What multiple Higgs doublets can do for you

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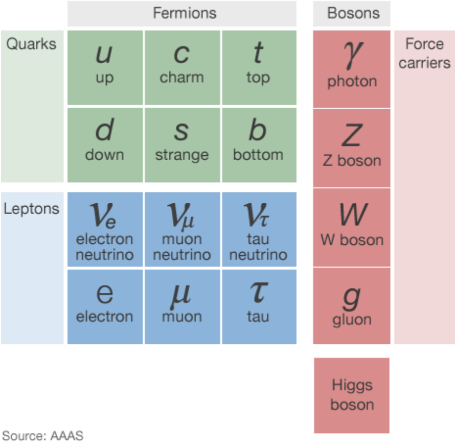
based on:
I. P. Ivanov, J. P. Silva, PRD 93, 095014 (2016)
A. Aranda, I. P. Ivanov, E. Jiménez, arXiv:1608.08922

1. Multi-Higgs-doublet models
   - Overstretching the SM Higgs
   - Fermion sector from NHDM symmetries
   - Astroparticle consequences

2. A hidden beauty: CP4-3HDM

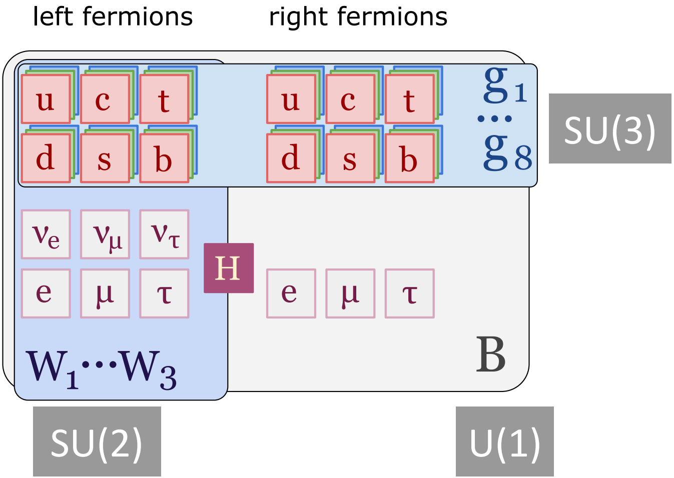
3. Conclusions

The usual picture of SM particle content:



Source: AAAS

Does not really represent which forces act on which particles...


left fermions
right fermions
u c t
d s b
u c t
d s b
g_1
g_8
SU(3)
ν_e ν_μ ν_τ
e μ τ
H
e μ τ
$W_1 \cdots W_3$
B
SU(2)
U(1)

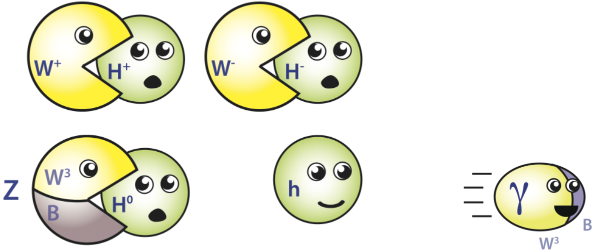
Brout-Englert-Higgs mechanism in SM

Standard Model: 3-in-1

- Start with matter fields, organize them into left doublets $Q_L = (u_L, d_L)$ and right singlets u_R, d_R,
- require local gauge symmetry $\to$ get gauge interactions for free $\to$ all fields are massless.
- add Higgs doublet with "mexican hat" potential $\to$ get massive W and Z, as well as fermions, without spoiling high-energy features.

