Anisotropic flow measurements in ALICE

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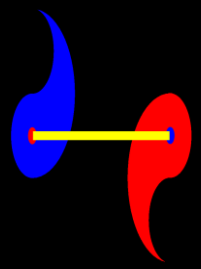
Lund, 03/05/2012

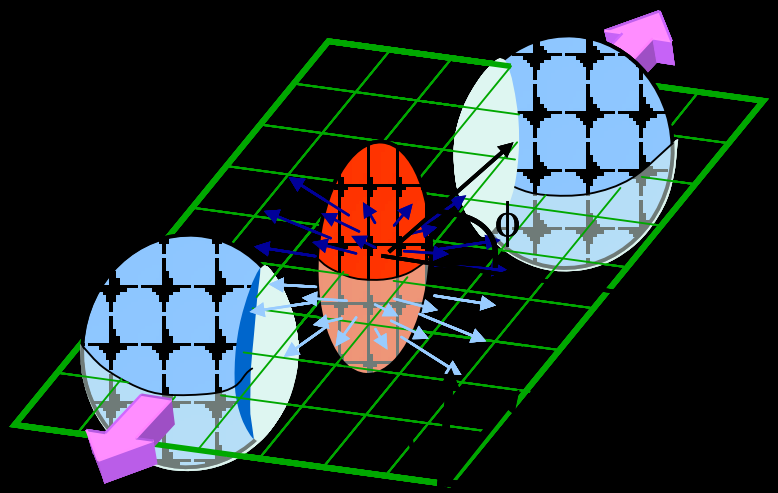


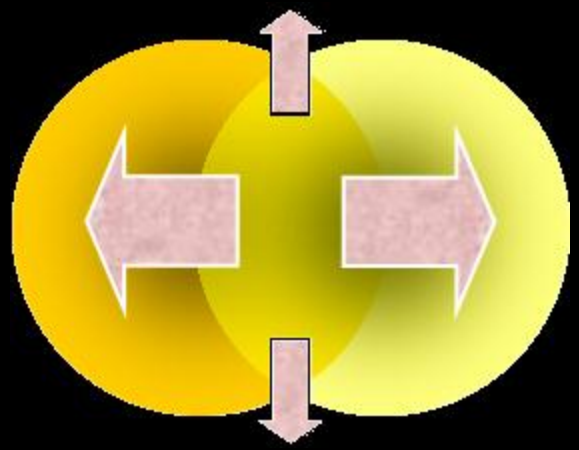
Outline

- Introduction to anisotropic flow
  - Theoretical definition
  - Centrality => Glauber model
  - Connection to system properties
- How do we measure anisotropic flow?
  - Multi-particle *Q*-cumulants
  - Discussion of various systematic biases
- Anisotropic flow results in ALICE
  - Elliptic flow paper
  - Higher harmonic paper

Introduction

Anisotropic flow





- Anisotropies in momentum space (S. Voloshin and Y. Zhang (1996)):

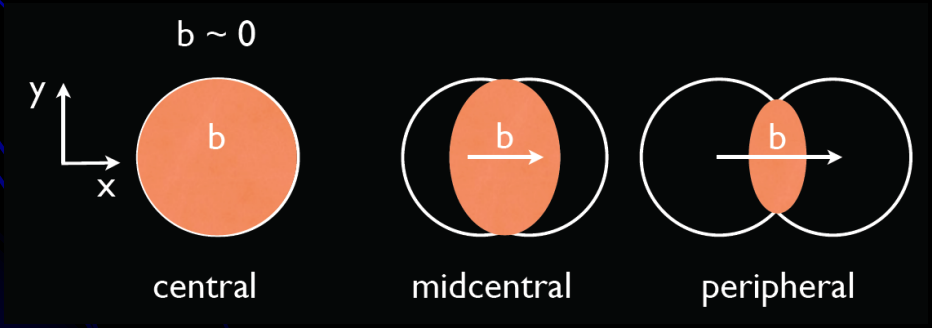
$$E\frac{d^3N}{d^3\vec{p}} = \frac{1}{2\pi}\frac{d^2N}{p_T\,dp_T\,dy}\left(1+\sum_{n=1}^{\infty}2v_n\cos\left(n\left(\phi-\Psi_{\mathrm{RP}}\right)\right)\right)$$

$$v_n = \langle\cos(n(\phi-\Psi_{\mathrm{RP}}))\rangle$$

- Harmonics v_n quantify anisotropic flow
  - v_1 is directed flow, v_2 is elliptic flow, v_3 is triangular flow, etc.

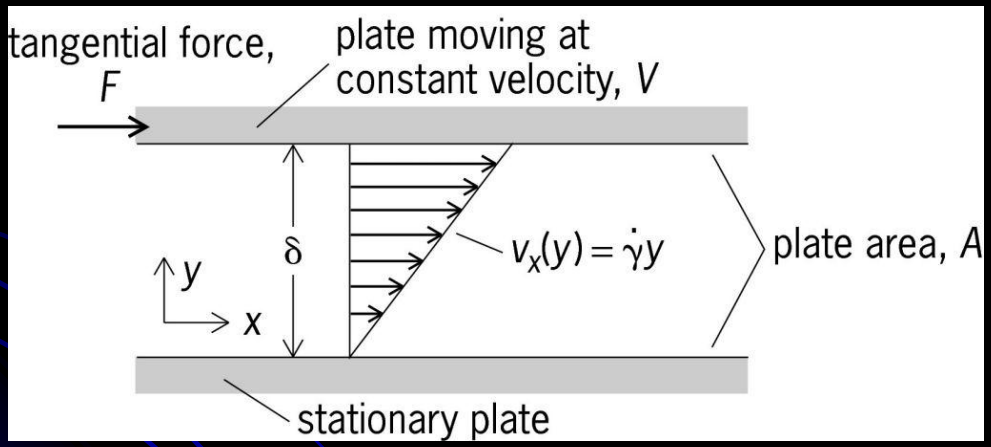
Anisotropic flow

- Anisotropic flow is geometrical quantity => need to classify all events in terms of initial geometry
  - Minimum bias flow analysis doesn't make any sense
- Another geometrical quantity available: Multiplicity
  - In head-on collisions more nucleons within nuclei interact than in the peripheral collisions => more particles are produced in the head-on collisions than in the peripheral
- Glauber model: Quantitative description of multiplicity distribution, centrality classes of events
  - Most central events: Centrality class 0-5%
  - Peripheral events: Centrality class 70-80%



Anisotropic flow

- By measuring event-by-event anisotropies in the resulting momentum distribution of detected particles, we are probing the properties of produced matter
- Example: Shear viscosity

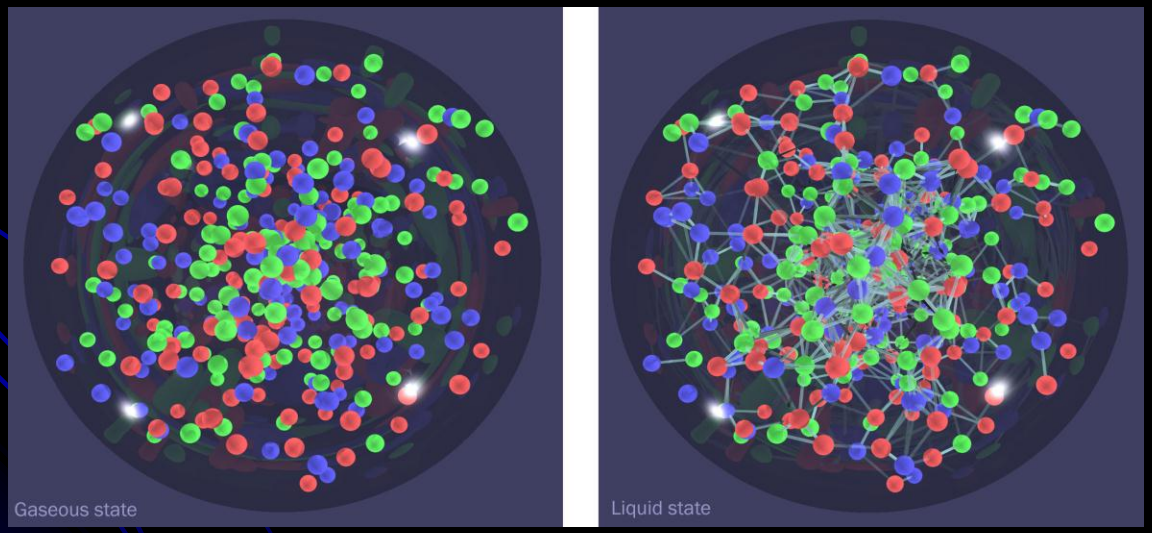


Shear viscosity characterizes quantitatively the resistance of the liquid or gas to displacement of its layers

- Shear viscosity works against anisotropic flow
  - Perfect liquid <=> shear viscosity negligible <=> anisotropic flow develops easily

Anisotropic flow

- In 2005 in Au-Au collisions at RHIC after 3 years of data taking the discovery of a new state of matter was reported
  - Expected: weakly interacting gas
  - Observed: strongly coupled liquid



Anisotropic flow

- Press coverage in 2005:

New State of Matter Is 'Nearly Perfect' Liquid

Physicists working at Brookhaven National Laboratory announced today that they have created what appears to be a new state of matter out of the building blocks of atomic nuclei, quarks and gluons. The researchers unveiled their findings--which could provide new insight into the composition of the universe just moments after the big bang--today in Florida at a meeting of the American Physical Society.

SCIENTIFIC AMERICAN

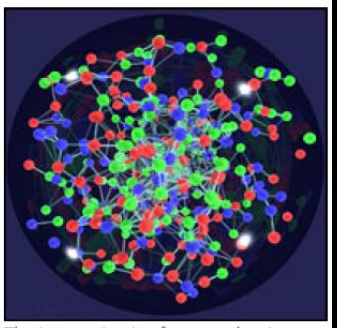
Image: BNL

There are four collaborations, dubbed BRAHMS, PHENIX, PHOBOS and STAR, working at Brookhaven's Relativistic Heavy Ion Collider (RHIC). All of them study what happens when two interacting beams of gold ions smash into one another at great velocities, resulting in thousands of subatomic collisions every second. When the researchers analyzed the patterns of the atoms' trajectories after these collisions, they found that the particles produced in the collisions tended to move collectively, much like a school of fish does. Brookhaven's associate laboratory director for high energy and nuclear physics, Sam Aronson, remarks that "the degree of collective interaction, rapid thermalization and extremely low viscosity of the matter being formed at RHIC make this the most nearly perfect liquid ever observed."

Early Universe was 'liquid-like'

Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.

BBC NEWS



The impression is of matter that is more strongly interacting than predicted

The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost perfect "liquid".

The work is expected to help scientists explain the conditions that existed just milliseconds after the Big Bang.

Early Universe was a liquid

Quark-gluon blob surprises particle physicists.

nature

by Mark Peplow
news@nature.com

The Universe consisted of a perfect liquid in its first moments, according to results from an atom-smashing experiment.

Scientists at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory on Long Island, New York, have spent five years searching for the quark-gluon plasma that is thought to have filled our Universe in the first microseconds of its existence. Most of them are now convinced they have found it. But, strangely, it seems to be a liquid rather than the expected hot gas.

Universe May Have Begun as Liquid, Not Gas

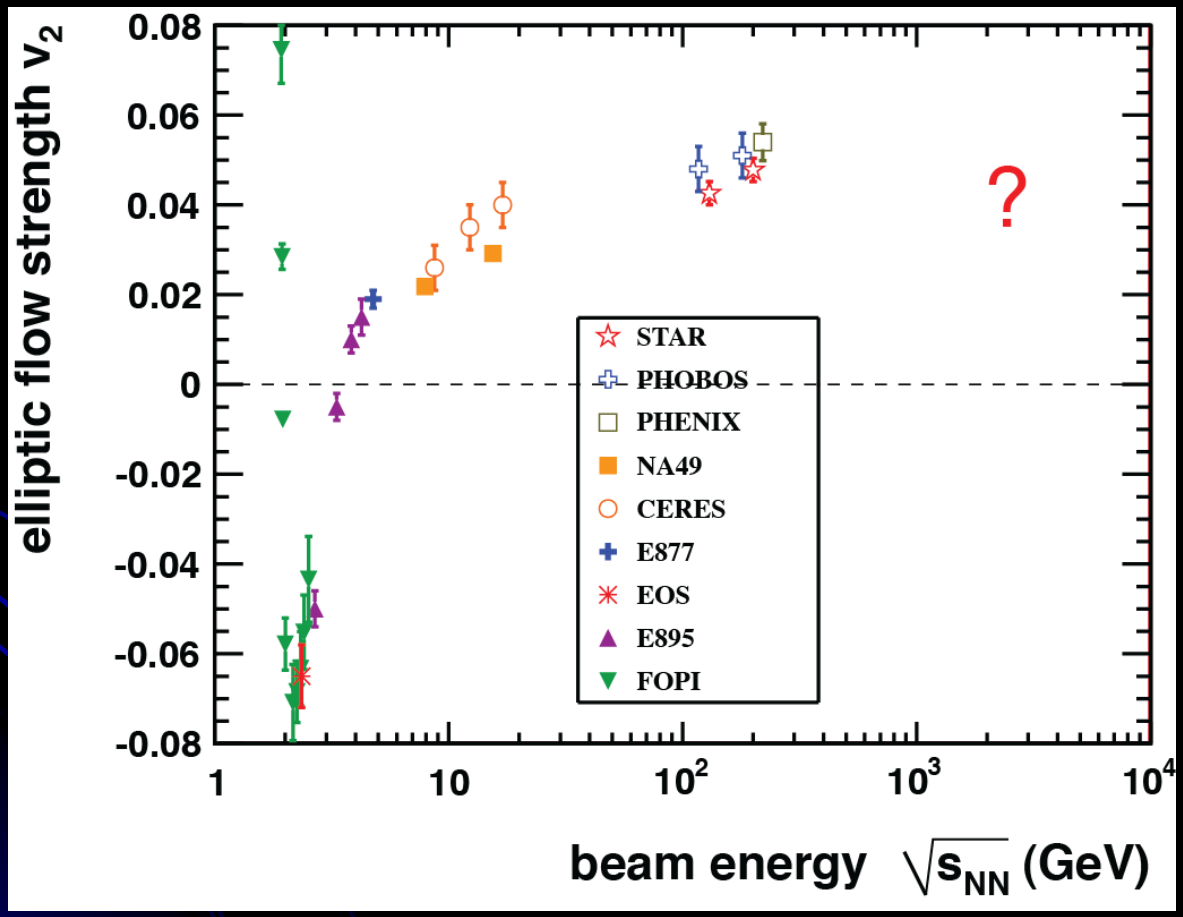
Associated Press
Tuesday, April 19, 2005; Page A05

The Washington Post

New results from a particle collider suggest that the universe behaved like a liquid in its earliest moments, not the fiery gas that was thought to have pervaded the first microseconds of existence.

Anisotropic flow

- Is the collective behavior of matter at LHC still like a perfect liquid or rather a viscous gas?



How do we measure anisotropic flow?

Is it really that trivial?

- Anisotropic flow recipe
  - Step 1: Measure/estimate reaction plane in an event
  - Step 2: Take azimuthal angles of all reconstructed particles in an event
  - Step 3: Evaluate anisotropic flow harmonics via the average

$$v_n = \langle \cos(n(\phi - \Psi_{\mathrm{RP}})) \rangle$$

- In experimental practice the above prescription will not work
  - We cannot neither measure directly nor estimate reaction plane reliably event-by-event
- Can we estimate anisotropic flow harmonics v_n without requiring the reaction plane?

Methods implemented for ALICE
(naming conventions)

- MC = Monte Carlo event plane
- SP = Scalar Product
- GFC = Generating Function Cumulants
- QC = Q-cumulants
- FQD = Fitting q-distribution
- LYZ = Lee-Yang Zero (sum and product)
- LYZEP = Lee-Yang Zero Event Plane

Anisotropic flow (exp)

- Theoretical definition not useful in practice

$$\langle v_n \rangle = \langle\langle \cos(n(\phi - \Psi_{\mathrm{RP}})) \rangle\rangle$$

- Alternative approach: Two- and multi-particle azimuthal correlations:

$$\begin{aligned} \left\langle\left\langle e^{in(\phi_1-\phi_2)} \right\rangle\right\rangle &= \left\langle\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}}-(\phi_2-\Psi_{\mathrm{RP}}))} \right\rangle\right\rangle \\ &= \left\langle\left\langle e^{in(\phi_1-\Psi_{\mathrm{RP}})} \right\rangle \left\langle e^{-in(\phi_2-\Psi_{\mathrm{RP}})} \right\rangle\right\rangle = \langle v_n^2 \rangle \end{aligned}$$

- Price to pay (#1): Systematic bias due to other sources of correlations (autocorrelations, few-particle nonflow correlations)
- Price to pay (#2): Systematic bias due to statistical flow fluctuations

Autocorrelations (1/2)

- We have to correlate only distinct particles, because autocorrelations are dominant and useless (really!) contribution in all averages. So:

$$\langle 2 \rangle \equiv \langle \cos n(\phi_1 - \phi_2) \rangle \ , \qquad \phi_1 \neq \phi_2$$

$$\langle 4 \rangle \equiv \langle \cos n(\phi_1 + \phi_2 - \phi_3 - \phi_4) \rangle \ , \qquad \phi_1 \neq \phi_2 \neq \phi_3 \neq \phi_4$$

- How to enforce above constrains in practice?
  - Brute force evaluation via nested loops? => not feasible
  - Formalism of generating functions? => only approximate

$$G_n(z) \equiv \prod_{j=1}^{M} \left(1 + \frac{z^* e^{in\phi_j} + z e^{-in\phi_j}}{M} \right)$$

$$\langle G_n(z) \rangle = \sum_{k=0}^{M/2} \frac{|z|^{2k}}{M^{2k}} \binom{M}{k} \binom{M-k}{k} \left\langle e^{in(\phi_1 + \cdots + \phi_k - \phi_{k+1} - \cdots - \phi_{2k})} \right\rangle$$

N. Borghini, P. M. Dinh and J.-Y. Ollitrault, "Flow analysis from multiparticle azimuthal correlations," PRC 64 (2001) 054901

Autocorrelations (2/2)

