

# $\gamma\gamma$ Interactions from Real to Virtual Photons<sup>1</sup>

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## Abstract

A ‘complete’ framework for  $\gamma\gamma/\gamma^*\gamma/\gamma^*\gamma^*$  interactions is presented. The emphasis is on providing a model for  $\gamma\gamma$  physics at all photon virtualities, including the difficult transition region  $Q^2 \sim m_\rho^2$ .

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$\gamma\gamma$  physics is interesting in its own right, because of the challenges it offers to our understanding of complex QCD-related phenomena. In addition,  $\gamma\gamma$  events may offer a significant background to other kinds of physics studies, such as SUSY searches. This note summarizes the model recently presented in [1]. It starts from the model for real photons in [2], but further develops this model and extends it also to encompass the physics of virtual photons. The physics has been implemented in the PYTHIA generator [3], so that complete events can be studied under realistic conditions.

A first step in a calculation is the flux of incoming bremsstrahlung photons. Here a machinery has been set up for the flux of transverse and longitudinal photons as a function of the photon virtualities  $Q_1^2$  and  $Q_2^2$  and their longitudinal momentum fractions  $y_1$  and

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$y_2$ . Assuming isotropic azimuthal angles,  $W^2$  can be calculated. The user can specify cuts on the range of these variables, in order to restrict the generation to interesting events. The flux is convoluted with  $\gamma^*\gamma^*$  cross sections dependent on  $Q_1^2$ ,  $Q_2^2$ ,  $W^2$  and the polarization states. Beamstrahlung involves only real photons, and is thereby considerably simpler. With the flux of such photons given by some external program, already the older machinery can cover this case.

Photon interactions are complicated since the photon wave function contains so many components, each with its own interactions. To first approximation, it may be subdivided into a direct and a resolved part. (In higher orders, the two parts can mix, so one has to provide sensible physical separations between the two.) In the former the photon acts as a pointlike particle, while in the latter it fluctuates into hadronic states. These fluctuations are of  $\mathcal{O}(\alpha_{\text{em}})$ , and so correspond to a small fraction of the photon wave function, but this is compensated by the bigger cross sections allowed in strong-interaction processes. For real photons therefore the resolved processes dominate the total cross section, while the pointlike ones take over for virtual photons.

The fluctuations  $\gamma \rightarrow q\bar{q} (\rightarrow \gamma)$  can be characterized by the transverse momentum  $k_\perp$  of the quarks, or alternatively by some mass scale  $m \simeq 2k_\perp$ , with a spectrum of fluctuations  $\propto dk_\perp^2/k_\perp^2$ . The low- $k_\perp$  part cannot be calculated perturbatively, but is instead parameterized by experimentally determined couplings to the lowest-lying vector mesons,  $V = \rho^0, \omega^0, \phi^0$  and  $J/\psi$ , an ansatz called VMD for Vector Meson Dominance. Parton distributions are defined with a unit momentum sum rule within a fluctuation [4], giving rise to total hadronic cross sections, jet activity, multiple interactions and beam remnants as in hadronic interactions. In interactions with a hadron or another resolved photon, jet production occurs by typical parton-scattering processes such as  $qq' \rightarrow qq'$  or  $gg \rightarrow gg$ .

States at larger  $k_\perp$  are called GVMD or Generalized VMD, and their contributions to the parton distribution of the photon are called anomalous. Given a dividing line  $k_0 \simeq 0.5$  GeV to VMD states, the anomalous parton distributions are perturbatively calculable. The total cross section of a state is not, however, since this involves aspects of soft physics and eikonalization of jet rates. Therefore an ansatz is chosen where the total cross section of a state scales like  $k_V^2/k_\perp^2$ , where the adjustable parameter  $k_V \approx m_\rho/2$  for light quarks. The spectrum of GVMD states is taken to extend over a range  $k_0 < k_\perp < k_1$ , where  $k_1$  is identified with the  $p_{\perp\text{min}}(s)$  cut-off of the perturbative jet spectrum in hadronic interactions,  $p_{\perp\text{min}}(s) \approx 1.5$  GeV at typical energies [3]. Above that range, the states are assumed to be sufficiently weakly interacting that no eikonalization procedure is required, so that cross sections can be calculated perturbatively without any recourse to Pomeron phenomenology. There is some arbitrariness in that choice, and some simplifications are required in order to obtain a manageable description.

A real direct photon in a  $\gamma p$  collision can interact with the parton content of the proton:  $\gamma q \rightarrow qg$  (QCD Compton) and  $\gamma g \rightarrow q\bar{q}$  (Boson Gluon Fusion). The  $p_\perp$  in this collision is taken to exceed  $k_1$ , in order to avoid double-counting with the interactions of the GVMD states. In  $\gamma\gamma$ , the equivalent situation is called single-resolved (or direct $\times$ resolved), where a direct photon interacts with the partonic component of the other, resolved photon. The  $\gamma\gamma$  direct (or direct $\times$ direct) process  $\gamma\gamma \rightarrow q\bar{q}$  has no correspondence in  $\gamma p$ .

