ACDM Model - The Standard Cosmological Model

1. Composition of the present Universe

Energy density is composed of non-relativistic matter (baryons, relativistic matter (photons, neutrino $m_\nu \lesssim 10^{-5}$ eV), Dark Energy (vacuum)

Realistic Friedmann equation

$$H^2 \equiv \left(\frac{a}{a_0}\right)^2 = \frac{8\pi}{3} \rho \left(\rho_m + \rho_{pr} + \rho_A + \rho_{curv}\right)$$

$$\frac{8\pi}{3} \rho_{curv} \overset{def}{=} -\frac{\kappa}{a^2}$$

critical density of flat universe ($\propto$ current density!)

$$\rho_c = \frac{3}{8\pi G} H_0^2$$

very small!

$$\approx 0.52 \cdot 10^{-5} \text{ GeV} \text{ cm}^{-3}$$

($\approx 5$ photons $\text{ m}^{-3}$)

$$\Omega_m = \frac{\rho_m}{\rho_c} \Rightarrow \Omega_{pr} = \frac{\rho_{pr}}{\rho_c} \Rightarrow \Omega_A = \frac{\rho_A}{\rho_c} \Rightarrow \Omega_{curv} = \frac{\rho_{curv}}{\rho_c} \Rightarrow \Omega_i = \frac{\rho_i}{\rho_c}$$

comes mainly from relic photons, $T_0 = 2.726$ K

The Stefan-Boltzmann law

$$\rho_b = \frac{5}{16\pi} \cdot \frac{T_0^4}{30} \approx 2.6 \cdot 10^{-10} \text{ GeV} \text{ cm}^{-3}$$

two transverse polarisations

CMB anisotropy data $\Rightarrow |\Delta_{curv}| < 0.02$ + other data

$$\Delta_{dm} \approx 0.27, \Delta_A \approx 0.03$$ (5% precision)

$$\Delta_B + \Delta_{dm} \Rightarrow \Delta_B = 0.046, \Delta_{dm} = 0.22$$

by electric neutrality $< ne > = < np >$

the main contribution to $\Delta_B$

$\rho_e \approx \frac{m_e}{m_p} \Delta_B \approx 2.5 \cdot 10^{-5}$
Spatially flat cosmological model with non-relativistic DM (cold DM) and Dark Energy (constant $\Lambda$-term) is called $\Lambda$CDM model.

$\Omega_i \rightarrow$ in the present epoch only!

\[ \rho_{DM} \propto a^{-4}, \quad \rho_{DE} \propto a^{-3}, \quad \rho_{curv} \propto a^{-2}, \quad \rho_{\Lambda} \propto \text{const} \]

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G \rho_c \left[ 2 \Omega_m \left( \frac{\dot{a}}{a} \right)^2 + 3 \Omega_{DE} \left( \frac{\dot{a}}{a} \right)^4 + 3 \Omega_{\Lambda} + \Omega_{curv} \left( \frac{\dot{a}}{a} \right)^2 \right] \]

$\Rightarrow$ should be employed with care: number of d.o.f. ($\rho_{DE}$) is different at different epochs!

For time-dependent DE: $\rho_{DE} = \rho_{DE}(a)$, $\rho_{DE} \neq 0$

Current constraint: $-0.8 \leq \omega_{DE} \leq -0.8$ (extremely crucial for both cosmology and particle physics)

\[ \begin{cases} \rho_{\Lambda} \text{ const} \Rightarrow \Lambda \text{-term} \\ \rho_{\Lambda} \neq \text{ const} \Rightarrow \text{ exotic form of matter, etc.} \end{cases} \]

Properties of cosmological evolution:

- $\Omega_{curv}$ was and will be small $\Rightarrow \Omega_m/\Omega_{curv} \propto a^2$ will decrease, will always dominate. $\sim \Omega_c$

- in Eq. 5, all terms, except for $\Lambda$-term, will die out!

$\Rightarrow$ in the future: $a \sim e^{H_{ES} t}$, $H_{ES} = \sqrt{8\pi/3} G \rho_c \Omega_m$

(f for $\rho_{DE}$ = const only!)

In the past: "radiation domination" $\Rightarrow$ "matter domination" $\Rightarrow$ "DE domination" $\Rightarrow$ "DE domination"

Consider each of these transitions in detail.