- We have a new analytic results to eliminate all autocorrelations!
- Q-vector (a.k.a. flow vector) Q_n evaluated in harmonic n:

$$Q_n = \sum_{i=1}^{M} e^{in\phi_i}$$

- Key result: Analytical expressions for multi-particle azimuthal correlations in terms of Q-vectors

$$\langle 2 \rangle = \frac{|Q_n|^2 - M}{M(M-1)}$$

$$\langle 4 \rangle = \frac{|Q_n|^4 + |Q_{2n}|^2 - 2 \cdot \mathbf{Re}\left[Q_{2n} Q_n^* Q_n^*\right] - 4(M-2) \cdot |Q_n|^2}{M(M-1)(M-2)(M-3)} + \frac{2}{(M-1)(M-2)}$$

R. Snellings, S. Voloshin, A.B. *"Flow analysis with cumulants: Direct calculations"*, PRC 83, 044913 (2011)

Few-particle nonflow

- Question: Can we suppress systematically unwanted contribution to measured azimuthal correlations which do not originate from the initial geometry?
  - Resonance decays
  - Track splitting during reconstruction
- Multi-particle cumulants can do the magic!
  - Originally, cumulants were introduced in flow analysis by Borghini, Dinh and Ollitrault
  - Studied by mathematicians and used in the other fields of physics already for a long time

Cumulants (1/6)

- In what follows X_i will denote the general i-th random observable
- The most general decomposition of 2-particle correlation reads

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2nd term above is 2-particle cumulant => it isolates the genuine 2-particle correlation in the system, which cannot be factorized further
- We cannot measure cumulants directly, however trivially:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

Cumulants (2/6)

- The most general decomposition of 3-particle correlation reads:

$$\begin{aligned}\langle X_1 X_2 X_3 \rangle &= \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \\ &+ \langle X_1 X_2 \rangle_c \langle X_3 \rangle + \langle X_1 X_3 \rangle_c \langle X_2 \rangle + \langle X_2 X_3 \rangle_c \langle X_1 \rangle \\ &+ \langle X_1 X_2 X_3 \rangle_c\end{aligned}$$

- Inserting previous results for 2-particle cumulants, it follows:

$$\begin{aligned}\langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle\end{aligned}$$

- In this way, one can isolate cumulants recursively for any number of random observables

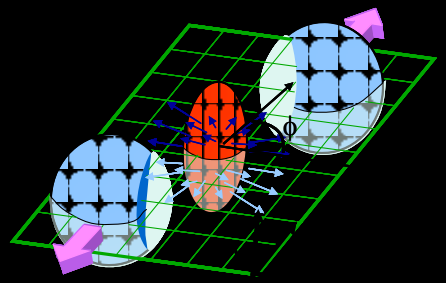
Ryogo Kubo, *"Generalized Cumulant Expansion Method"*

Cumulants (3/6)

- Choice for random observables in the context of anisotropic flow analysis (Ollitrault *et al*)

$$X_1 \equiv e^{in\phi_1}, \qquad X_2 \equiv e^{in\phi_2}$$
$$X_3 \equiv e^{-in\phi_3}, \qquad X_4 \equiv e^{-in\phi_4}$$

- In reality:
  - a) Reaction plane fluctuates randomly e-b-e



  - b) Detector has “close to uniform acceptance” in azimuthal angle
- All-event averages of single random variables vanish (e.g. <<X_1>>) => expressions for cumulants simplify tremendeously

Cumulants (4/6)

- Cumulants expressed in terms of azimuthal correlations:

$$
\begin{aligned}
QC\{2\} &= \langle\langle 2\rangle\rangle \\
QC\{4\} &= \langle\langle 4\rangle\rangle - 2\langle\langle 2\rangle\rangle^2 \\
QC\{6\} &= \langle\langle 6\rangle\rangle - 9\langle\langle 2\rangle\rangle\langle\langle 4\rangle\rangle + 12\langle\langle 2\rangle\rangle^3 \\
QC\{8\} &= \langle\langle 8\rangle\rangle - 16\langle\langle 6\rangle\rangle\langle\langle 2\rangle\rangle - 18\langle\langle 4\rangle\rangle^2 \\
&+ 144\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle^2 - 144\langle\langle 2\rangle\rangle^4
\end{aligned}
$$

- In the case all correlations are expressed analytically in terms of *Q*-vectors => *Q*-cumulants (QC)

Cumulants (5/6)

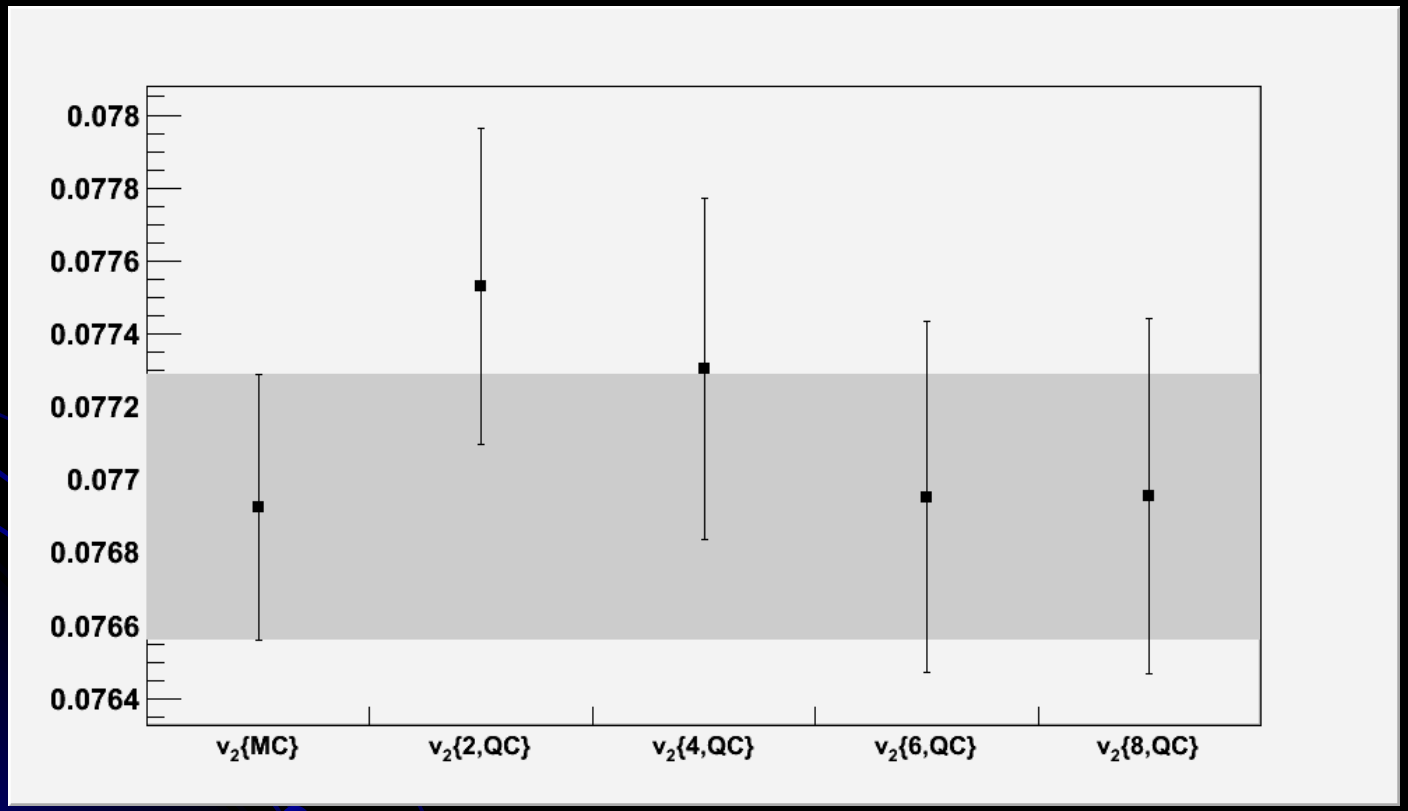
- When only flow correlations are present in the system their contribution to QCs is well understood and quantified (neglecting e-b-e flow fluctuations):

$$
\begin{aligned}
QC\{2\} &= v^2 \\
QC\{4\} &= -v^4 \\
QC\{6\} &= 4v^6 \\
QC\{8\} &= -33v^8
\end{aligned}
$$

- These relations hold for any harmonic
- Thing to note and remember: Flow contribution to QCs have a distinct signature **(+,-,+,-)**
  - In order to interpret dominant contribution to QCs as a flow this signature is a necessary condition (not sufficient, though)

Cumulants (6/6)

- Proof of the principle: Using Therminator events (realistic Monte Carlo generator of heavy-ion events, has both anisotropic flow and all resonances in the standard model)



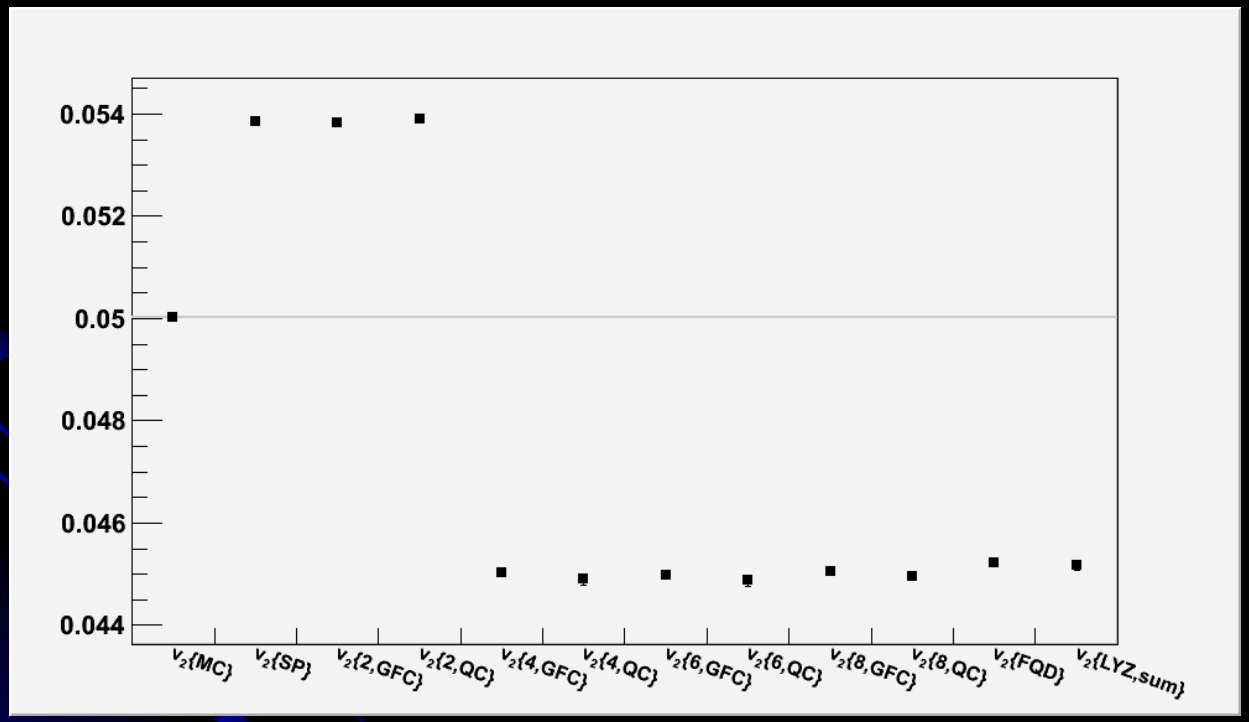
In this regime multi-particle QCs are precision method

Flow fluctuations

- In the limit <v> >> sigma^2:

$$\begin{aligned} 2\text{–particle}: \; v_2\{2\} &= \langle v_2 \rangle + \sigma_{v_2}^2/(2\langle v_2 \rangle) \\ 2\text{k–particle}: \; v_2\{2k\} &= \langle v_2 \rangle - \sigma_{v_2}^2/(2\langle v_2 \rangle), \quad (k > 1) \end{aligned}$$

- Example: v_2 = 0.05 +/- 0.02 (Gaussian), M = 500, N = 10^6



True flow value is in-between 2- and multi-particle estimates

Differential flow (1/2)

- Main task: Calculating differential flow of particles of interest (POIs)
  - Example question: What is $v_2(p_t)$?
  - Direct differential flow analysis of POIs by using 2- and multi-particle correlations is not feasible due to limited statistics of POIs, i.e. one cannot blindly use in each p_t bin something like

$$v_{n,\mathrm{POI}} = \left\langle e^{in(\phi_{1,\mathrm{POI}} - \phi_{2,\mathrm{POI}})} \right\rangle^{1/2}$$

  - Such estimate will be in majority of the cases statistically unstable (i.e. useless)

Differential flow (2/2)

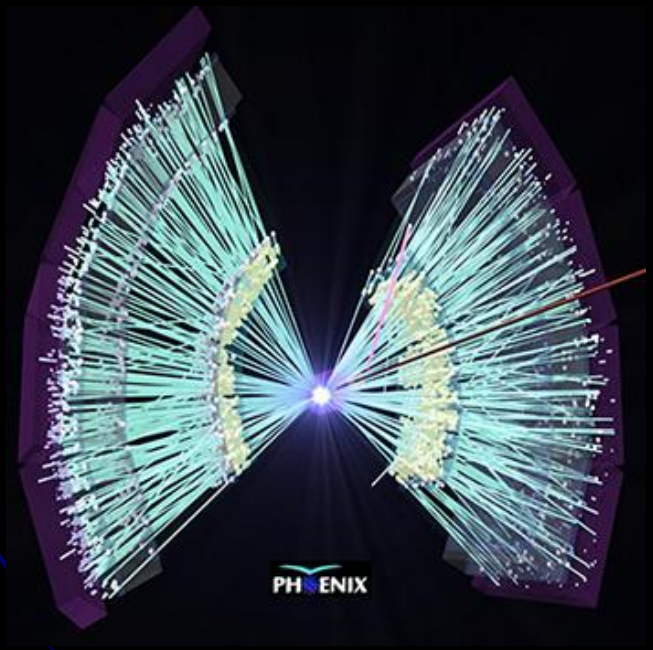
- Instead, one uses as reference particles (RPs) some abundant particles (e.g. all charged particles) and evaluates:

$$\frac{\left\langle e^{in(\phi_{1,\mathrm{POI}}-\phi_{2,\mathrm{RP}})}\right\rangle}{\left\langle e^{in(\phi_{1,\mathrm{RP}}-\phi_{2,\mathrm{RP}})}\right\rangle^{1/2}} = \frac{v_{n,\mathrm{POI}} \times v_{n,\mathrm{RP}}}{(v_{n,\mathrm{RP}} \times v_{n,\mathrm{RP}})^{1/2}} = v_{n,\mathrm{POI}}$$

- Because for RPs we took some abundant particles, both correlators on the LHS are statistically stable now, and RP dependence by construction should cancel out
  - This idea can be straightforwardly generalized to multi-particle correlations

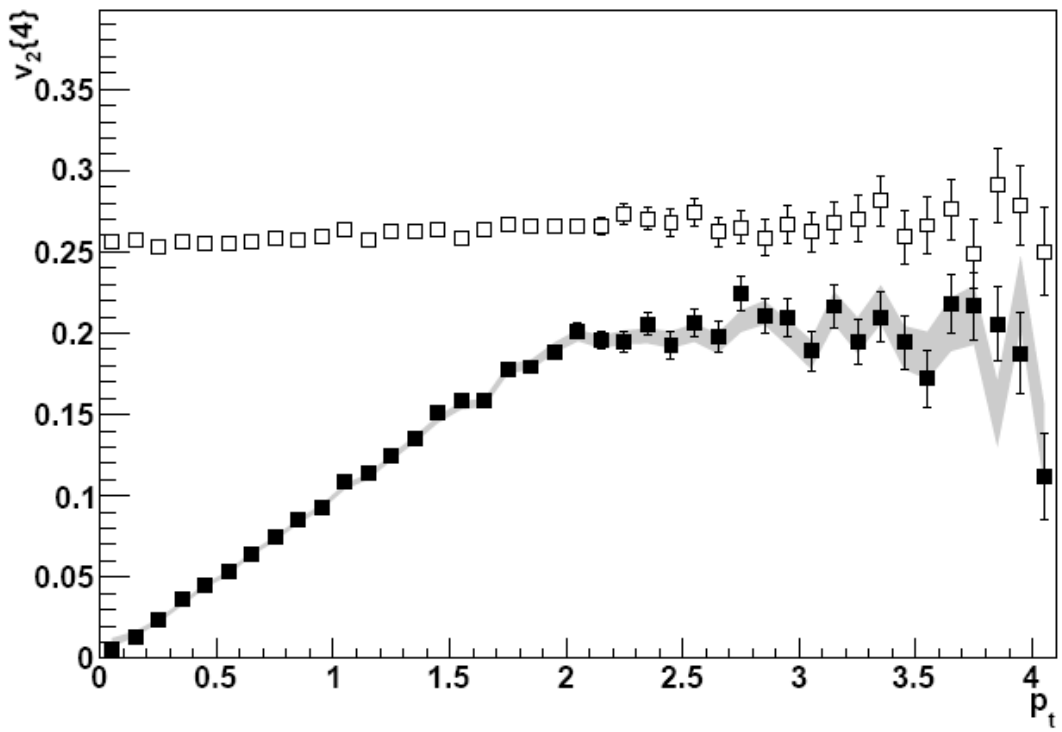
Non-uniform acceptance (1/2)

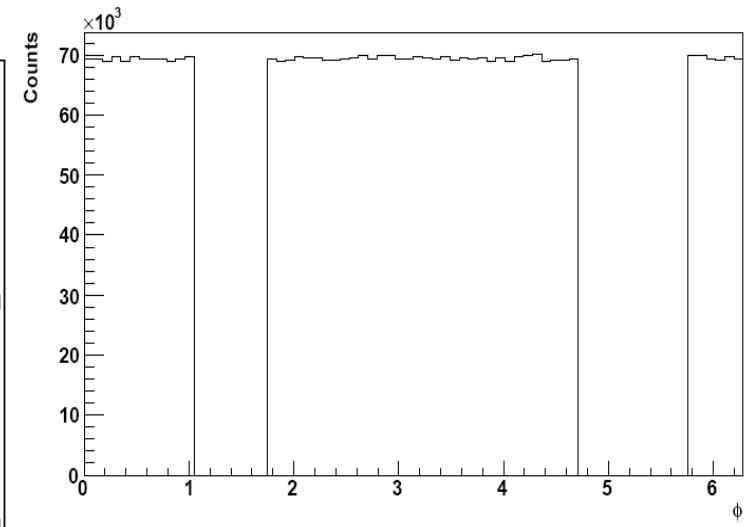
- If a detector has non-uniform acceptance in azimuthal angle, than in each event we have trivial anisotropies in momentum distributions of detected particles => this has nothing to do with anisotropic flow!
  - Can we disentangle one anisotropy from another?



