Exercises: Mellin transforms and asymptotic expansions

Consider the integral:

\[ I(\lambda) = \int_0^\infty \frac{e^{-1/t}}{t(1+\lambda t)} \, dt \]

Show that \( I \) has an asymptotic expansion as \( \lambda \to \infty \) which is

\[ I(\lambda) = \sum_{n=0}^\infty (-1)^n \Gamma(n+1) \lambda^{-1-n} \]

Hint: Use the Parseval formula for Mellin-transforms and the change of contour as described in the appendix of Hughes, Random walks and random environments, vol. 1.