Transition $B$: Deceleration $\rightarrow$ Acceleration

\[ \ddot{a} = \frac{8\pi}{3} G \rho_c \cdot \left( \frac{\dot{a}^2}{a} + \rho_{DE} \dot{a}^2 \right) \Rightarrow \ddot{a} = a \frac{8\pi}{3} G \rho_c (2 \dot{a} \rho_{DE} - \rho_{DE} \dot{a}^2) \]

at the present epoch $\ddot{a} > 0$, since $2 \dot{a} \rho_{DE} - \rho_{DE} \dot{a}^2$ expands and accelerates.
transition occurred when \( \left( \frac{\rho_c}{\rho_{eq}} \right)^3 = \frac{2\Omega_m}{\Omega_m} \), \( \rho_{eq} = \rho_c / \Omega_m - 1 \) \[
\Rightarrow \quad \rho_{eq} = \left( \frac{2\Omega_m}{\Omega_m} \right)^{1/3} - 1 \approx 0.46
\]
\( \rho_c \propto a^{-3}, \quad \rho_{eq} \text{ const} \quad \Rightarrow \text{ abrupt transition!}
\]

\text{transition A: }

The earliest epoch - the "radiation domination"

\( \text{Very important for the growth of density perturbations.} \)

\( \text{Crude estimate (neglecting } D_\Lambda \text{ and } D_{
u em} \text{) gives:} \)

\[ 2\rho_c + 1 = \frac{\rho_c}{\rho_{eq}} \rightarrow \frac{\Delta m}{\Delta \rho_{eq}} \approx 10^4 \quad \rightarrow \text{very early epoch!} \]

\text{Temperature: } \quad T_{eq} = T_0 (1 + 2\rho_c) \times 10^4 K \approx 1 eV \quad \checkmark

At this stage, \( \nu_1, \nu_2, \nu_3 \) are also relativistic, and may not be negligible! We will see that they do not interact between themselves and matter at \( T = T_{eq} \) \[
\Rightarrow \quad \text{thermal distribution}
\]

Later we will show that \( T_0 = \left( \frac{y}{11} \right)^{1/3} \frac{\pi}{16} \) \[ \quad 3 \text{ species: } e^+ e^-, \nu \text{ neutrinos, } \nu_0 = 3.2 \cdot \frac{\pi^2}{16} \frac{\pi}{16} \]

\( \Rightarrow \) The density of relativistic matter

\[ \rho_{\text{rad}} = \rho_{\text{eq}} + \rho_0 = \rho_{\text{eq}} \left[ 2 + \frac{2!}{4} \left( \frac{y}{11} \right)^{1/3} \right] \frac{\pi^2}{30} T^4 \rightarrow \quad T_{eq} \approx T \rightarrow \text{temperature of Universe}
\]

\[ \Rightarrow \quad \rho_{\text{rad}} = 0.68 \left( \frac{\rho_c}{\rho_{eq}} \right) \frac{\Delta m}{\Delta \rho_{eq}} \rho_c \times 0.68
\]

\( \text{for the non-relativistic matter} \quad \Rightarrow \quad \rho_m = \left( \frac{\rho_c}{\rho_{eq}} \right)^3 \frac{\rho_c}{\rho_{eq}} \frac{\Delta m}{\Delta \rho_{eq}} \)

\( \Omega_m = 0.6 \Omega_m \rightarrow \frac{\Delta m}{\Delta \rho_{eq}} = 2.5 \cdot 10^{-5} \Rightarrow \rho_{eq} = 2.5 \cdot 10^{-5} \rho_{eq} h^{-2} \approx 3.2 \cdot 10^{-3} \)

\[ T_{eq} = (1 + 2\rho_c)T_0 = 5.6 \Omega_m h^2 \times 0.76 \text{ eV}, \quad \Omega_m = 0.28 h = 0.705
\]

\( \uparrow \text{refine the estimate above!} \)
neglecting non-relativistic matter, the evolution is driven only by photons and neutrinos, so the effective number of \( \sum n_{\text{eff}} = 2 + \frac{21}{4} \left( \frac{4}{11} \right)^{3/2} = 3.36 \)

\[
\ell_H = \frac{1}{H(t)} \quad \text{where} \quad H = \frac{1}{t} \approx \frac{M_{\text{Pl}}}{\sqrt{8\pi T_{\text{eq}}} \ell_{\text{H}}} = \frac{M_{\text{Pl}}}{1.66 \times 10^{-5}} \]

\[
\Rightarrow \quad \ell_{\text{eq}} = \frac{1}{2H_{\text{eq}}} = \frac{M_{\text{Pl}}}{8\pi T_{\text{eq}}} \ell_{\text{H}} = 2.3 \times 10^{36} \text{ GeV}^{-1} \approx 2.3 \times 10^{12} \text{ years} \quad \text{Thousand BB}
\]

"Radiation-to-matter transition is a continuous process at the time scale \( \sim H_{\text{eq}} \) (its duration is comparable to \( \ell_{\text{eq}} \)) since

\[
\frac{\rho_{\text{rad}}}{\rho_{\text{mat}}} \propto a \text{ depends rather weakly on } a \text{ (typical change is the Hubble time)}
\]

Even though the matter density surpassed the radiation density at \( t = \ell_{\text{eq}} \), the Universe remained optically thick to radiation until recombination at \( \ell_{\text{rec}} \approx 10^{18} \text{ when the Universe was } 3 \text{ billion years old.}

Present-day constraints on cosmological parameters

see \texttt{arXiv:1001.4744}