Non-uniform acceptance (2/2)

- Generalized *Q*-cumulants can correct for non-uniform acceptance very well





Technical details => Appendix C in R. Snellings, S. Voloshin, A.B. *"Flow analysis with cumulants: Direct calculations"*, PRC 83, 044913 (2011)

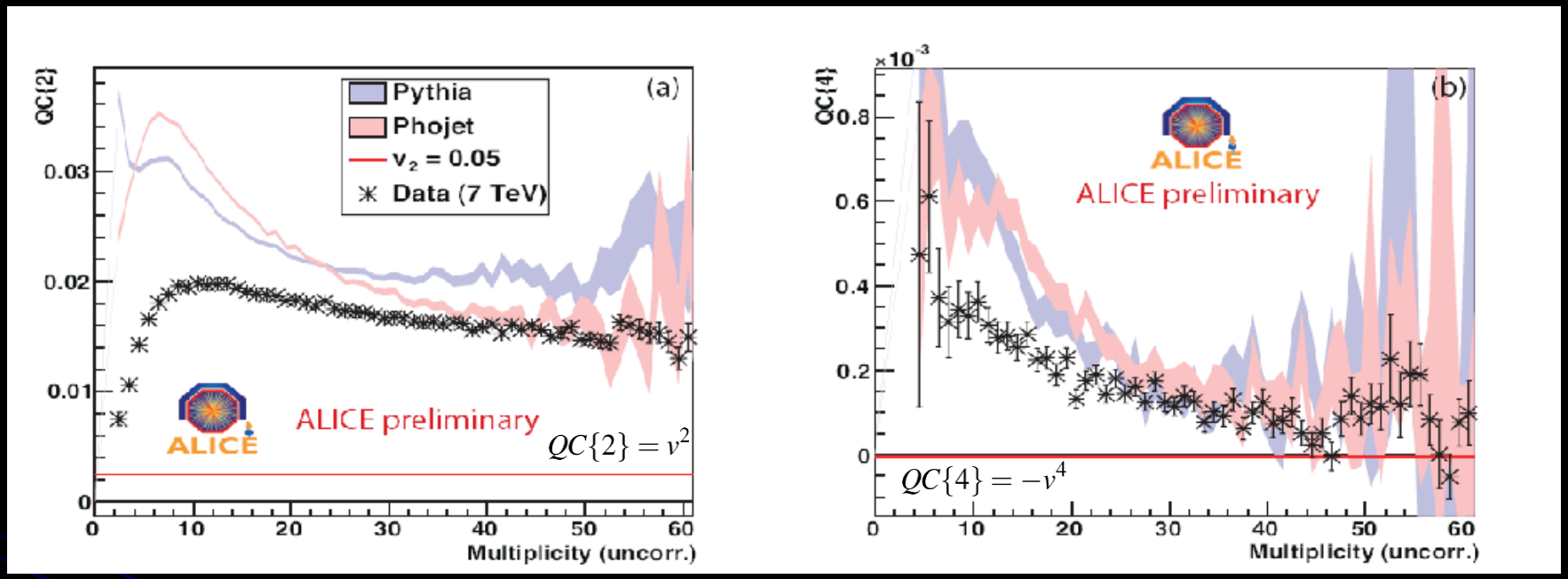
Grey band => $v_2\{MC\}$; open markers => $v_2\{4\}$ from isotropic *Q*-cumulants; filled markers => $v_2\{4\}$ from generalized *Q*-cumulants

Data analysis

Elliptic flow in *pp*?

- Effects of collectivity in high multiplicity *pp* collisions as well?
  - Various theoretical predictions indicate possible elliptic flow values between 0.03 and 0.15 in *pp* collisions @ LHC energies
- Testing the software and getting experienced with ALICE analysis framework

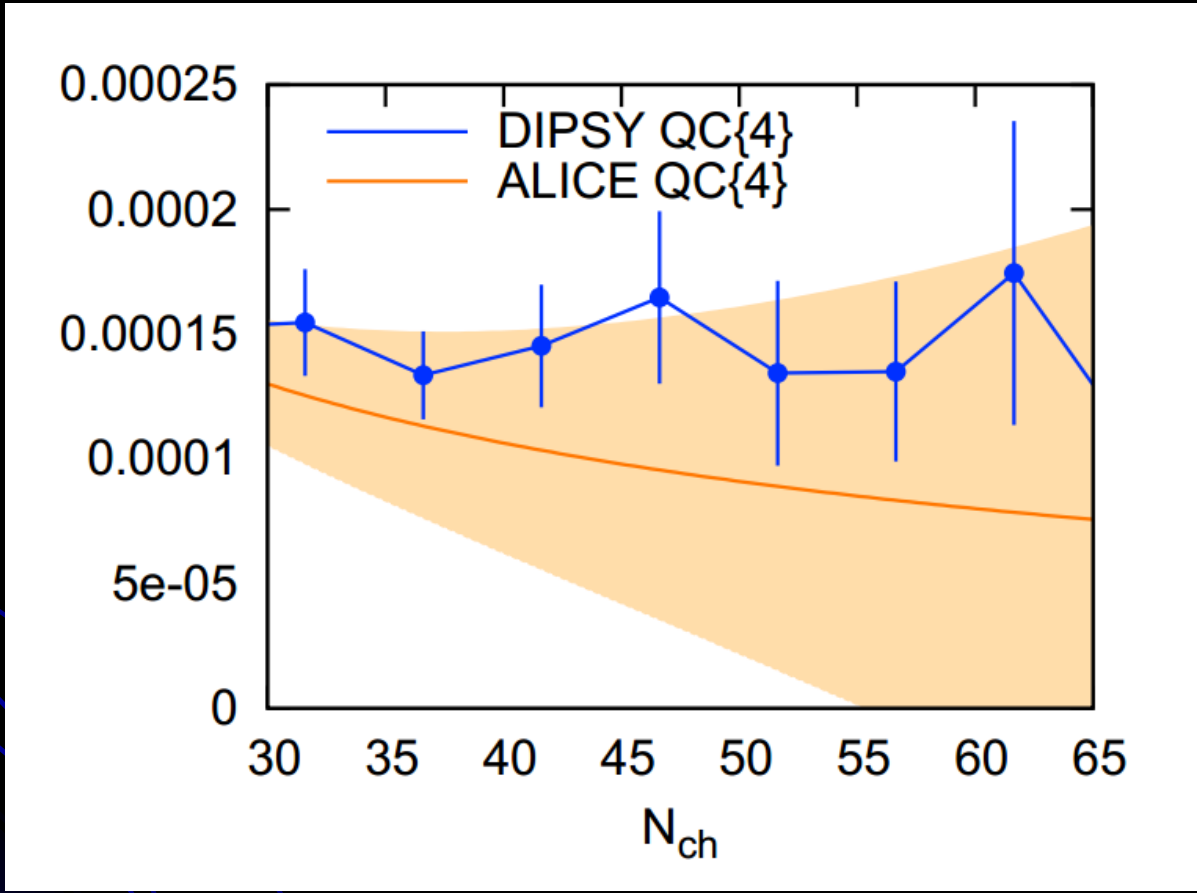
Elliptic flow in *pp*?



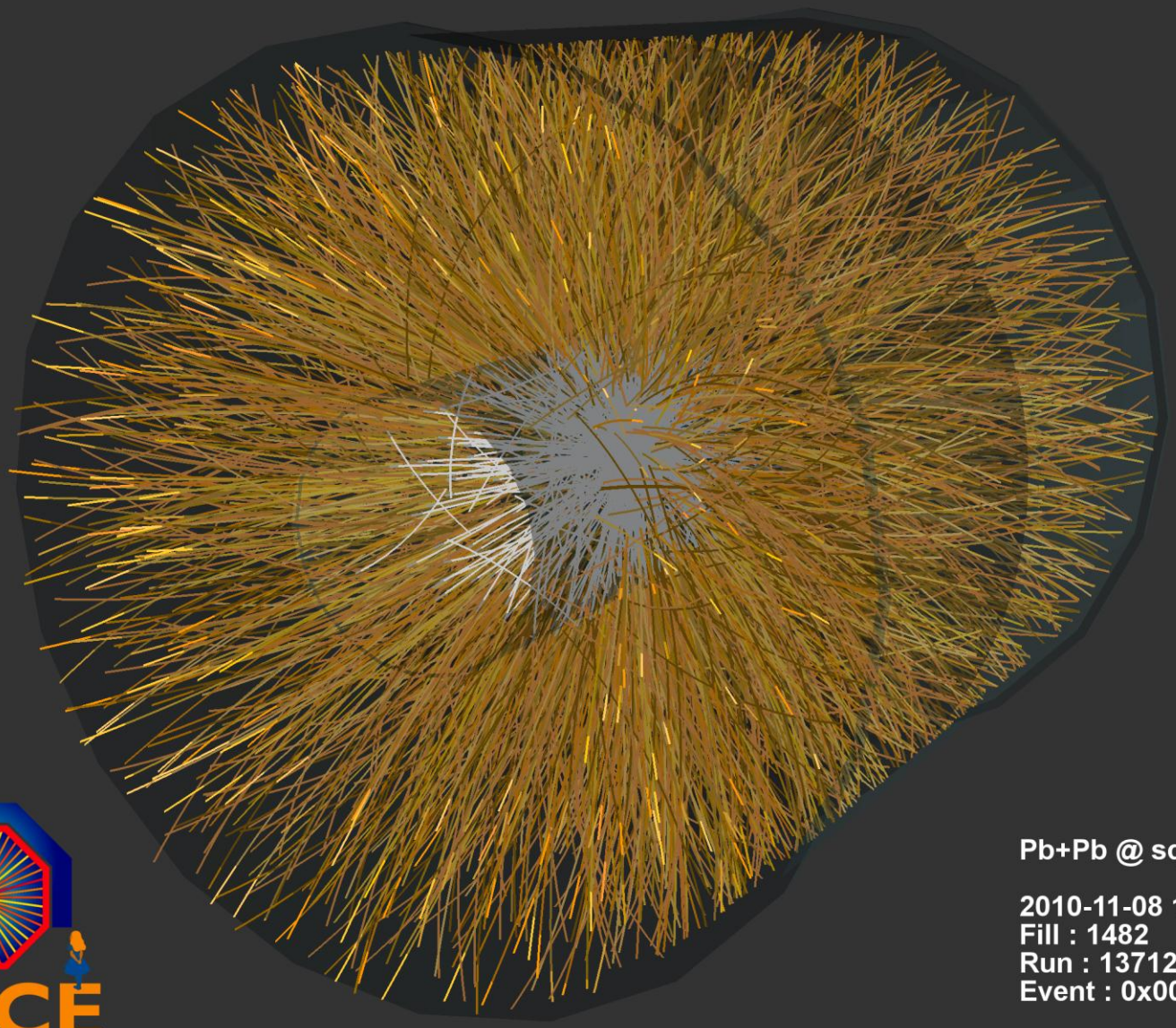
- Both 2- and 4-particle correlations decrease with multiplicity: Typical for non-collective behavior
- Pythia and Phojet are overestimating the strength of the correlations (and these two generators are dominated by jets and resonances)
- 4-p cumulant comes with a “wrong sign” => its dominant contribution is not coming from flow
- Current status – **We do not see elliptic flow in *pp***

Elliptic flow in *pp*?

- Comparison to DIPSY:



E. Avsar, Y. Hatta, C. Flensburg, J. Y. Ollitrault and T. Ueda, "Eccentricity and elliptic flow in pp collisions at the LHC," J. Phys. G **38** (2011) 124053

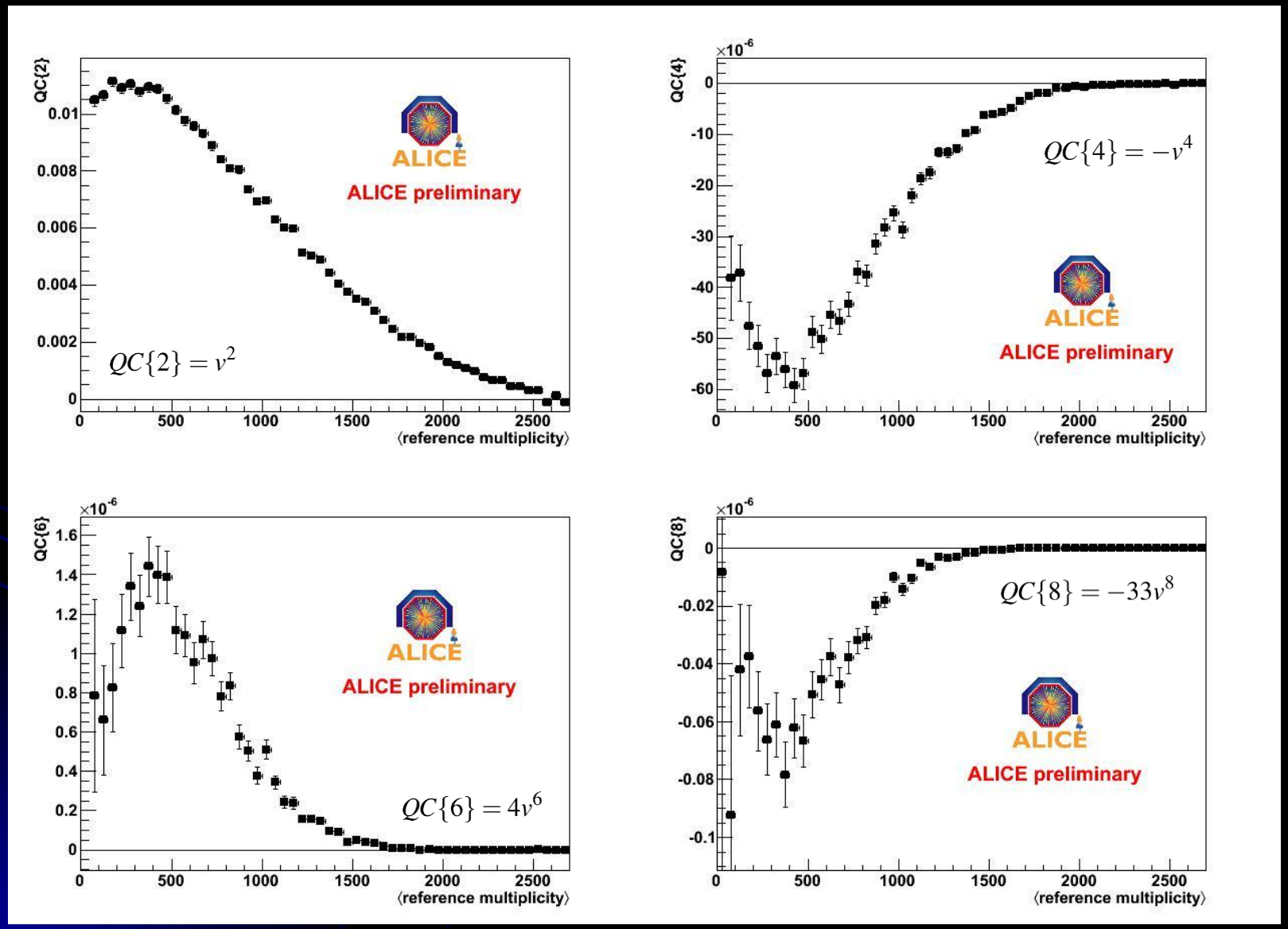


Pb+Pb @ sqrt(s) = 2.76 ATeV

2010-11-08 11:29:52
Fill : 1482
Run : 137124
Event : 0x0000000042B1B693

ALICE

Flow at first sight!



Elliptic flow paper

Selected for a Viewpoint in *Physics*

PRL **105,** 252302 (2010) PHYSICAL REVIEW LETTERS week ending 17 DECEMBER 2010

Elliptic Flow of Charged Particles in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

K. Aamodt *et al.**

(ALICE Collaboration)

(Received 18 November 2010; published 13 December 2010)

We report the first measurement of charged particle elliptic flow in Pb-Pb collisions at $\sqrt{s_{NN}} =$ 2.76 TeV with the ALICE detector at the CERN Large Hadron Collider. The measurement is performed in the central pseudorapidity region ($|\eta| < 0.8$) and transverse momentum range $0.2 < p_t < 5.0$ GeV/c. The elliptic flow signal v_2, measured using the 4-particle correlation method, averaged over transverse momentum and pseudorapidity is 0.087 ± 0.002(stat) $\pm$ 0.003(syst) in the 40%–50% centrality class. The differential elliptic flow $v_2(p_t)$ reaches a maximum of 0.2 near $p_t = 3$ GeV/c. Compared to RHIC Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV, the elliptic flow increases by about 30%. Some hydrodynamic model predictions which include viscous corrections are in agreement with the observed increase.