Electroweak symmetry breaking

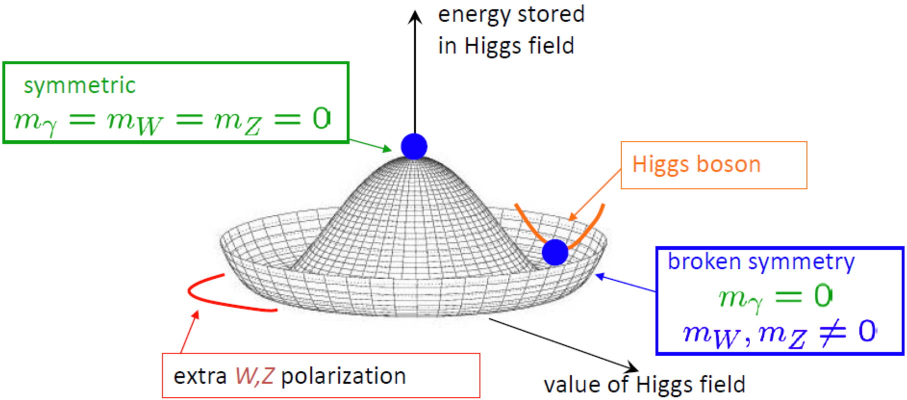
Brout-Englert-Higgs mechanism of electroweak symmetry breaking:



Bosons partially mix:

$$W_1,\ W_2,\ W_3,\ B,\ (H^+,\ H^0) \to W^\pm,\ Z,\ \gamma,\ h\,.$$

Electroweak symmetry breaking



Overstretching the Higgs mechanism

The scalar sector of SM is overly minimalistic. We ask the single Higgs doublet to accomplish three simultaneous tasks:

- give masses of W and Z via $|D_\mu\phi|^2$,
- give masses of down-quarks and leptons via $\bar{Q}_L\phi d_R$.
- give masses of up-quarks via $\bar{Q}_L\tilde{\phi}u_R$, where $\tilde{\phi} \equiv i\sigma_2\phi^*$.

The gauge/fermion structure of SM does not require the Higgs sector to be minimal! It can well be rich and it can extend the idea of “generations” to scalars.

Overstretching the Higgs mechanism

With so many tasks to do, the Higgs sector of the SM is completely "exhausted":

- does not explain the hierarchical fermion masses and the mixing patterns nor offers any relation among them (flavour sector = the ugly part of the SM).
- "boring" flavor properties of the Higgs boson exchange: no FCNC, no LFV,
- *CP*-violation must be inserted by hand,
- unable to generate astroparticle and cosmological phenomena we observe: no DM, insufficient cosmological EWPT $\to$ no sizable baryon asymmetry.

All these features can be successfully reproduced with the SM fermions and gauge interactions but with an extended scalar sector, in particular, in N-Higgs-doublet models (NHDM).

Quark masses and mixing

For three generations $d_i = (d, s, b)$, $u_i = (u, c, t)$, Yukawa interactions are written as

$$\bar{Q}_{Li}\Gamma_{ij}\phi d_{Rj} + \bar{q}_{Li}\Delta_{ij}\tilde{\phi} u_{Rj} + h.c. \to \bar{d}_{Li}(M_d)_{ij} d_{Rj} + \bar{u}_{Li}(M_u)_{ij} u_{Rj} + h.c.$$

where the 3×3 mass matrices are

$$M_d = \Gamma \frac{v}{\sqrt{2}}, \quad M_u = \Delta \frac{v^*}{\sqrt{2}}.$$

They can be diagonalized: $V_{dL}^\dagger M_d V_{dR} = D_d$, $V_{uL}^\dagger M_u V_{uR} = D_u$, but then the charged current matrix can become non-trivial:

$$\bar{u}_{Li}\gamma^\mu W_\mu^+ d_{Li} \to \bar{u}_{Li}\gamma^\mu W_\mu^+ V_{ij} d_{Lj}, \quad \text{where} \quad V_{ij} = V_{uL}^\dagger V_{dL} \neq \delta_{ij}.$$

Diagonalizing M_d, M_u, we also diagonalize $hf\bar{f}$-interactions $\to$ no FCNC or LFV in Higgs exchanges.

