Written exam problems, part 2

Problem No 4.4

Neglecting dark energy and spatial curvature and assuming that the effective number of relativistic degrees of freedom determining the energy density is the constant $g_* = 3.36$, find the exact law of evolution $a = a(t)$ at temperatures of order the equality temperature T_{eq}. Find the age of the Universe at the moment when its temperature equals to $T_{eq} = 5.6\,\Omega_M\,h^2$ eV. Show that at $h = 0.705$ and $\Omega_M = 0.27$ the numerical value is $t_{eq} = 57$ thousand years.

Problem No 5.2

Show that at temperature exceeding the mass and chemical potential, $T \gg m$, μ, the difference of the number densities of particles and their antiparticles of a given helicity is

$$\Delta n = \mu\frac{T^2}{3} \qquad \text{Bose,} \qquad \Delta\mathrm{n} = \mu\frac{\mathrm{T}^2}{6} \qquad \text{Fermi}$$

Hints: (1) Recall that chemical potentials of particles and their antiparticles differ by sign only; (2) Take linear in μ term in the integral (5.8) with $m = 0$, integrate by parts and use formulas presented in the end of Section 5.1.

Problem No 6.5

Find equilibrium recombination temperature for helium. Hint: Do not forget that helium nucleus has to catch two electrons to make a neutral helium atom.

Problem No 6.11

Find the Hubble rate at temperature $T_r = 0.26$ eV taking into account the contribution of radiation into the energy density. With the latter refinement, show that the age of the Universe at this temperature is actually $t_r = 3.7 \cdot 10^5$ years.

Problem No 6.12

Show that the mean free time of a photon with respect to scattering off neutral atoms exceeds the Hubble time at recombination and afterwards. This means that the presence of neutral hydrogen and helium does not affect CMB photons.