DOI: 10.1103/PhysRevLett.105.252302 PACS numbers: 25.75.Ld, 25.75.Gz, 25.75.Nq

The goal of ultrarelativistic nuclear collisions is the creation and study of the quark-gluon plasma (QGP), a state of matter whose existence at high energy density is predicted by quantum chromodynamics. One of the experimental observables that is sensitive to the properties of this matter is the azimuthal distribution of particles in the plane perpendicular to the beam direction. When nuclei collide at finite impact parameter (noncentral collisions), the geometrical overlap region and therefore the initial matter distribution is anisotropic (almond shaped). If the matter

scribe flow at RHIC predict an increase of the elliptic flow at the LHC ranging from 10% to 30%, with the largest increase predicted by models which account for viscous corrections [15–18] at RHIC energies. In models with viscous corrections, v_2 at RHIC is below the ideal hydrodynamic limit [12,17] and therefore can show a stronger increase with energy. In hydrodynamic models the charged particle elliptic flow as a function of transverse momentum does not change significantly [7,14], while the p_t-integrated elliptic flow increases due to the rise in

Elliptic flow paper

- Submitted only **10 days** after the first heavy-ion collisions at LHC!
  - At the moment still the **most cited** LHC physics paper (including p-p)! Medal share at 27.04.2012:

1. **Elliptic flow of charged particles in Pb-Pb collisions at 2.76 TeV.**
(186) The ALICE Collaboration (K Aamodt (Bergen U.) *et al.*). Nov 2010. 10 pp.
Published in **Phys.Rev.Lett. 105 (2010) 252302**
e-Print: **arXiv:1011.3914 [nucl-ex]**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Abstract and Postscript and PDF from arXiv.org; Journal Server - Phys.Rev.Lett.
Detailed record - Cited by 186 records

2. **Search for Supersymmetry in pp Collisions at 7 TeV in Events with Jets and Missing Transverse Energy.**
(161) CMS Collaboration (Vardan Khachatryan (Yerevan Phys. Inst.) *et al.*). Jan 2011.
CMS-SUS-10-003,CERN-PH-EP-2010-084.
Published in **Phys.Lett. B698 (2011) 196-218**
e-Print: **arXiv:1101.1628 [hep-ex]**
References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
Abstract and Postscript and PDF from arXiv.org; Journal Server - Phys.Lett.; Fermilab Today Result of the Week
Detailed record - Cited by 161 records

3. **Suppression of Charged Particle Production at Large Transverse Momentum in Central Pb--Pb**
(148) **Collisions at $\sqrt{s_{NN}} = 2.76$ TeV.**
ALICE Collaboration (Kenneth Aamodt (Bergen U.) *et al.*). Dec 2010. 16 pp.
CERN-PH-EP-ALICE-2010-004.
Published in **Phys.Lett. B696 (2011) 30-39**
e-Print: **arXiv:1012.1004 [nucl-ex]**
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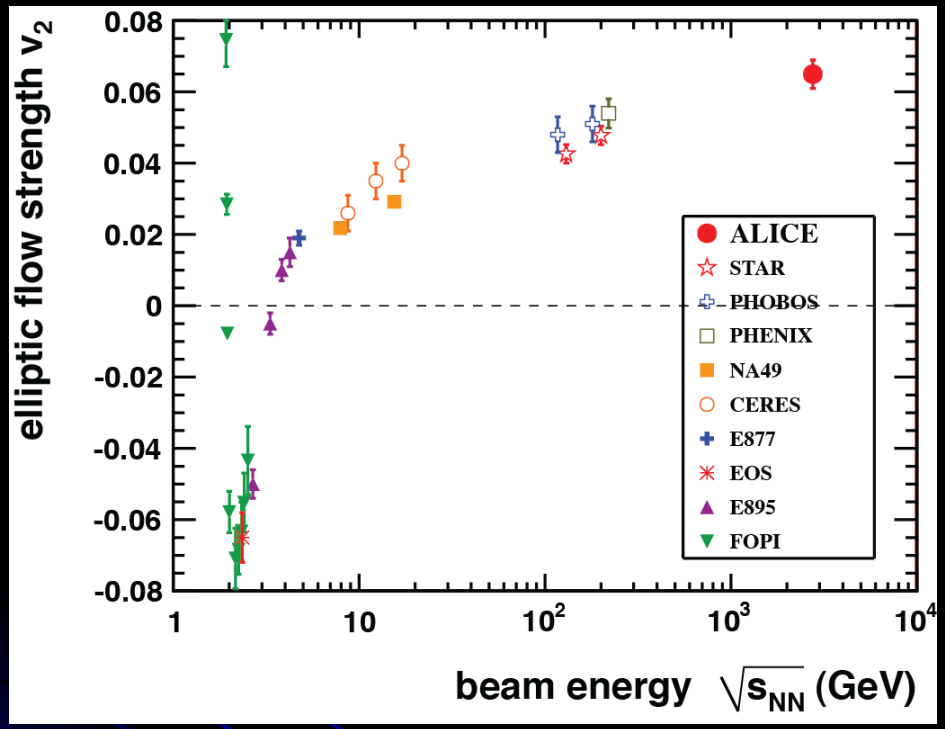
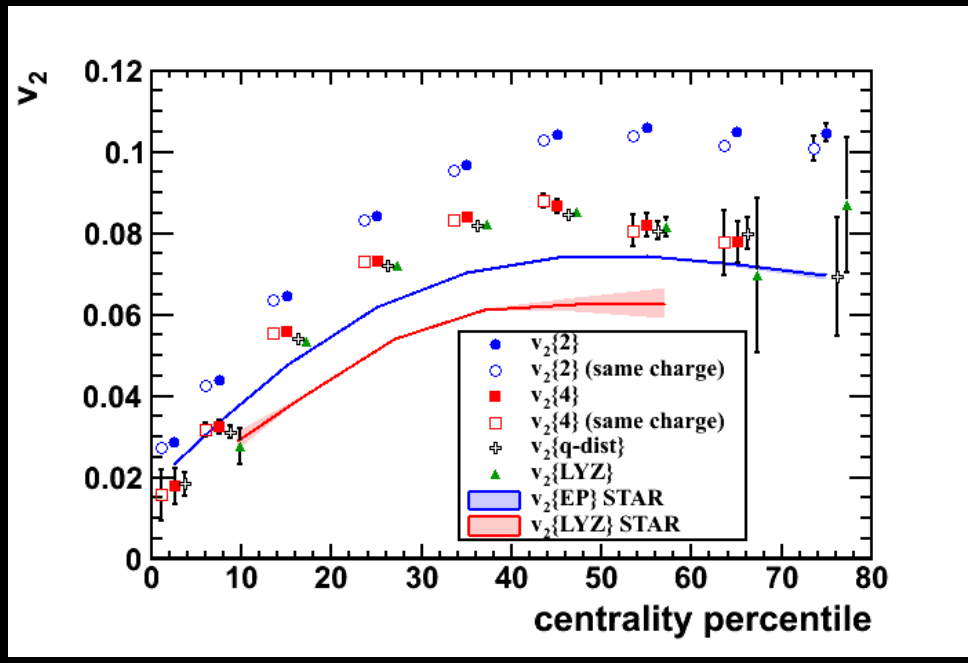
1st look at the LHC data



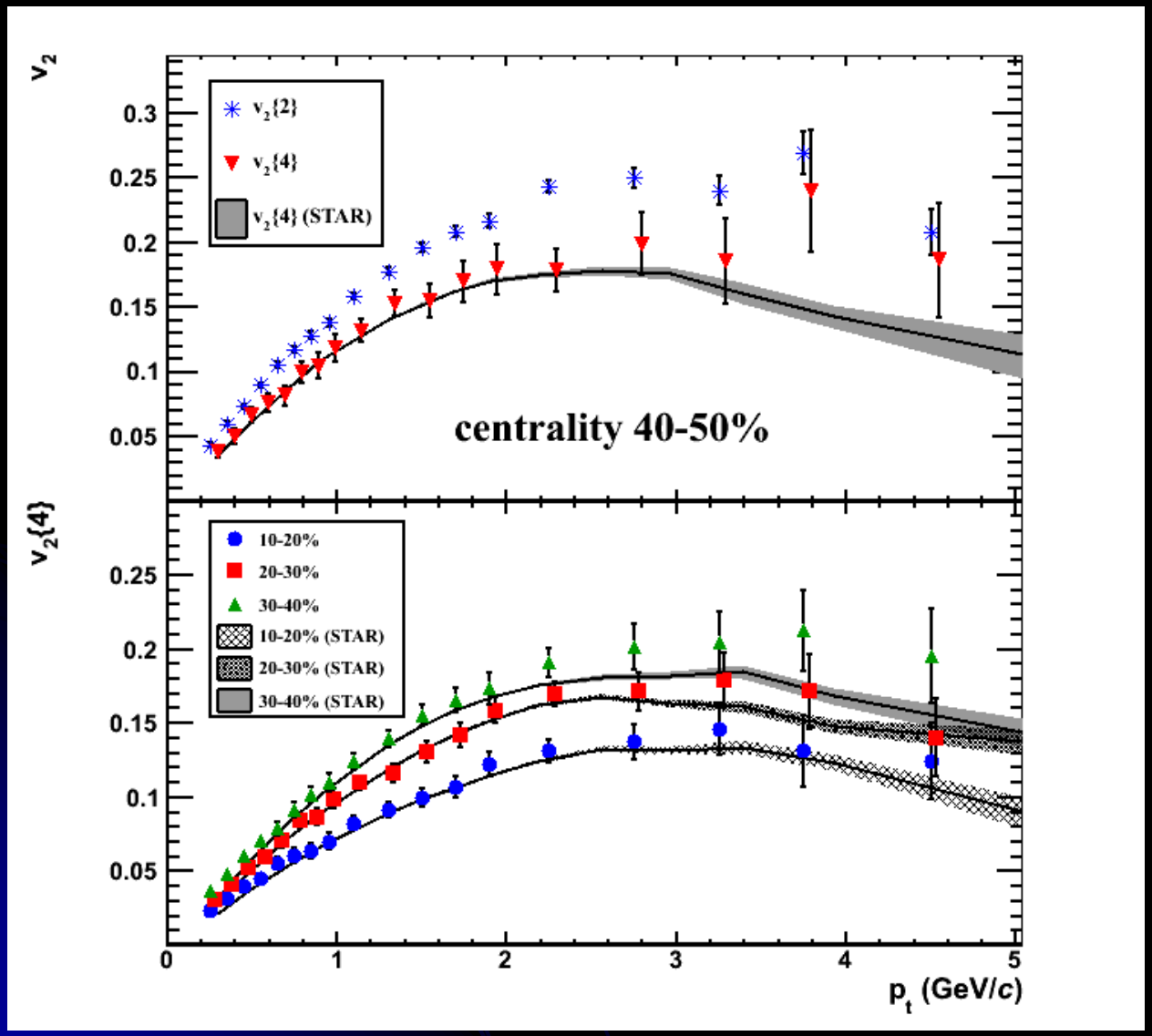
Phys. Rev. Lett. 105, 252302 (2010)

Cited by now 186 times!



Elliptic flow increases by ~ 30% when compared to RHIC energies

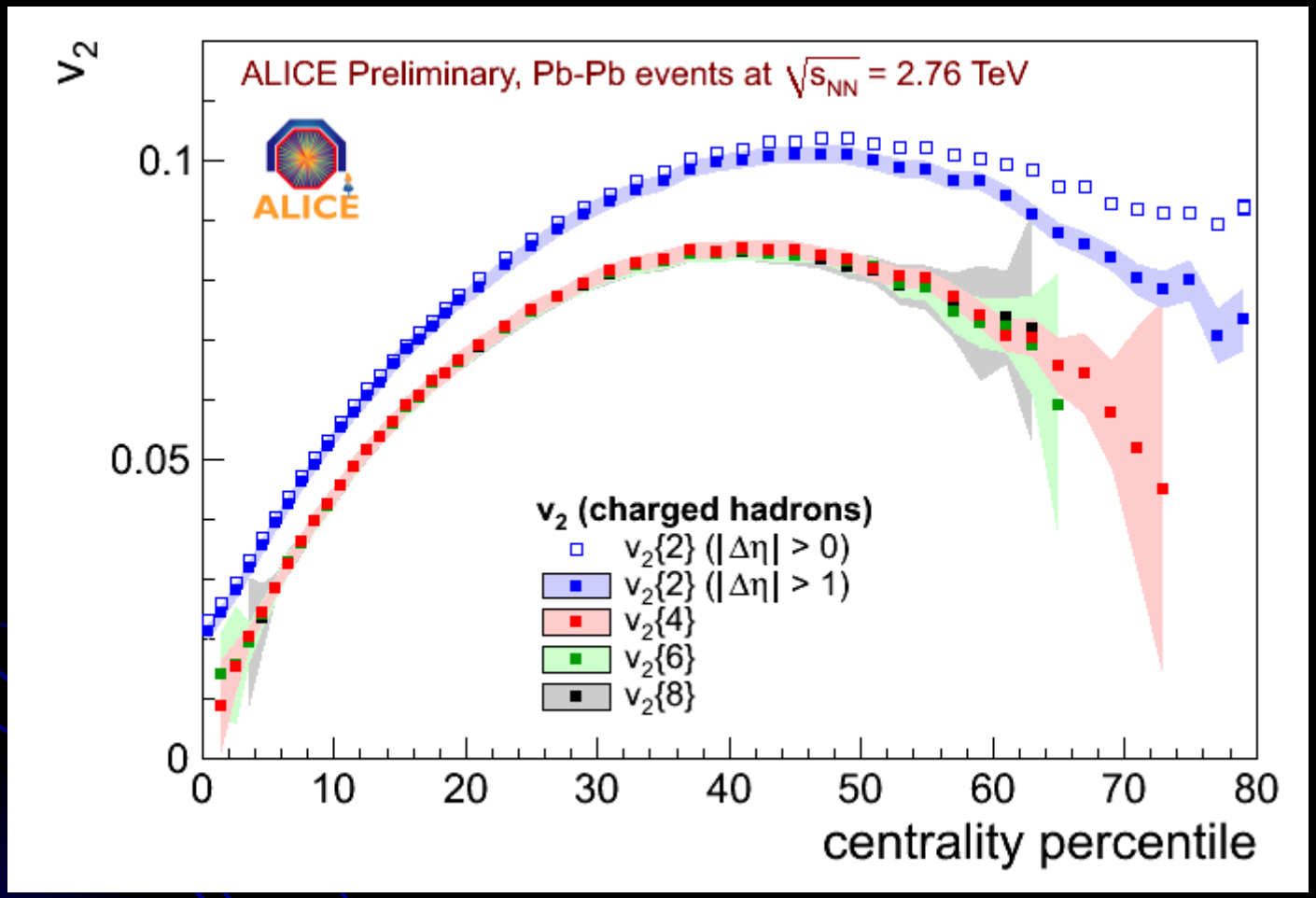
1st look at the LHC data



Phys. Rev. Lett. 105, 252302 (2010)

p_t dependence of elliptic flow at LHC close to the one at RHIC!

Exploiting all statistics….



- The difference between 2- and multi-particle estimates is due to fluctuations in the initial geometry
- $v_2\{2\}$ might still have some nonflow bias leftover (not in the systematical uncertainty here). With eta gap nonflow is suppressed, not eliminated completely

“Higher harmonics” paper

PRL **107,** 032301 (2011) PHYSICAL REVIEW LETTERS week ending 15 JULY 2011

Higher Harmonic Anisotropic Flow Measurements of Charged Particles in Pb-Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV

K. Aamodt *et al.**
(ALICE Collaboration)

(Received 19 May 2011; published 11 July 2011)

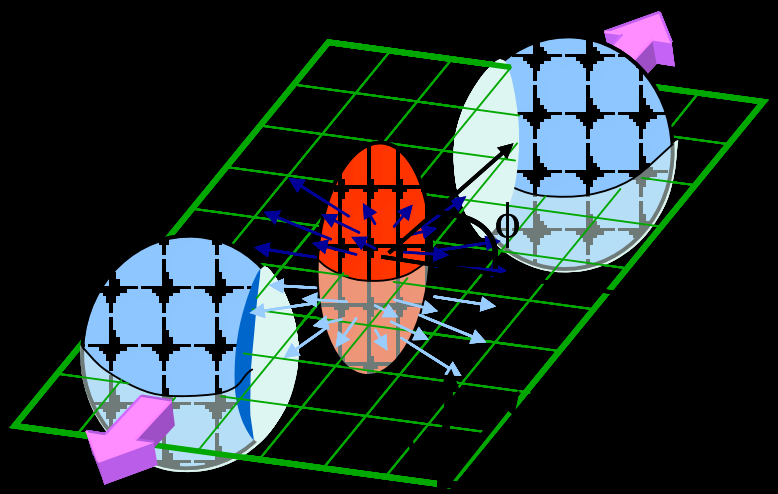
We report on the first measurement of the triangular v_3, quadrangular v_4, and pentagonal v_5 charged particle flow in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV measured with the ALICE detector at the CERN Large Hadron Collider. We show that the triangular flow can be described in terms of the initial spatial anisotropy and its fluctuations, which provides strong constraints on its origin. In the most central events, where the elliptic flow v_2 and v_3 have similar magnitude, a double peaked structure in the two-particle azimuthal correlations is observed, which is often interpreted as a Mach cone response to fast partons. We show that this structure can be naturally explained from the measured anisotropic flow Fourier coefficients.

