of time $(T)$.

Full Lorentz group $\rightarrow$ proper fundamental representation.

Free Dirac fermions obey

\[ i \gamma^\mu \partial_\mu \psi = m \psi \]

Dirac equation.

Dirac matrices

\[ \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \sigma^\mu & 0 \end{pmatrix} \]

in the chiral representation.

\[ \begin{pmatrix} 0 & i \sigma^\mu \partial_\mu \\ i \sigma^\mu \partial_\mu & 0 \end{pmatrix} \psi_L = m \psi_L \]

in the massless case.

The wavefunctions of the helicity operators $\begin{pmatrix} P_0 \sigma^\mu \\ i P_\mu \end{pmatrix}$ with eigenvalues $-1$ and $+1$, respectively. So in the massless case $\chi_{L,R}$ spinors only. This is the way how neutrino is described in the Standard Model (P is broken!)

\[ \chi_L = P_+ \psi_L = \frac{1+i\gamma^5}{2} \psi_L \]

\[ \psi = (\chi_L)^T, \psi_R = (\chi_R)^T \]

where $\chi_L = (0, \chi_R)$. 

$\chi_L, \chi_R$ are eigenvectors of the helicity operator $\frac{P_0}{P_\mu}$ with eigenvalues $-1$ and $+1$, respectively. So in the massless case, the theory can be constructed in terms of left (or right) Weyl spinors only.
Classification of matter field w.r.t. gauge interactions:

- $SU(3)_C$: $q_{l, R}$ are identical and form triplets.
- $SU(2)_W$: $q_{l, R}$ form singlets (i.e., do not interact under $SU(2)$). $l_{l, R}$ form doublets.
- $Q_R$ and $l_{R} (ER)$ form singlets (only left neutrino $\nu$ is described in this).

\[
\begin{align*}
Q_1 &= (u_1^+ d_1^-)_{\nu} \\
Q_2 &= (c_1^+ s_1^-)_{\nu} \\
Q_3 &= (t_1^+ b_1^-)_{\nu} \\
U_n &= \{ d_{l, R}, c_{l, R}, t_{l, R}, \nu \} \quad n = 1, 2, 3 \\
D_n &= \{ l_{l, R}, e_{l, R}, \nu \} \quad n = 1, 2, 3
\end{align*}
\]

Higgs field: single under $SU(3)_C$, double under $SU(2)_W$ carries $Y = +1$ w.r.t. $U(1)$.

The general form of the SM Lagrangian explicitly covariant under gauge group $G_{SM}$ reads:

\[
\mathcal{L}_{SM} = -\frac{1}{4} \text{Tr} G_{\mu\nu} G^{\mu\nu} - \frac{1}{4} \text{Tr} V_{\mu\nu} V^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} \bar{\nu}_L D^\nu \nu_L \nu_R + \frac{1}{2} \bar{E}_L D^\nu E_L E_R +
\]

\[
+ \bar{D}_L D^\nu D^\mu H - \lambda \left( \frac{H^+ H}{2} \right)^2 + \frac{\tilde{Y}_{\nu}}{v^2} \bar{\nu} \nu \bar{\nu} \nu + \text{h.c.}
\]

**strength tensors:**

\[
\begin{align*}
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\
V_{\mu\nu} &= \partial_\mu V_\nu - \partial_\nu V_\mu - ig [V_\mu, V_\nu] \\
G_{\mu\nu} &= \partial_\mu G_\nu - \partial_\nu G_\mu - ig [G_\mu, G_\nu]
\end{align*}
\]

\[
\begin{align*}
\text{Tr} [G_{\mu\nu} G^{\mu\nu}] &= \frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu} \quad \text{Tr} [V_{\mu\nu} V^{\mu\nu}] = \frac{1}{2} \text{Tr} V_{\mu\nu} V^{\mu\nu}
\end{align*}
\]

with:

- $C_\mu = \partial_\mu C_\nu - \partial_\nu C_\mu + \frac{g}{2} \varepsilon_{abc} C_\mu C^a C^b$
- $C^\mu = C_\mu C_\nu - \partial_\nu C_\mu + \frac{g}{2} \varepsilon_{abc} C_\mu C^a C^b$
- $V^\mu_{\mu} = \partial_\mu V^\nu_\mu - \partial_\nu V^\mu_\mu + \frac{g}{2} \varepsilon_{abc} V^\mu_\mu V^a_\nu V^b_\mu$
- $V^\mu_\mu = \partial_\mu V^\nu_\mu - \partial_\mu V^\mu_\nu + \frac{g}{2} \varepsilon_{abc} V^\mu_\mu V^a_\nu V^b_\mu$

- gives rise to self-interactions of gluons $G_{SU(3)}$ and $SU(2)$, respectively.

- covariant derivatives $D_\mu f = (\partial_\mu - ig \varepsilon_{abc} \frac{1}{2} C_\mu C^a - ig \frac{1}{2} B_\mu) f$

In $SU(2)$ gauge interactions are diagonal in generations:

- for $q$: $T^3_a = \frac{1}{2}$
- for $l$: $T^3_a = 0$