Quark masses and mixing

With N Higgs doublets ϕ_a, the procedure is the same:

$$\sum_a \left(\bar{Q}_{Li} \Gamma^{(a)}_{ij} \phi_a d_{Rj} + \bar{q}_{Li} \Delta^{(a)}_{ij} \tilde{\phi}_a u_{Rj} \right) + h.c.$$

Vacuum coresponds to a vev alignment $\langle \phi^0_a \rangle = v_a/\sqrt{2}$, which gives

$$M_d = \frac{1}{\sqrt{2}} \sum_a \Gamma^{(a)} v_a \,, \quad M_u = \frac{1}{\sqrt{2}} \sum_a \Delta^{(a)} v_a^* \,.$$

- Dangerous tree-level FCNC $\to$ can be avoided with natural flavour conservation via discrete symmetries [Weinberg, Glashow, 1977; Pachos, 1977].
- Individual $\Gamma^{(a)}$ and $\Delta^{(a)}$ can be very simple, symmetry-constrained. In M_d and M_u, this symmetry is ruined but can leave traces in masses/mixing.
- If complex vevs v_a get a relative phase, then CP-violation can appear spontaneously for real Γ and Δ [T.D.Lee, 1973, Branco, 1979].

Quark masses and mixing

Starting from late 1970: intensive NHDM model-building, aiming at deriving properties of quark and lepton masses and mixing from symmetry considerations — and relating quark mixing angles with masses.

- simplest texture-based approaches: 2 generations [Weinberg, 1977; Wilczek, Zee, 1977] with $\sin\theta_C \approx \sqrt{m_d/m_s}$, 3-generation Ansatz [Fritzsch, 1978] with full expression of V_{ij} via mass ratios and phases.
- global permutation symmetry groups S_3 or S_4: [Pakvasa, Sugawara, 1978, 1979, + Yamanaka, 1982] $\to$ in early 80's, perfectly agreed on the CKM matrix.
- rephasing + permutation global symmetry groups $\Delta(54)$ which makes $\Gamma^{(a)}$ and $\Delta^{(a)}$ very simple [Segre, Weldon, Wyers, 1979]: mass hierarchy may come from vev hierarchy $v_1 \ll v_2 \ll v_3$, though it is difficult to arrange that in the scalar potential.

Scalar dark matter

More Higgses — more fun!

- some Higgs fields participate in EWSB → massive W and Z, massive fermions,
- other can be inert: no coupling to fermions, and no role in W and Z masses. If inert scalars are protected by a new "parity" (do not get vev) → the lightest parity-odd scalar is stable → scalar DM. This does not require any fine-tuning!

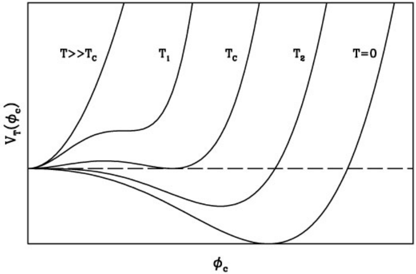
Example: Inert doublet model = 2HDM with exact $\mathbb{Z}_2$-symmetry [Deshpande, Ma, 1978; Barbieri et al, 2006, Lopez Honorez et al, 2006].

Two Higgs doublets ϕ and ϕ_D, with $\langle\phi_D\rangle = 0$. Extra Higgses from ϕ_D: $H^\pm$, H, A, with the lightest one being the DM + interesting collider phenomenology.

Cosmological phase transition

Baryon asymmetry requires a strong first-order thermal electroweak phase transition in early Universe.

$$\frac{v(T_c)}{T_c} \gtrsim 1\,.$$



The SM could satisfy it only for light Higgs ($m_h \lesssim 50$ GeV) [Kajantie et al, 1996; Csikor, 1999].

Extra scalars modify the Higgs potential and can produce strong EWPT, from [Turok, Zadrozny, 1992] on, recent detailed work [Dorsch, Huber, No, 2013]. Can be probed in principle with future GW observatories!

CP4-3HDM:

a three-Higgs-doublet model with order-4 *CP*-symmetry

Freedom of defining CP

In QFT, the discrete transformations such as CP are not uniquely defined *a priori* [e.g. Feinberg, Weinberg, 1959].
For example, in NHDM with doublets ϕ_i, $i = 1, \dots, N$, the transformation

$$\phi_i \xrightarrow{CP} X_{ij}\phi_j^* \,, \quad X \in U(N)\,,$$

with any X can play the role of "the CP transformation" [e.g. Branco, Lavoura, Silva, 1999]. The "standard" convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent.