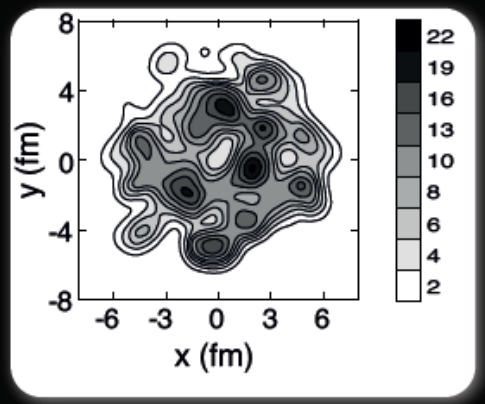
DOI: 10.1103/PhysRevLett.107.032301 PACS numbers: 25.75.Ld, 05.70.Fh, 25.75.Gz

The quark-gluon plasma is a state of matter whose existence at high-energy density is predicted by quantum chromodynamics. The creation of this state of matter in the laboratory and the study of its properties are the main goals

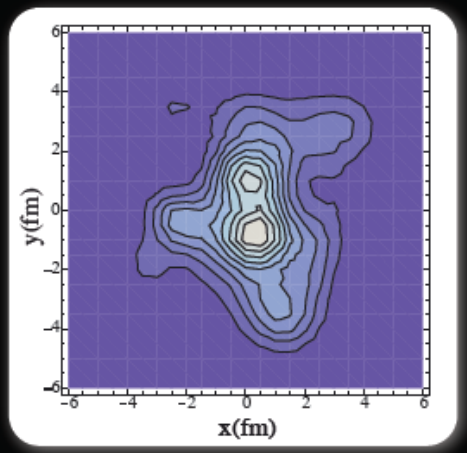
odd Fourier coefficients are zero by symmetry. However, due to fluctuations in the matter distribution, including contributions from fluctuations in the positions of the participating nucleons in the nuclei, the plane of symmetry

Intermezzo (1/2)





Y. Hama, et al.



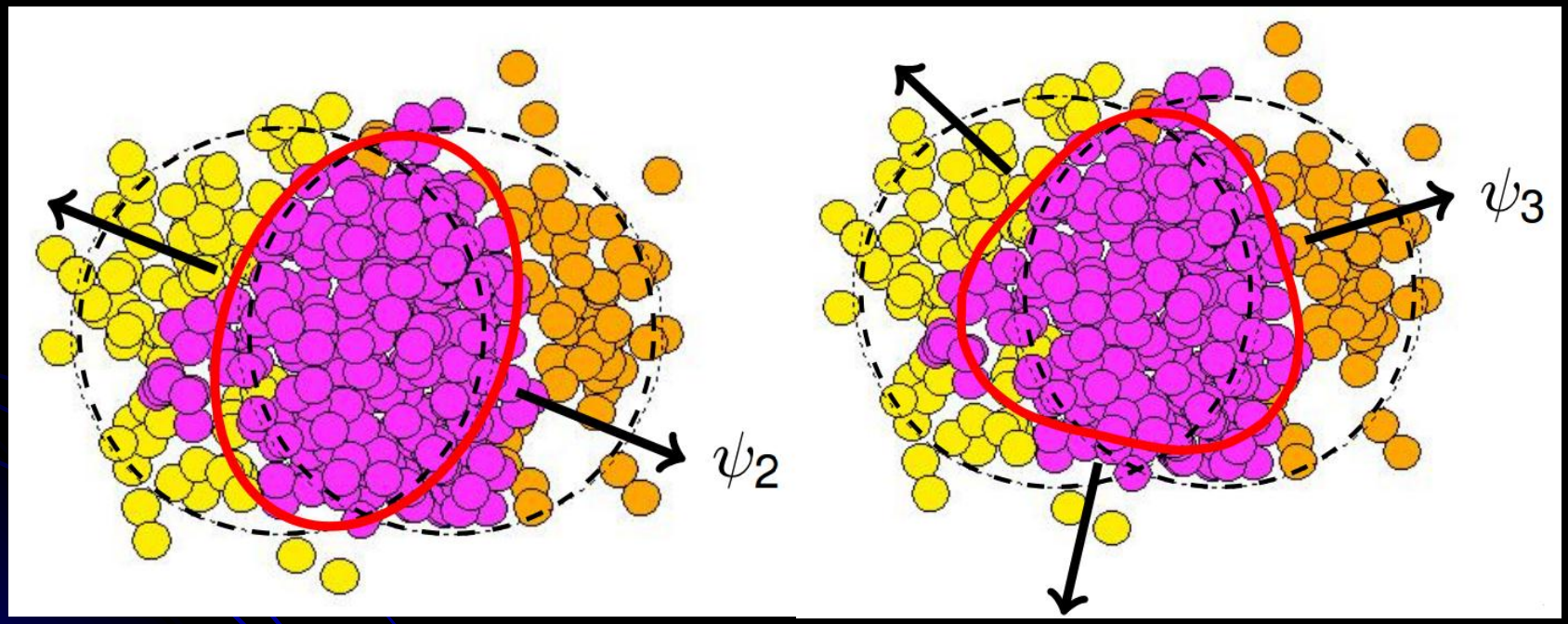
G. Qin, H. Petersen, S. Bass, and B. Muller

- **TOP:** Highly idealized picture of HI collision, lot of symmetries present. In particular, it is equally probable for particles to be emitted in phi and phi + 180° => v_1, v_3, v_5, … are all zero
- **RIGHT:** More realistic picture, all harmonics are present, general Fourier series needed:

$$r(\varphi) = \frac{v_0}{2\pi} + \frac{1}{\pi}\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]$$

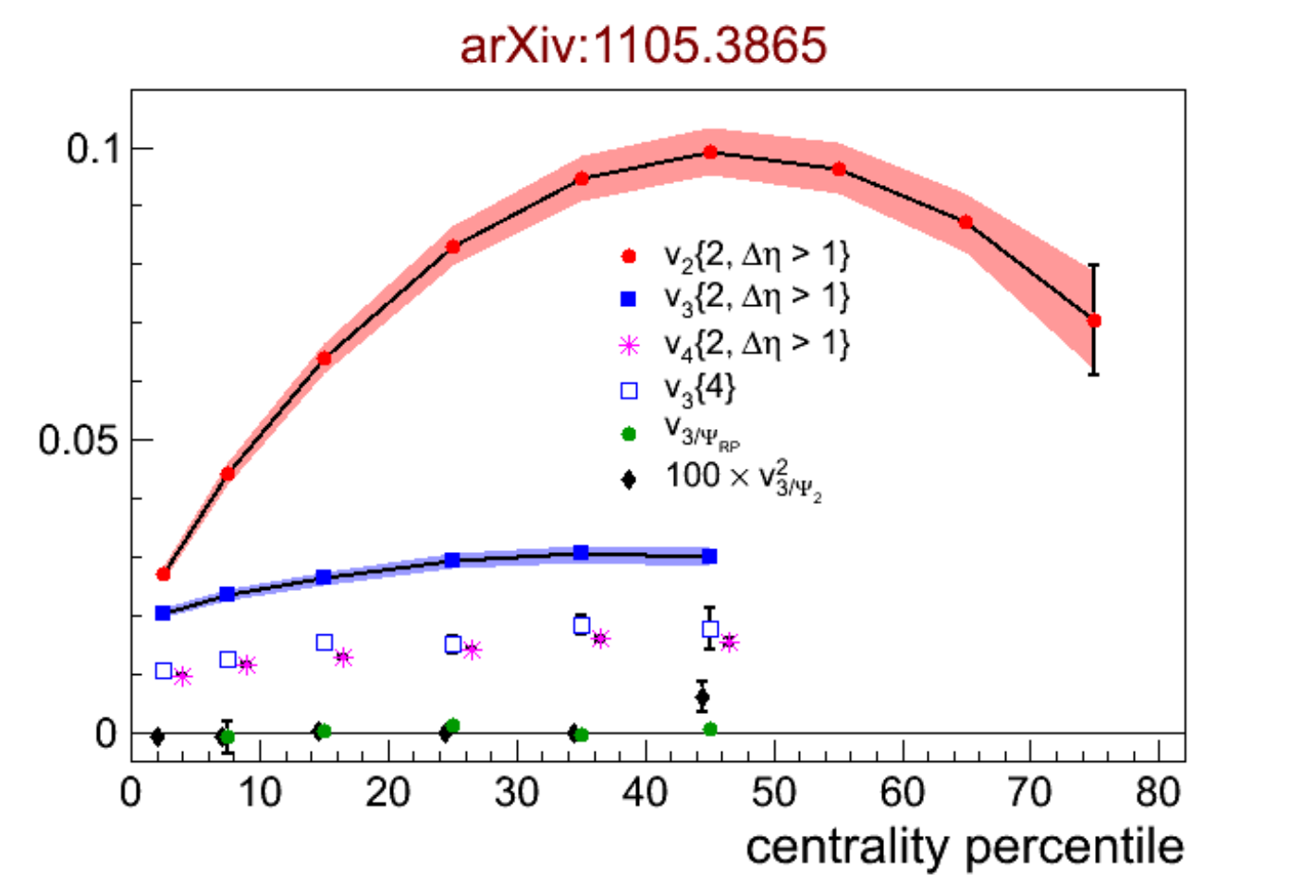
Intermezzo (2/2)

- What are the "symmetry planes"?
  - Example: Symmetry planes of v_2 and v_3



- With multi-particle azimuthal correlations in mixed harmonics, we can measure the correlation of different symmetry planes

Charged particle v_3



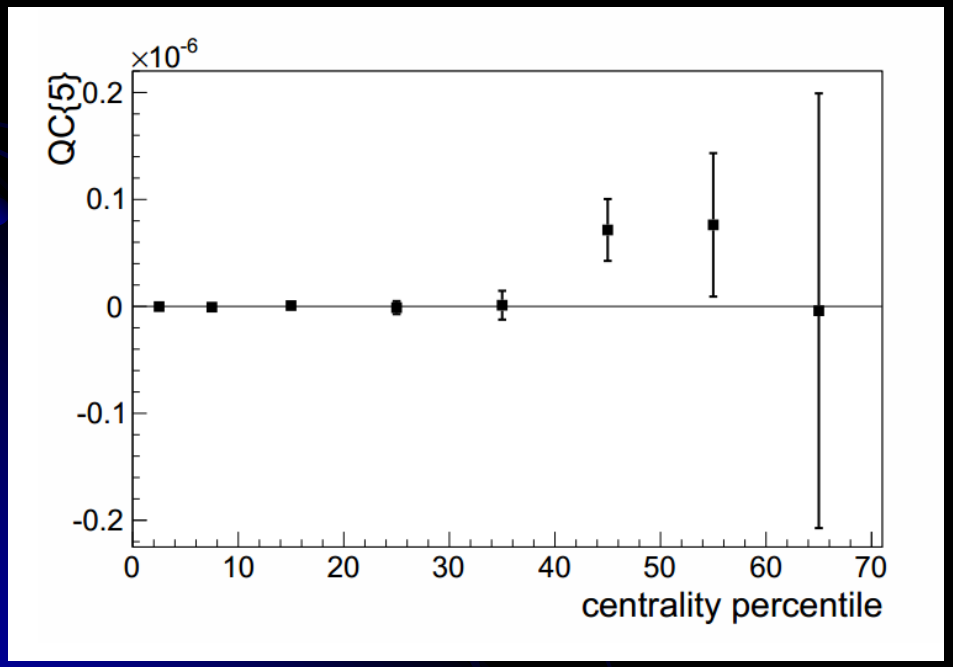
- **Phys.Rev.Lett. 107 (2011) 032301**
- v_3 is not 0 and it develops along its own symmetry plane
- Symmetry plane of v_2 is not the symmetry plane of v_3

QC{5}

- For the detector with uniform acceptance:

$$QC\{5\} = \langle \cos(3\phi_1 + 3\phi_2 - 2\phi_3 - 2\phi_4 - 2\phi_5) \rangle \overset{\text{in theory}}{=} v_3^2 v_2^3 \cos[6(\Psi_3 - \Psi_2)]$$

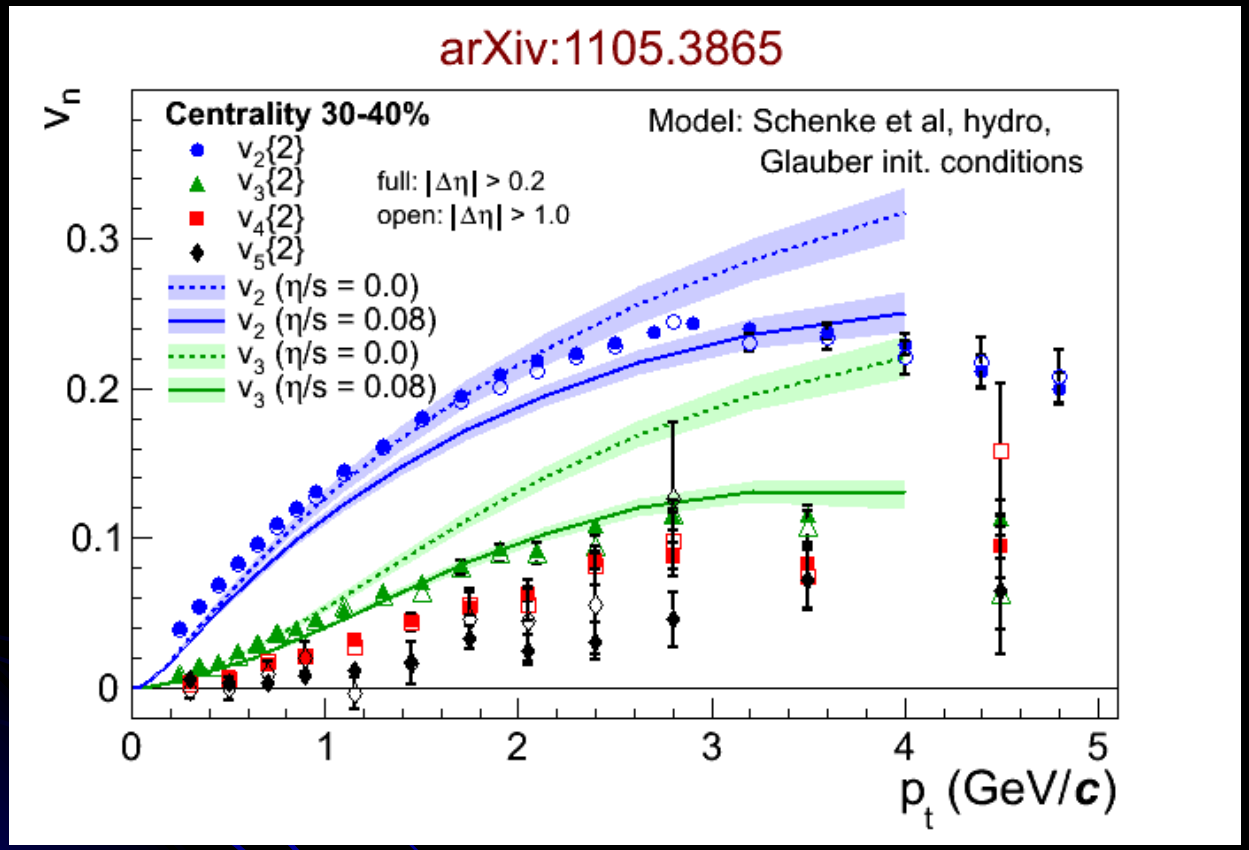
- QC{5} *vs* centrality for the ALICE data (unofficial):



1.) For most- and mid-central events measured QC{5} is zero
2.) For most- and mid-central events v_2 and v_3 measured independently (via QC{2} and QC{4}) are not zero

⇒ <cos[6(Psi3 – Psi2)]> must be 0 in accordance with above equation, i.e. **symmetry planes of v_3 and v_2 are not correlated for most- and mid-central events**

Comparison to models



Within this model overall magnitude of v_2 and v_3 seems to be fine, but the details of p_t dependence are not well described

- More quantitative statement: The magnitude of $v_2(p_t)$ is described better with eta/s = 0, while for v_3(pt) eta/s = 0.08 provides a better description
- This model fails to describe well v_2 and v_3 simultaneously
- **Produced matter in Pb-Pb collisions at LHC continues to behave as a nearly perfect liquid**

Thanks!

Backup

Multiparticle azimuthal correlations

- Typically nonflow correlations involve only few particles. Based purely on combinatorial grounds:

$$\delta_2 \sim 1/M, \qquad \delta_4 \sim 1/M^3$$

- One can use 2- and 4-particle correlations to estimate flow only if:

$$v_n^2 \gg 1/M \quad \Rightarrow \quad v_n \gg 1/M^{1/2}$$
$$v_n^4 \gg 1/M^3 \quad \Rightarrow \quad v_n \gg 1/M^{3/4} \qquad \text{much milder!}$$

- It is possible to obtain flow estimate from the genuine multiparticle correlation (Ollitrault *et al).* In this case one reaches the theoretical limit of applicability:

$$v_n \gg 1/M$$

- Can we now relax once we have devised multiparticle correlations to estimate flow experimentally?

Cumulants: A principle

- Ollitrault *et al*: Imagine that there are only flow and 2-particle nonflow correlations present. Than contributions to measured 2- and 4-particle correlations read

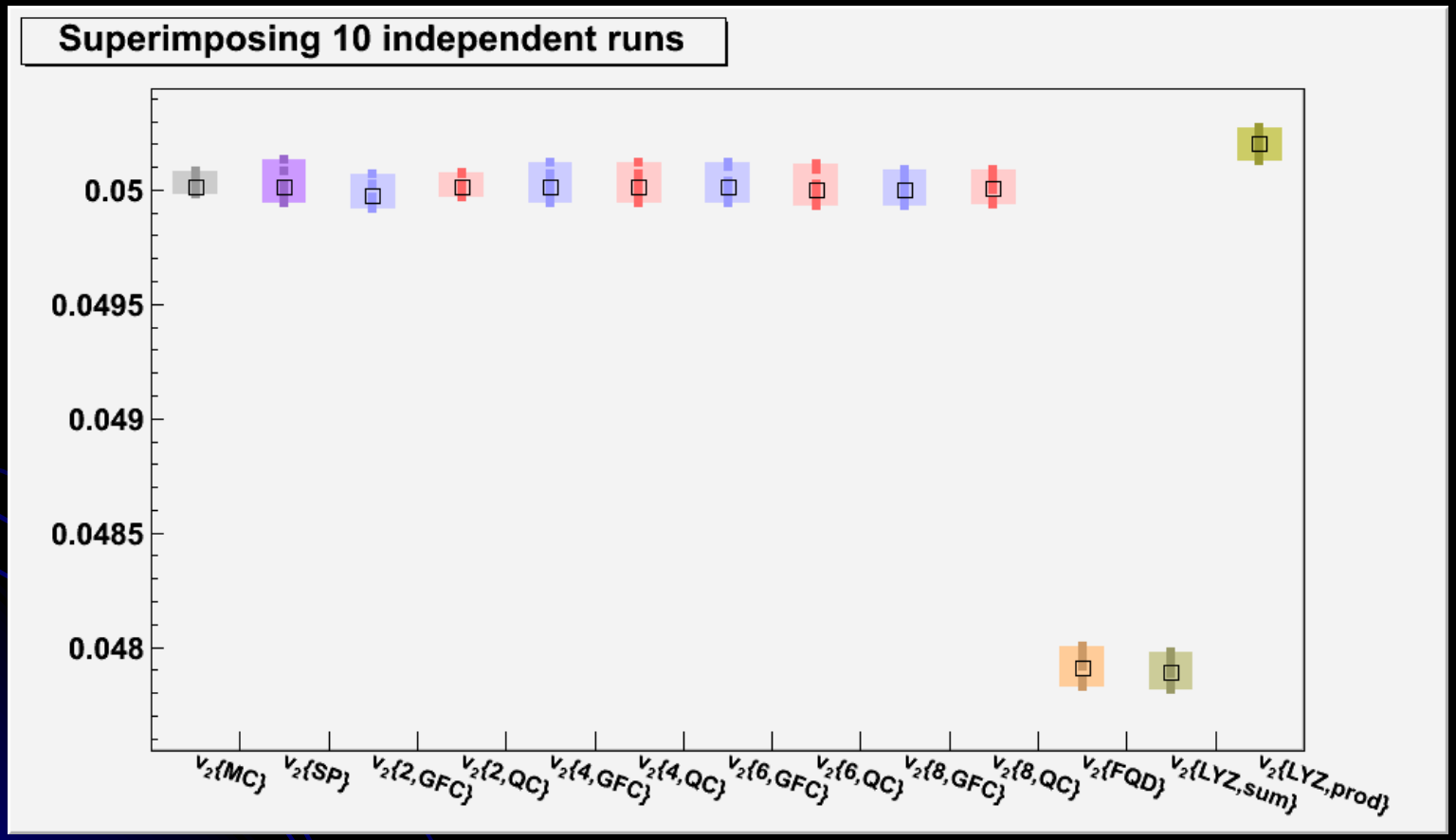
$$\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle = v_n^2+\delta_2$$
$$\left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = v_n^4+4v_n^2\delta_2+2\delta_2^2$$

- By definition, for detectors with uniform acceptance 2nd and 4th order cumulant are given by

$$\begin{aligned} c_n\{2\} &\equiv \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle = v_n^2+\delta_2 \\ c_n\{4\} &\equiv \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle - 2\left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle^2 \\ &= v_n^4+4v_n^2\delta_2+2\delta_2^2-2(v_n^2+\delta_2)^2 \\ &= -v_n^4 \end{aligned}$$

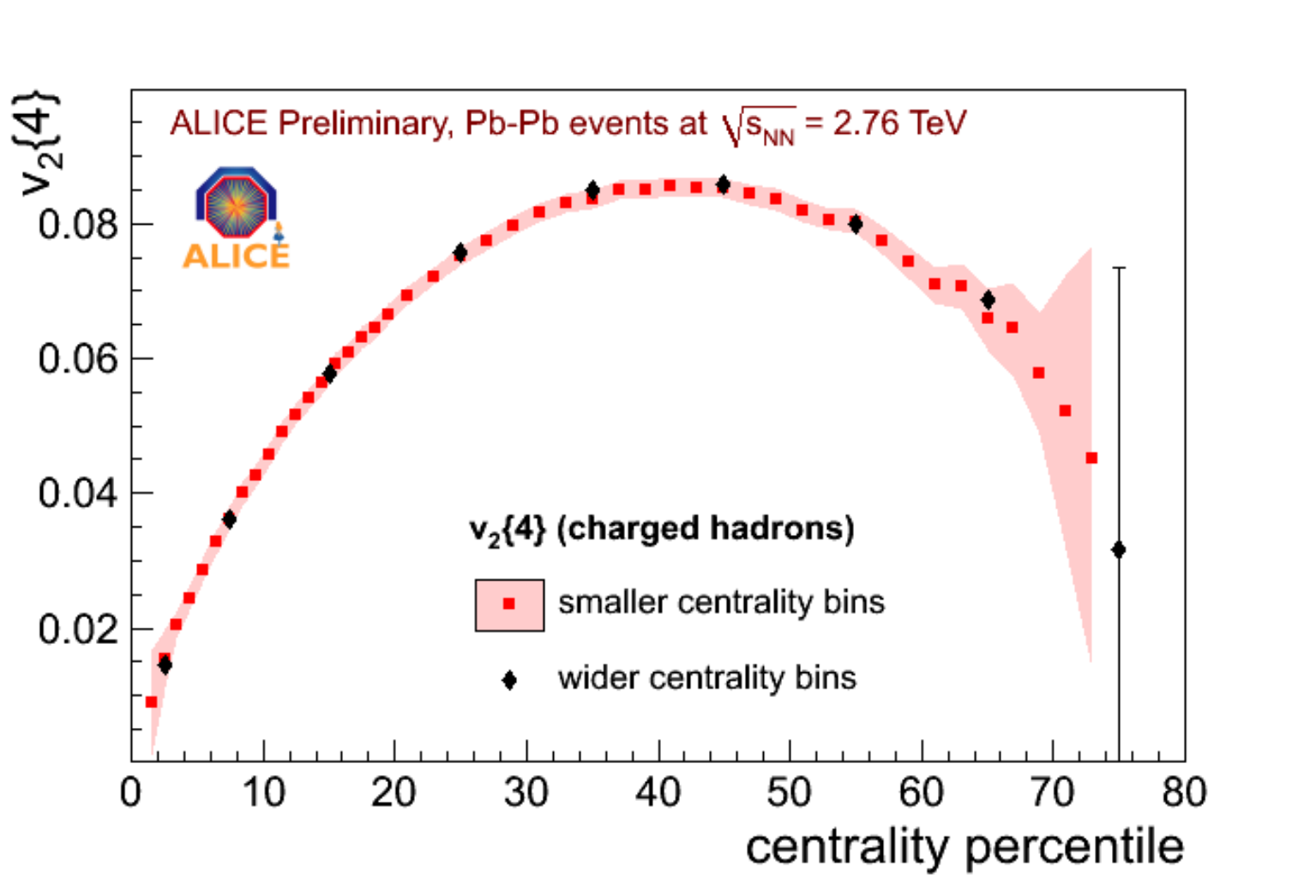
Extracting subdominant harmonic

- Example: input $v_2 = 0.05$, $v_4 = 0.10$, $M = 500$, $N = 10 \times 10^6$ and estimating subdominant harmonic v_2



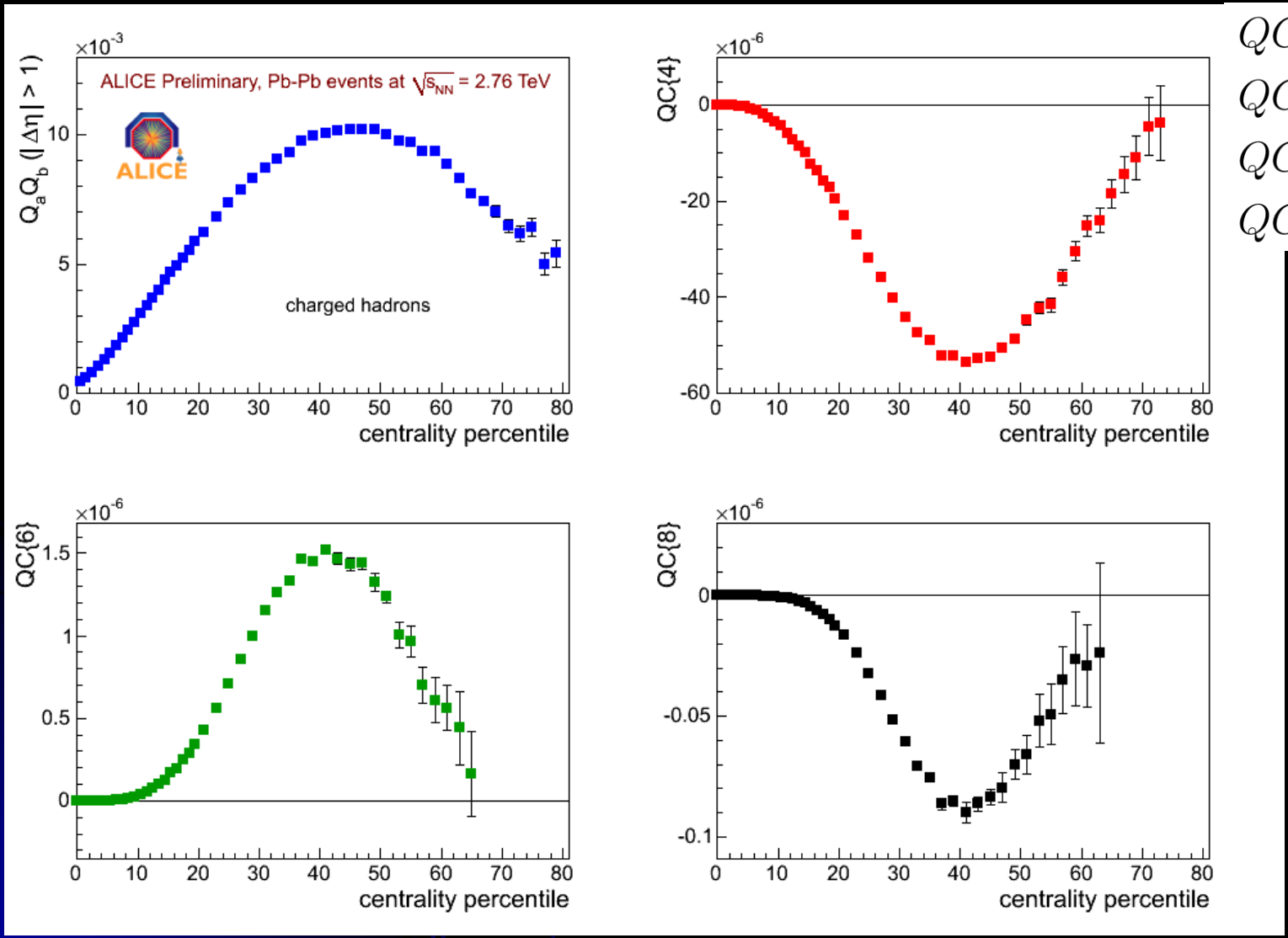
FQD and LYZ (sum) are biased and we still have to tune the LYZ product

Smaller *vs* wider centrality bins



Comparison with results for $v_2\{4\}$ in wider centrality bins => systematic bias due to various fluctuations is negligible at this scale

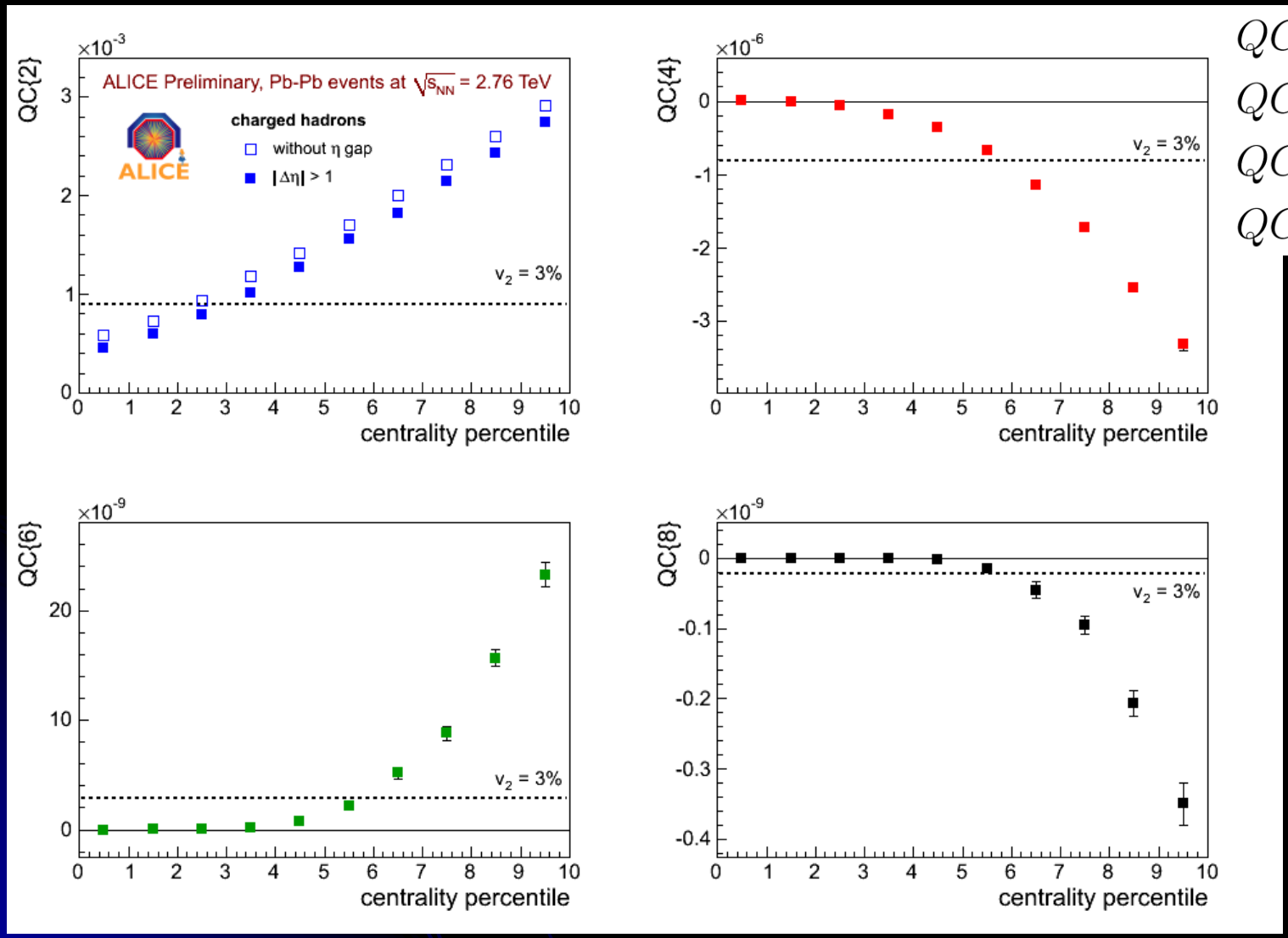
1st look at data: Cumulants



$$QC\{2\} = v^2$$
$$QC\{4\} = -v^4$$
$$QC\{6\} = 4v^6$$
$$QC\{8\} = -33v^8$$

Clear nontrivial flow signature (+,-,+,-) in the measured cumulants!

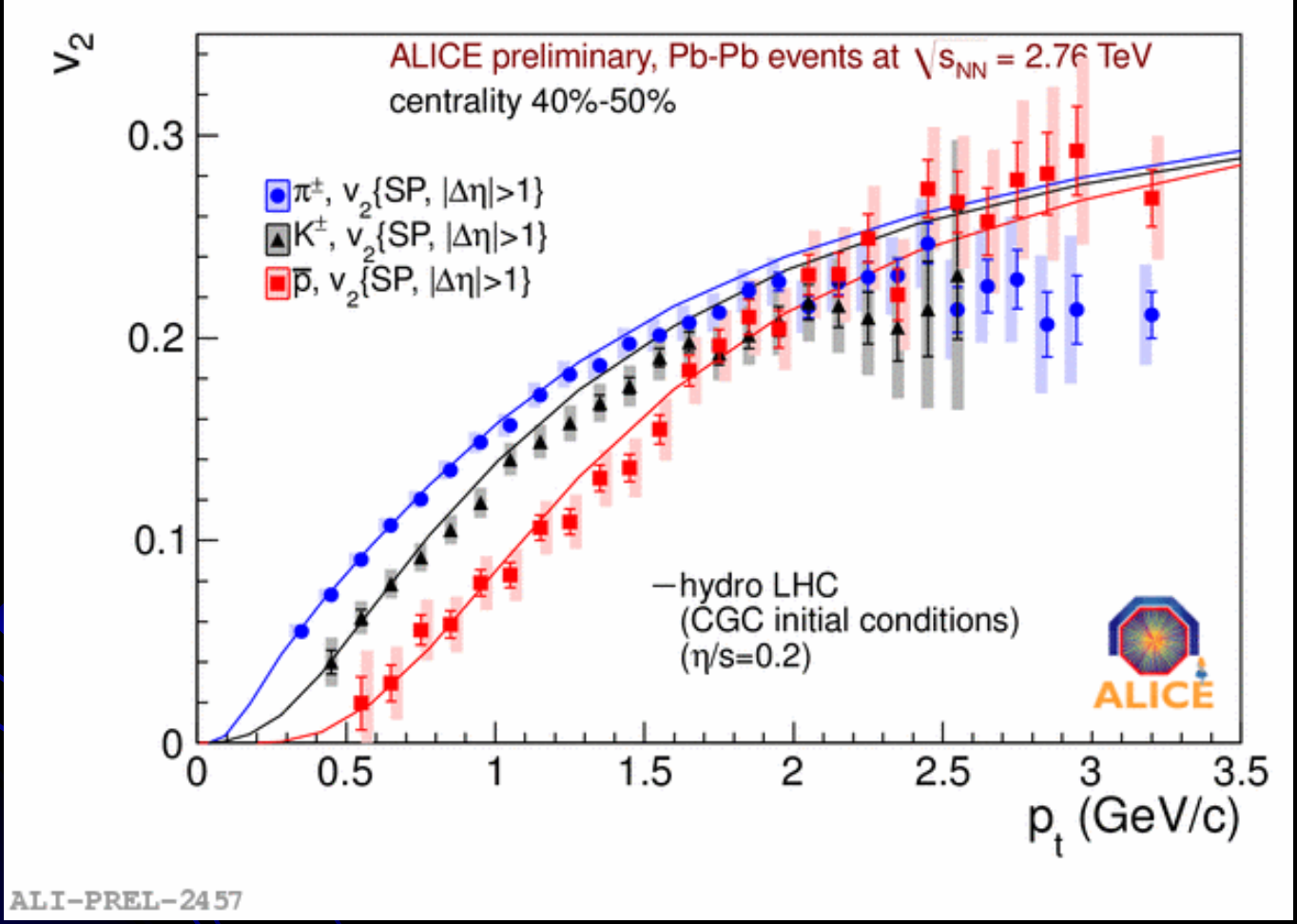
Cumulants (closer look at most central)



$$QC\{2\} = v^2$$
$$QC\{4\} = -v^4$$
$$QC\{6\} = 4v^6$$
$$QC\{8\} = -33v^8$$

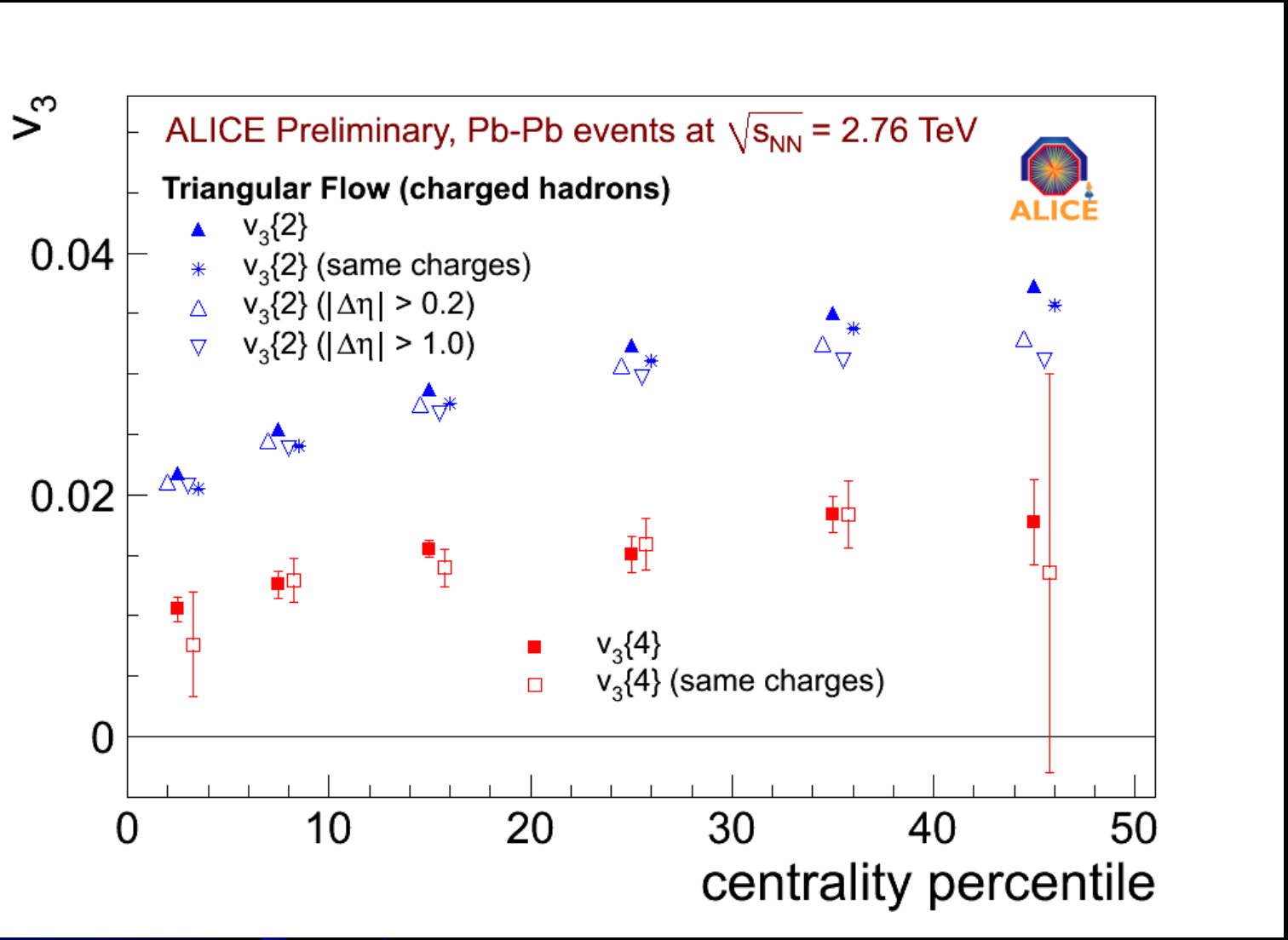
For centralities beyond 5% elliptic flow is already bigger than 3%!