I will show that in models with several gauge-blind scalars the freedom of defining CP is even larger. In particular, it can lead to scalars which are *CP*-half-odd:

$$J: \quad \Phi(\mathbf{x}, t) \xrightarrow{CP} i\,\Phi(-\mathbf{x}, t)\,.$$

Notice: (1) no conjugation, (2) CP4: order-4 transformation, $J^2 \neq \mathbb{I}$, $J^4 = \mathbb{I}$.

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CP4-3HDM

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right]$$
$$+\lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)\,,$$

with all parameters real, and

$$V_1 = \lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

with real $\lambda_{5,6}$ and complex $\lambda_{8,9}$. It is invariant under order-4 CP:

$$J:\ \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}.$$

Its square, $J^2 = XX^* = \mathrm{diag}(1, -1, -1)$ and $J^4 = \mathbb{I}$.
This model has no other symmetries [Ivanov, Keus, Vdovin, 2012].

Physical scalars

CP-conserving minimum: $v_i = (v,\, 0,\, 0)$.

Physical scalars: h_{SM}, degenerate $H^\pm_{2,3}$, and two pairs of degenerate neutrals: the heavier H and A and the ligher h and a, with masses

$$M^2,\, m^2 = -m_{22}^2 + \frac{v^2}{2}\left(\lambda_3 + \lambda_4 \pm \sqrt{\lambda_5^2 + \lambda_6^2}\right).$$

However, these real neutrals are not CP-eigenstates:

$$H \xrightarrow{CP} A, \quad A \xrightarrow{CP} -H\,, \quad h \xrightarrow{CP} -a, \quad a \xrightarrow{CP} h\,.$$

Can we combine them into neutral complex CP-eigenstate fields?

$$\Phi = \frac{1}{\sqrt{2}}(H - iA)\,, \quad \varphi = \frac{1}{\sqrt{2}}(h + ia)\,, \quad \Phi \xrightarrow{CP} i\Phi\,, \quad \varphi \xrightarrow{CP} i\varphi\,.$$

Conserved quantum number: not CP-parity but CP-charge q defined mod 4.

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To C, or not to C — that is the question

$\Phi(\mathbf{x},t) \xrightarrow{CP} i\Phi(-\mathbf{x},t)$ looks like P transformation rather than CP.

Where did we lose the C in CP?

Back to basics

Single complex scalar field:

$$\phi(\mathbf{x}, t) = \int \tilde{dp} \left[a(\mathbf{p}) e^{-ipx} + b^\dagger(\mathbf{p}) e^{ipx} \right] .$$

If we define $\phi \xrightarrow{CP} \phi^* \Rightarrow a(\mathbf{p}) \xrightarrow{CP} b(-\mathbf{p})$, $b(\mathbf{p}) \xrightarrow{CP} a(-\mathbf{p})$, which means

$$b^\dagger|0\rangle \text{ is antiparticle of } a^\dagger|0\rangle .$$

Two complex mass-degenerate scalars $\phi_i(x)$, $i = 1, 2$. If $\phi_1 \xrightarrow{CP} \phi_2^*$, $\phi_2 \xrightarrow{CP} \phi_1^*$, then $a_1(\mathbf{p}) \leftrightarrow b_2(-\mathbf{p})$, $b_1(\mathbf{p}) \leftrightarrow a_2(-\mathbf{p})$ under CP, which means

$$b_2^\dagger|0\rangle \quad \text{is antiparticle of} \quad a_1^\dagger|0\rangle ,$$
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By choosing X in $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, we assign who is antiparticle of whom.

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Basis change

Within the same $\phi_1 \xrightarrow{CP} \phi_2^*$, $\phi_2 \xrightarrow{CP} \phi_1^*$ example, define η and ξ as

$$\begin{pmatrix} \eta \\ \xi \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \eta \xrightarrow{CP} \eta^*, \ \xi \xrightarrow{CP} -\xi^*.$$

Remember: η and ξ are mass-degenerate.

Combine two CP-even fields $\mathrm{Re}\eta$ and $\mathrm{Im}\xi \to \Phi = \mathrm{Re}\eta - i\,\mathrm{Im}\xi$.
Combine two CP-odd fields $\mathrm{Re}\xi$ and $\mathrm{Im}\eta \to \tilde{\Phi} = \mathrm{Re}\xi - i\,\mathrm{Im}\eta$.

$$\Phi \xrightarrow{CP} \Phi, \quad \tilde{\Phi} \xrightarrow{CP} -\tilde{\Phi}.$$

Conjugation disappeared.