Elliptic flow of identified particles



Mikolaj Krzewicki, **arXiv:1107.0080 [nucl-ex]**

Charged particle v_3



One of the ways to estimate nonflow

Fitted q-distributions

- Quick remainder: Take in each event azimuthal angles of all particles and evaluate the *Q*-vector (a.k.a. flow vector) in harmonic n

$$Q_n \equiv \sum_{i=1}^{M} e^{in\phi_i}$$

- Then evaluate event-by-event modulus of reduced flow vector q_n and fill the histogram. The resulting distribution is fitted with the known theoretical distribution in which flow harmonic v_n appears as one of the parameters

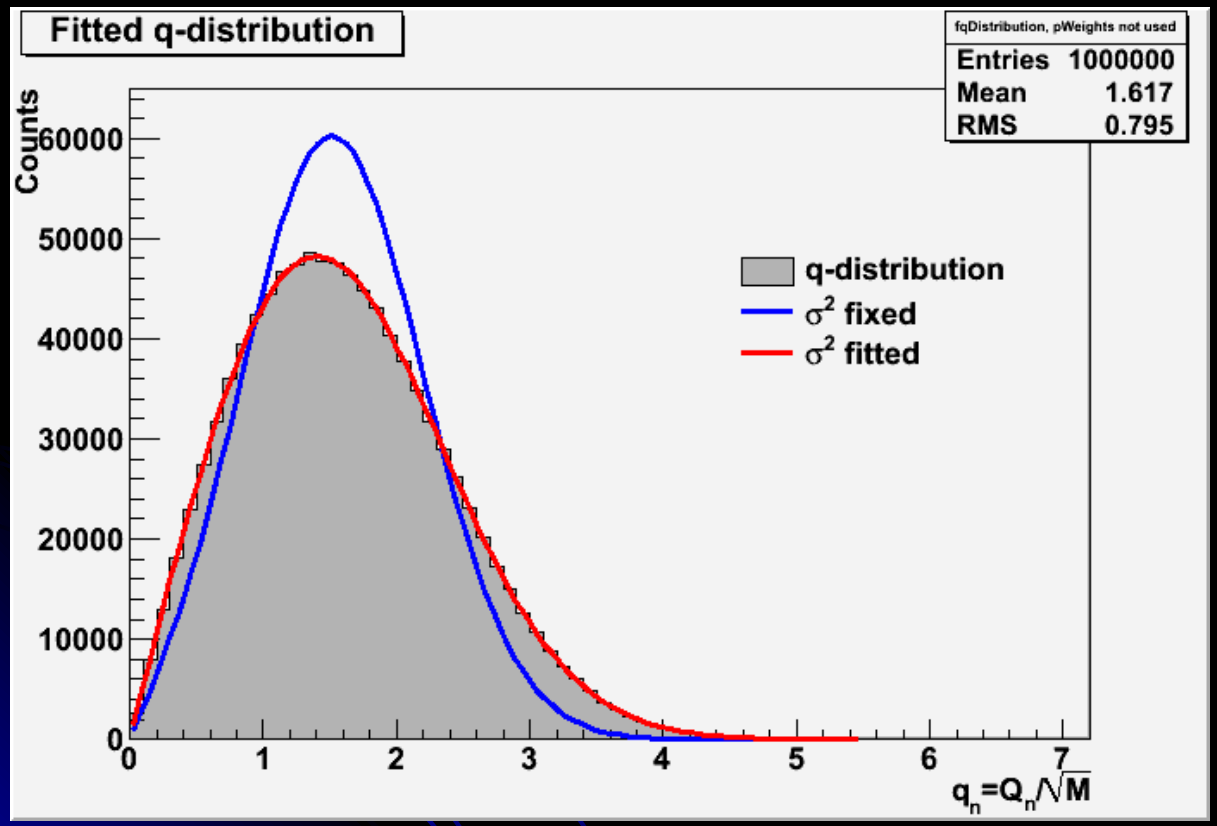
$$q_n \equiv \frac{Q_n}{\sqrt{M}}$$

$$\frac{dN}{dq_n} = \frac{q_n}{\sigma_n^2} e^{-\frac{v_n^2 M + q_n^2}{2\sigma_n^2}} I_0\left(\frac{q_n v_n \sqrt{M}}{\sigma_n^2}\right)$$

- The second fitting parameter is σ_n^2, which encapsulates the joint contribution from nonflow and flow fluctuations => see next page
- Credits: Theoretical distribution above first derived by Voloshin and Zhang => see Z.Phys.C70:665-672,1996

Fitted q-distributions

- MC example: Input v_2 = 0.05, M = 250, each particle taken twice to simulate 2-particle nonflow:



$$\sigma_n^2 \equiv \frac{1}{2}(1 + M\sigma_{\text{tot}}^2)$$

$$\sigma_{\text{tot}}^2 \equiv \delta_n + 2\sigma_{v_n}^2$$

Eqs. (20) and (21) from http://arxiv.org/abs/0809.2949

Fit works very well, but only if we also fit for σ_n^2 (red curve)! If instead we assume blindly that there is no nonflow and/or flow fluctuations, i.e. if we fix σ_n^2 to ½ (see above formulas), the fit fails miserably (blue curve)!

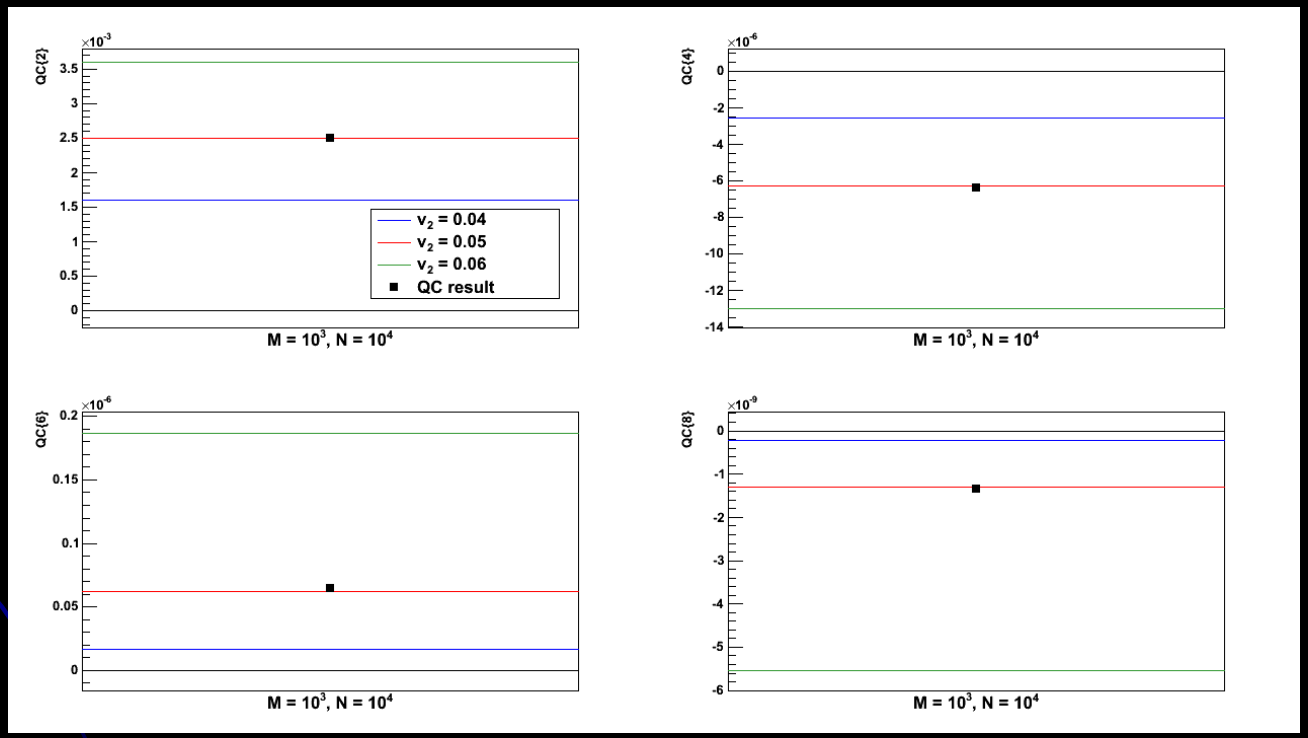
Heavy-ion regime

- For multiparticle methods the statistical error scales roughly as (k is the number of correlated particles):

$$s_k \sim \frac{1}{\sqrt{N}} \frac{1}{M^{k/2}} \frac{1}{v\{k\}^{k-1}}$$

- Example 1: heavy-ion regime ($M = 10^3$, $v = 5\%$, $N = 10^4$ = only few hours of data taking):

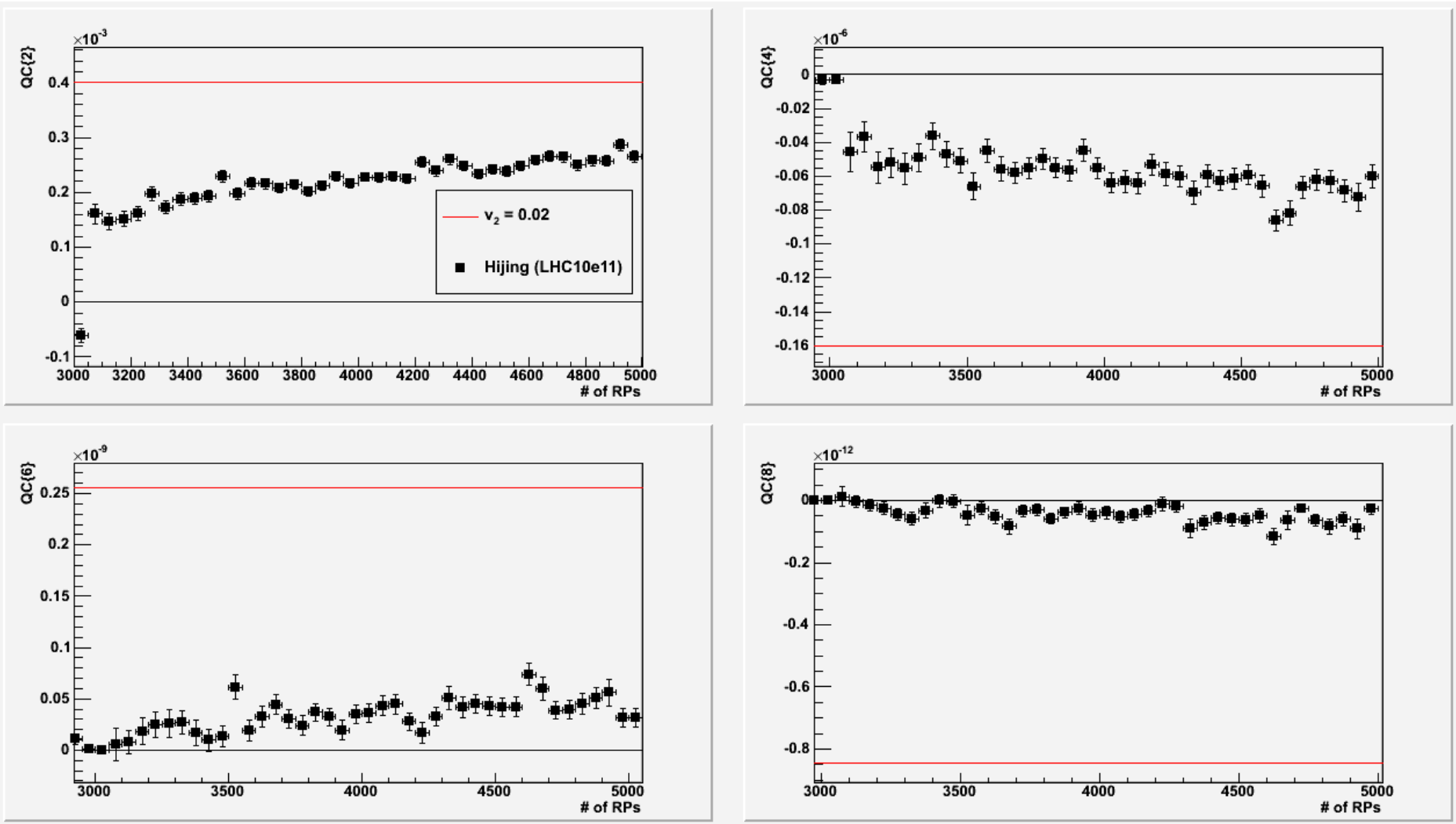
$$QC\{2\} = v^2$$
$$QC\{4\} = -v^4$$
$$QC\{6\} = 4v^6$$
$$QC\{8\} = -33v^8$$



Only 10^4 events are needed for a measurement on the scale on which flow signal appears!

Hijing central

<M> = 4121.02, N = 74617 merged 50 bins into one.



Nothing is flowing here

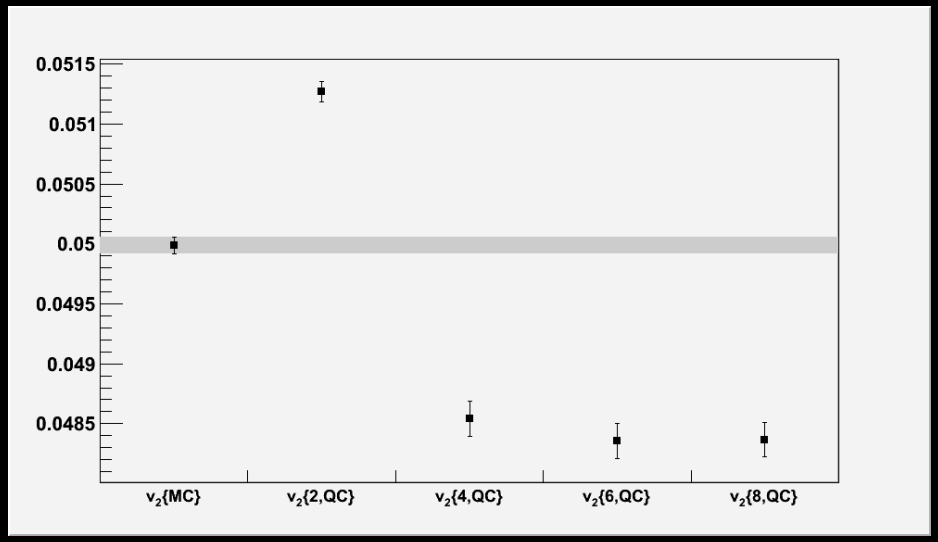
Statistical flow fluctuations

- By using multiparticle correlations to estimate flow harmonics we are actually estimating the averages of various powers of flow harmonics

$$\langle\langle 2\rangle\rangle = \langle v^2\rangle, \quad \langle\langle 6\rangle\rangle = \langle v^6\rangle$$
$$\langle\langle 4\rangle\rangle = \langle v^4\rangle, \quad \langle\langle 8\rangle\rangle = \langle v^8\rangle$$

- But what we are after is really $\langle v\rangle$
- With the fairly general assumption that $\sigma_v \ll \langle v\rangle$ and by working up to σ_v^2

$$v\{2\} = \langle v\rangle + \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$
$$v\{4\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$
$$v\{6\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$
$$v\{8\} = \langle v\rangle - \frac{1}{2}\frac{\sigma_v^2}{\langle v\rangle}$$