Basis change

One can now describe passage from $(\phi_1, \phi_2) \to (\eta, \xi) \to (\Phi, \tilde{\Phi})$ via

$$\begin{pmatrix} \Phi \\ \tilde{\Phi} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2^* \end{pmatrix} .$$

This basis change "undoes" the conjugation.

This is a norm-preserving non-holomorphic map on $\mathbb{C}^2$, not the usual basis change. It is a valid $O(4)$-rotation in $\mathbb{R}^4$ spanned by $(\mathrm{Re}\eta, \mathrm{Im}\eta, \mathrm{Re}\xi, \mathrm{Im}\xi)$.

If complex fields are gauge-blind (do not carry any non-zero gauge quantum number), the group of basis changes increases from $U(2)$ to $O(4)$.

For mass-degenerate gauge-blind scalars,
conjugating or not under *CP* is a matter of basis choice.

In all transformations, we never redefined the *CP* transformation itself.

CP4

CP-half-odd scalars arise in a similar way.

- Starting point: $\phi_1 \xrightarrow{CP} i\phi_2^*$, $\phi_2 \xrightarrow{CP} -i\phi_1^*$.
- Operators: $a_1 \xrightarrow{CP} ib_2\,,\ b_2 \xrightarrow{CP} ia_1$ and $a_2 \xrightarrow{CP} -ib_1\,,\ b_1 \xrightarrow{CP} -ia_2$.
- Basis change:

$$\begin{pmatrix} \Phi \\ \varphi^* \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2^* \end{pmatrix}.$$

- Result: $\Phi \xrightarrow{CP} i\Phi$, $\varphi \xrightarrow{CP} i\varphi$. For operators

$$a_\Phi = (a_1 + b_2)/\sqrt{2} \xrightarrow{CP} ia_\Phi\,, \quad b_\Phi^\dagger = (a_2^\dagger + b_1^\dagger)/\sqrt{2} \xrightarrow{CP} ib_\Phi^\dagger\,.$$

An example of doubling of mass eigenstates beyond Kramers degeneracy (a possibility mentioned e.g. in Weinberg, vol. 1, app. 2C).

Yukawas for CP4-3HDM

Can CP-half-odd scalars have CP-conserving Yukawa interactions?
Yes, provided the CP mixes the fermion families.

In CP4-3HDM, the charged lepton sector (here, $\phi_a \equiv \phi_a^0$):

$$-\mathcal{L}_Y = \bar{\ell}_{Li}\Gamma^a_{ij}\ell_{Rj}\,\phi_a + \bar{\ell}_{Ri}(\Gamma^a_{ij})^\dagger\ell_{Lj}\,\phi_a^*\,.$$

Take $\phi_a \xrightarrow{CP} X_{ab}\phi_b^*$, with the same X as before, and $\ell_i \xrightarrow{CP} Y^*_{ij}\ell^c_j$. Then,

$$Y^\dagger\Gamma_1^* Y = \Gamma_1\,,\quad -iY^\dagger\Gamma_2^* Y = \Gamma_3\,,\quad iY^\dagger\Gamma_3^* Y = \Gamma_2\,.$$

We switch to the fermion basis in which

$$Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} \\ 0 & e^{-i\alpha} & 0 \end{pmatrix}\,,$$

CP-conservation requires that fermions be degenerate: $m_2 = m_3$.

Yukawas for CP4-3HDM

- case 1: $\alpha = \pm\pi/4 + \pi k$, order-8 transformation.

$$\Gamma_1 = \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_2^* \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & g_{23} \\ 0 & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \pm g_{32}^* \\ 0 & \mp g_{23}^* & 0 \end{pmatrix}.$$

- case 2: $\alpha = \pm\pi/2$, order-4 transformation.

$$\Gamma_1 = \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & g_3 \\ 0 & -g_3^* & g_2^* \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & g_{12} & g_{13} \\ g_{21} & 0 & 0 \\ g_{31} & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \pm \begin{pmatrix} 0 & -g_{13}^* & g_{12}^* \\ g_{31}^* & 0 & 0 \\ -g_{21}^* & 0 & 0 \end{pmatrix}.$$

Yukawas for CP4-3HDM

Explicitly, in case 1 [notation: $(e, \mu, \tau) = (\ell_1, \ell_2, \ell_3)$]:

$$-\mathcal{L}_Y = (\bar{\mu}\tau)(g\Phi - \tilde{g}\varphi) + (\bar{\tau}\gamma_5\mu)(\tilde{g}^*\Phi + g^*\varphi) + h.c.,$$

where

$$g = \frac{c_\gamma g_{23} - s_\gamma g_{32}^*}{\sqrt{2}}, \quad \tilde{g} = \frac{s_\gamma g_{23} + c_\gamma g_{32}^*}{\sqrt{2}}, \quad \tan 2\gamma = -\lambda_6/\lambda_5 .$$

Notice that fermion biliears are CP-half-odd, and that insertion of γ_5 introduces an extra "CP-oddness" as in the usual case:

$$\bar{\mu}\tau \xrightarrow{CP} -i\bar{\mu}\tau, \quad \bar{\tau}\mu \xrightarrow{CP} i\bar{\tau}\mu, \quad \bar{\mu}\gamma_5\tau \xrightarrow{CP} i\bar{\mu}\gamma_5\tau, \quad \bar{\tau}\gamma_5\mu \xrightarrow{CP} -i\bar{\tau}\gamma_5\mu .$$

CP-half-odd scalars do not have to be inert.

Clash of two definitions

Lagrangian contains: $(\bar{\mu}\gamma^\mu\mu + \bar{\tau}\gamma^\mu\tau)A_\mu$ and $(\bar{e}\mu + \bar{\tau}e)\Phi + h.c.$ (case 2).

Do Yukawa couplings of Φ generate particle-antiparticle asymmetry?

No unambiguous answer exists! Depends on the particle-antiparticle assignment.

- "Formal" definition: just as above. Field μ creates μ^- and annihilates τ^+, field $\bar{\tau}$ creates τ^+ and annihilates μ^-. Then $(\bar{e}\mu + \bar{\tau}e)\Phi$ is particle-antiparticle symmetric. But then $\gamma^* \to \tau^+\mu^-$ and $\tau^-\mu^+$.
- "Physical" definition: we require, as usual, that $\gamma^* \to \ell^+\ell^-$. Then, Φ decays produce only μ^- and τ^+ and strongly violate C-symmetry.
- Agnostic position: we refrain from assigning particle-antiparticle pairs. Then it is unclear what CP-conservation/violation should mean.

It sounds pure semantics within the toy model. But it may have unusual consequences for realistic models with the broken CP4.

Remarks on phenomenology

- Does a model based on CP4 lead to any phenomenological signal which cannot be mimicked by any usual CP-conserving model?

$$(\mathcal{CP})a^\dagger_\Phi a^\dagger_\Phi(\mathcal{CP})^{-1} = -a^\dagger_\Phi a^\dagger_\Phi\,.$$

  We get a *CP*-odd pair of two identical bosons $\to$ any pheno signal?

- A variation on inert doublet model based on CP4 instead of $CP + \mathbb{Z}_2$: fully calculable, can be readily tested against collider data and DM searches.
- If CP4 is spontaneous broken, can CP4-3HDM with Yukawas reproduce the quark sector? How is the clash between two definitions resolved?
- Any specific predictions for the neutrino sector?

Conclusions

- Multi-Higgs-doublet models is a conceptually simple bSM framework with very rich phenomenology.
- CP4-3HDM is the simplest model featuring CP-half-odd scalars: $\Phi \xrightarrow{CP} i\Phi$. Their origin is the extra freedom of basis change arising in models with mass-degenerate gauge-blind scalars.
- No fine-tuning is required: the gauge blindness, mass degeneracy, and CP-half-oddness appear naturally within CP4-3HDM.
- The CP-half-odd scalars do not have to be inert: they can couple to fermions in CP-conserving way.
- The model is compact, analytically tractable, and can have interesting phenomenological consequences.