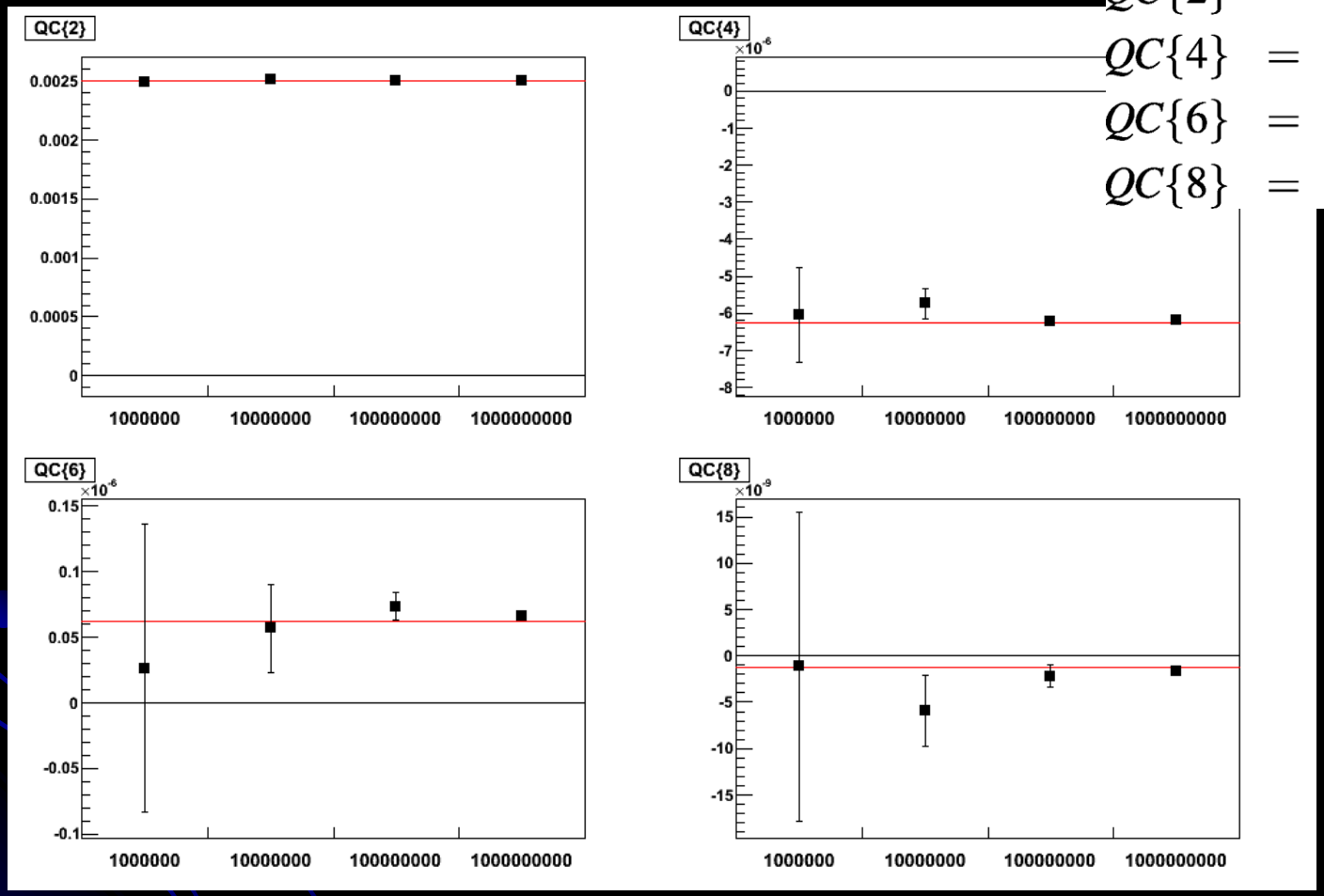


v_2 in [0.03,0.07] (uniform), $M = 500$, $N = 2 \times 10^5$

Low sensitivity for low multiplicity

- Example 2: *pp* regime ($M = [40,50]$, $v = 5\%$):

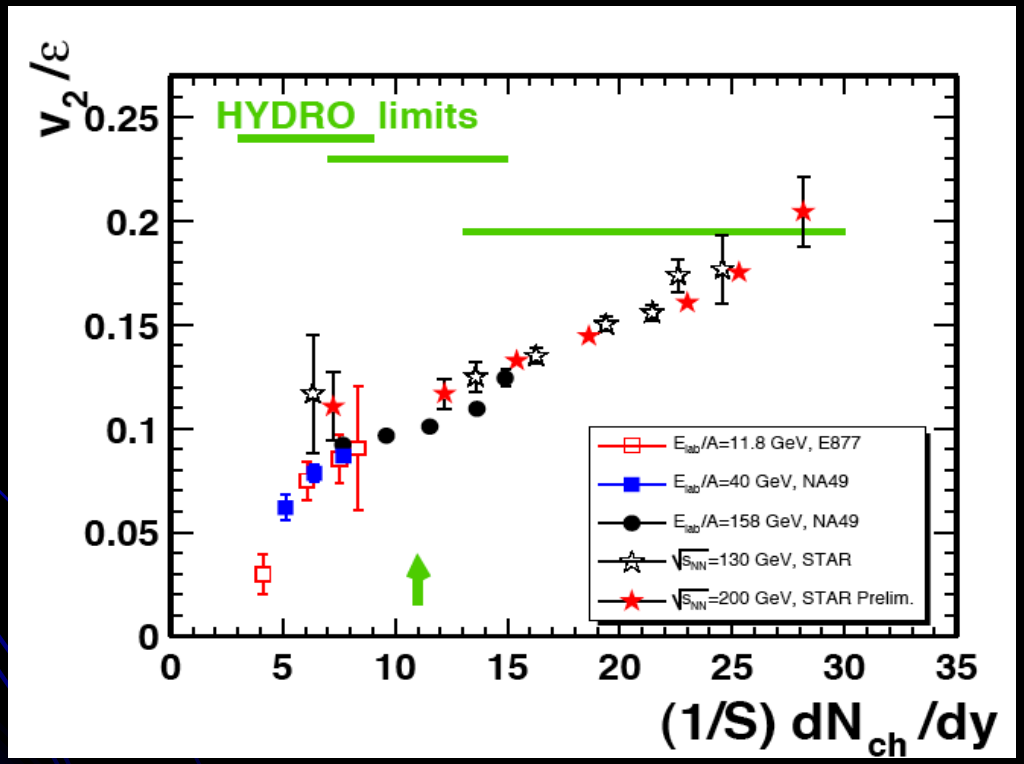
$$QC\{2\} = v^2$$
$$QC\{4\} = -v^4$$
$$QC\{6\} = 4v^6$$
$$QC\{8\} = -33v^8$$



Even in the ideal case when only flow correlations are present we need 10^9 events to measure v_2 of 5% for centrality bin $[40,50]$ independently with all cumulants up to order 8, for smaller flow values statistics is even larger

Anisotropic flow (th)

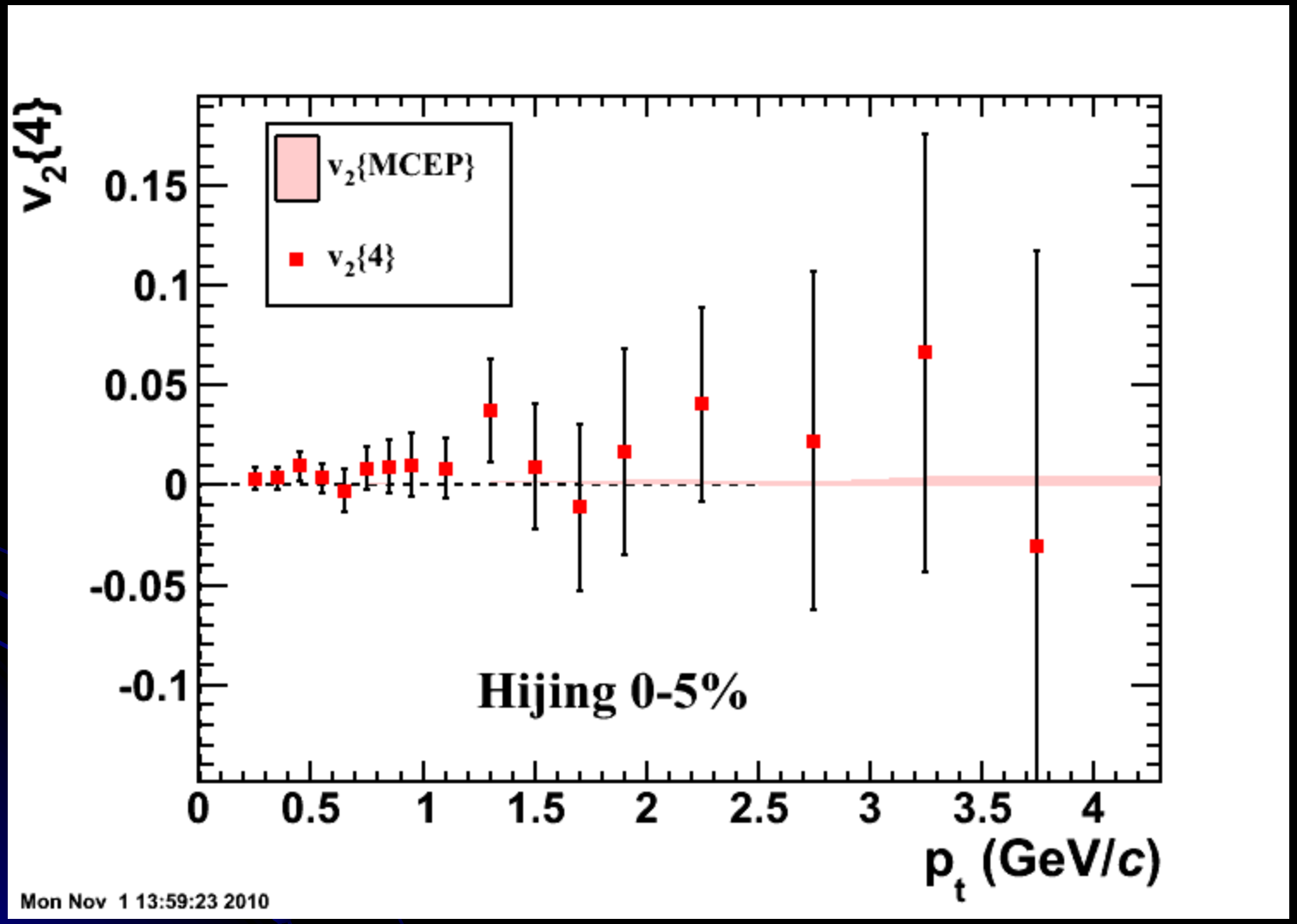
- Why theorists are interested in anisotropic flow? For instance harmonic v_2 (so-called elliptic flow) is sensitive to the equation of state of produced matter in heavy-ion collisions.
  - In particular v_2 can be used to discriminate if the quark-gluon plasma is a perfect liquid



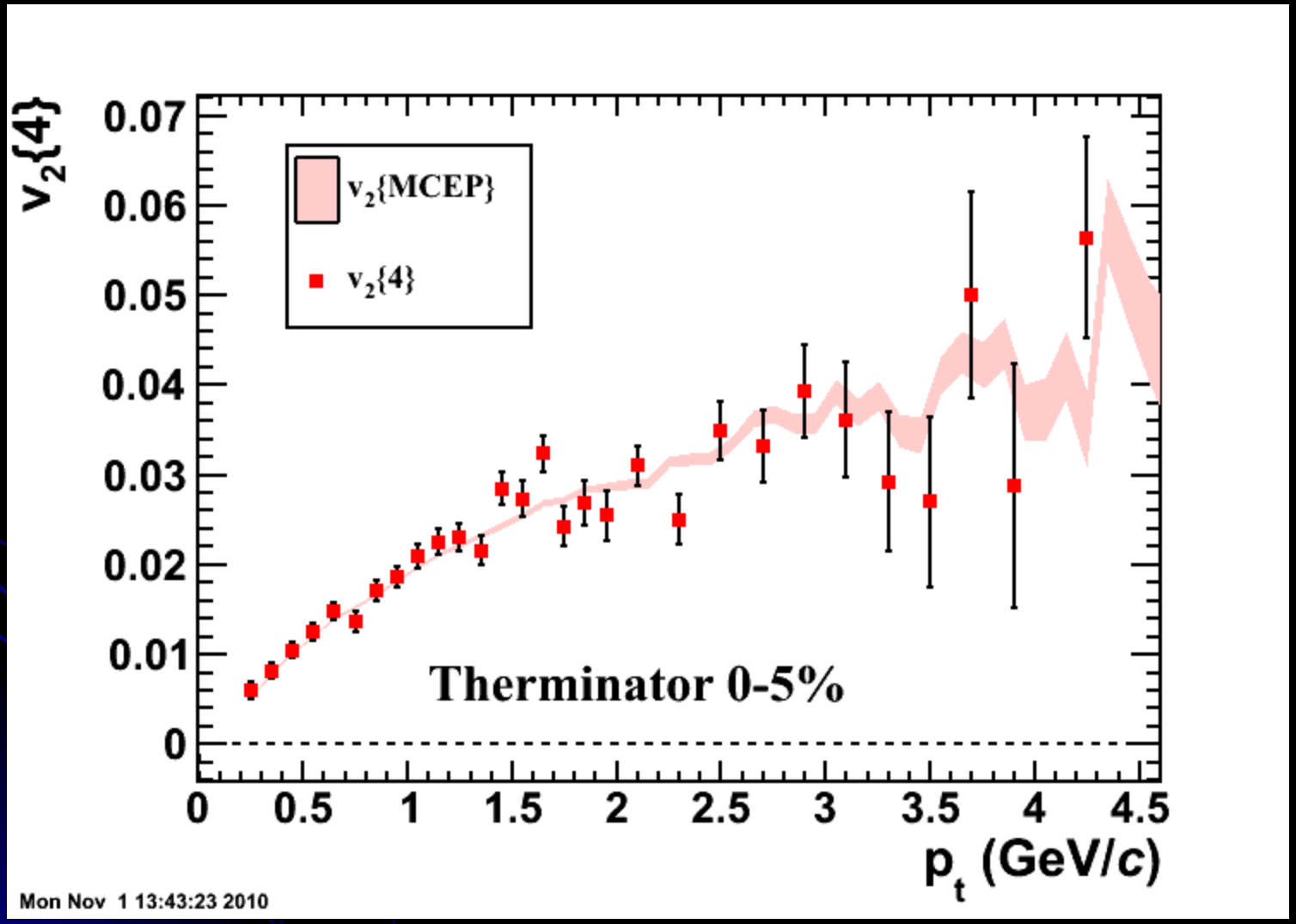
Conclusions/Summary

- Integrated elliptic flow at LHC energies 30% larger than at RHIC - increase compatible with estimates from hydrodynamic models
  - p_t dependence of elliptic flow at LHC energies close to the one at RHIC energies => increase of 30% in integrated flow is due to increase in radial flow
- Elliptic flow for centralities beyond 5% is already larger than 3% (much larger than the value of any other measured harmonic in any other centrality)
  - $v_2\{4\}$ peaks at ~ 8.5% in midcentral collisions at LHC
- Triangular flow is significant and its centrality dependence is in agreement with hydro model predictions
  - Each harmonic has its own participant plane along which it develops
  - Participant plane of v_2 is not the participant plane of v_3
  - $v_3\{4\}$ about two times smaller than $v_3\{2\}$ => v_3 originates predominantly from event by event fluctuations of the initial spatial geometry (Ollitrault *et al*, arXiv:1104.4740)
- Hydrodynamic prediction with Glauber initial conditions (Schenke *et al*, arXiv:1102.0575) does not describe well simultaneously the details of p_t dependence of v_2 and v_3

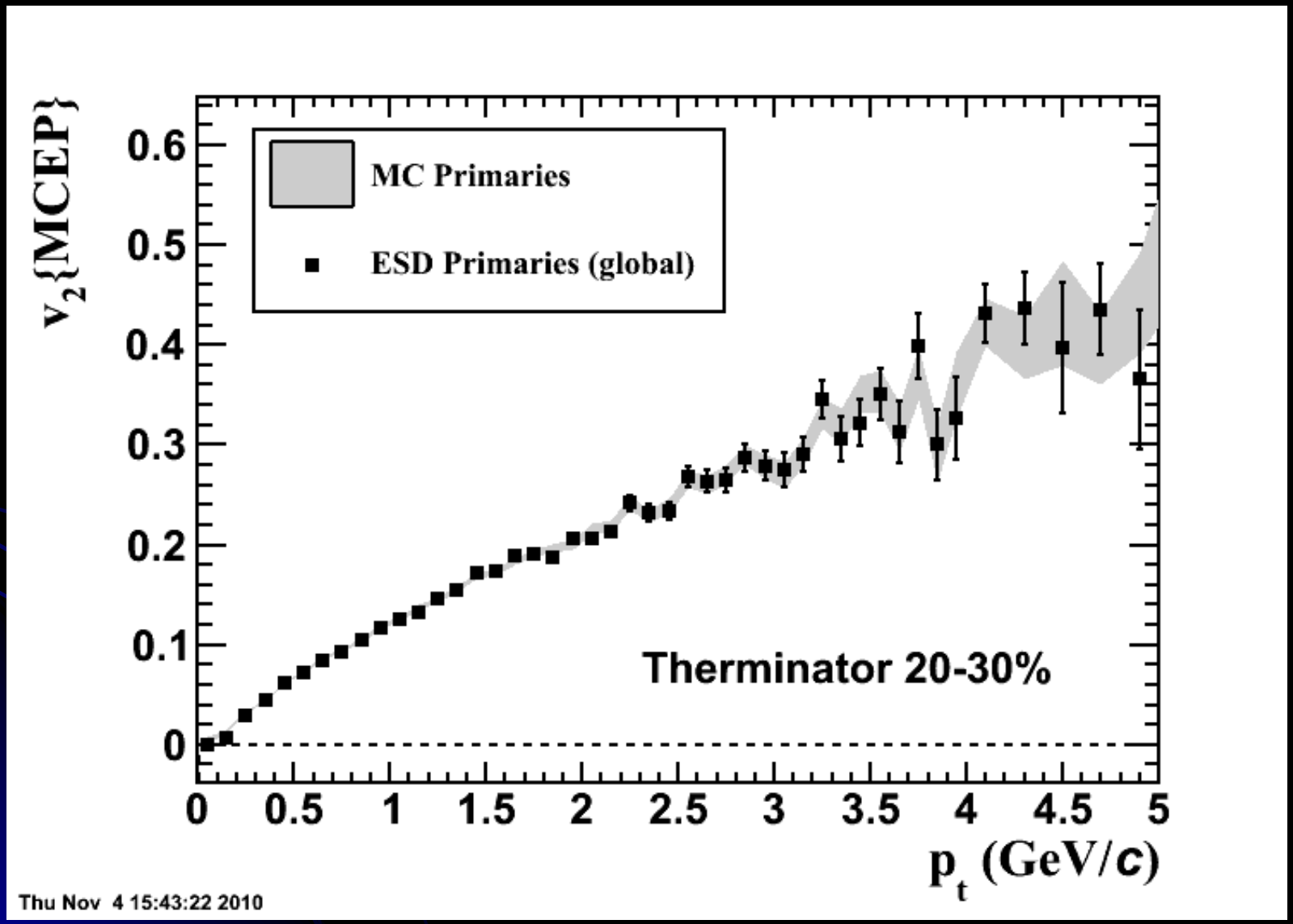
Very small flow (1/2)



Very small flow (2/2)



Reference: MC primaries (1/2)



Reference: MC primaries (2/2)