The space of  $\gamma\gamma$  processes thus is three-dimensional, with axes given by the  $k_{\perp 1}$ ,  $k_{\perp 2}$  and  $p_\perp$  scales. Here each  $k_{\perp i}$  is a measure of the virtuality of a fluctuation of a photon, and  $p_\perp$  corresponds to the most virtual rung on the ladder between the two photons, possibly excepting the endpoint  $k_{\perp i}$  ones. So, to first approximation, the coordinates along the

$k_{\perp i}$  axes determine the characters of the interacting photons while  $p_{\perp}$  determines the character of the interaction process. Double counting should be avoided by trying to impose a consistent classification. Thus, for instance,  $p_{\perp} > k_{\perp i}$  with  $k_{\perp 1} < k_0$  and  $k_0 < k_{\perp 2} < k_1$  gives a hard interaction between a VMD and a GVMD photon, while  $k_{\perp 1} > p_{\perp} > k_{\perp 2}$  with  $k_{\perp 1} > k_1$  and  $k_{\perp 2} < k_0$  is a single-resolved process (direct $\times$ VMD; with  $p_{\perp}$  now in the parton distribution evolution).

If the photon is virtual, it has a reduced probability to fluctuate into a vector meson state, and this state has a reduced interaction probability. This can be modelled by a traditional dipole factor  $(m_V^2/(m_V^2 + Q^2))^2$  for a photon of virtuality  $Q^2$ , where  $m_V \rightarrow 2k_{\perp}$  for a GVMD state. Putting it all together, the cross section of the GVMD sector then scales like

$$\int_{k_0^2}^{k_1^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \frac{k_V^2}{k_{\perp}^2} \left( \frac{4k_{\perp}^2}{4k_{\perp}^2 + Q^2} \right)^2. \quad (1)$$

For a virtual photon the DIS process  $\gamma^* q \rightarrow q$  is also possible, but by gauge invariance its cross section must vanish in the limit  $Q^2 \rightarrow 0$ . At large  $Q^2$ , the single-resolved processes can be considered as the  $\mathcal{O}(\alpha_s)$  correction to the lowest-order DIS process, but the single-resolved ones survive for  $Q^2 \rightarrow 0$ . There is no unique prescription for a proper combination at all  $Q^2$ , but we have attempted an approach that gives the proper limits and minimizes double-counting. For large  $Q^2$ , the DIS  $\gamma^* \gamma$  cross section is proportional to the structure function  $F_2^{\gamma}(x, Q^2)$  with the Bjorken  $x = Q^2/(Q^2 + W^2)$ . Since normal parton distribution parameterizations are frozen below some  $Q_0$  scale and therefore do not obey the gauge invariance condition, an ad hoc factor  $(Q^2/(Q^2 + m_{\rho}^2))^2$  is introduced for the conversion from the parameterized  $F_2^{\gamma}(x, Q^2)$  to a  $\sigma_{\text{DIS}}^{\gamma^* \gamma}$ :

$$\sigma_{\text{DIS}}^{\gamma^* \gamma} \simeq \left( \frac{Q^2}{Q^2 + m_{\rho}^2} \right)^2 \frac{4\pi^2 \alpha_{\text{em}}}{Q^2} F_2^{\gamma}(x, Q^2) = \frac{4\pi^2 \alpha_{\text{em}} Q^2}{(Q^2 + m_{\rho}^2)^2} \sum_{q, \bar{q}} e_q^2 x q(x, Q^2). \quad (2)$$

Here  $m_{\rho}$  is some non-perturbative hadronic mass parameter, for simplicity identified with the  $\rho$  mass.

In order to avoid double-counting between DIS and single-resolved events, a requirement  $p_{\perp} > \max(k_{\perp 1}, Q)$  is imposed on the latter. In the remaining DIS ones, denoted lowest order (LO) DIS, thus  $p_{\perp} < Q$ . This would suggest a subdivision  $\sigma_{\text{LO DIS}}^{\gamma^* \gamma} = \sigma_{\text{DIS}}^{\gamma^* \gamma} - \sigma_{1\text{-res}}^{\gamma^* \gamma}$ , with  $\sigma_{\text{DIS}}^{\gamma^* \gamma}$  given by eq. (2) and  $\sigma_{1\text{-res}}^{\gamma^* \gamma}$  by the perturbative matrix elements. In the limit  $Q^2 \rightarrow 0$ , the DIS cross section is now constructed to vanish while the direct is not, so this would suggest  $\sigma_{\text{LO DIS}}^{\gamma^* \gamma} < 0$ . However, here we expect the correct answer not to be a negative number but an exponentially suppressed one, by a Sudakov form factor. This modifies the cross section:

$$\sigma_{\text{LO DIS}}^{\gamma^* \gamma} = \sigma_{\text{DIS}}^{\gamma^* \gamma} - \sigma_{1\text{-res}}^{\gamma^* \gamma} \longrightarrow \sigma_{\text{DIS}}^{\gamma^* \gamma} \exp\left(-\frac{\sigma_{1\text{-res}}^{\gamma^* \gamma}}{\sigma_{\text{DIS}}^{\gamma^* \gamma}}\right). \quad (3)$$

Since we here are in a region where the DIS cross section is no longer the dominant one, this change of the total DIS cross section is not essential.

The space of  $\gamma^* \gamma^*$  processes is now five-dimensional:  $Q_1, Q_2, k_{\perp 1}, k_{\perp 2}$  and  $p_{\perp}$ . As before, an effort is made to avoid double-counting, by having a unique classification of each region in the five-dimensional space. Some double-counting remain in the region of large  $x \approx Q^2/(Q^2 + W^2)$ , where an ad hoc suppression factor  $(1 - x)^3$  is introduced for the resolved components.

In total, our ansatz for  $\gamma^*\gamma^*$  interactions at all  $Q^2$  contains 13 components: 9 when two VMD, GVMD or direct photons interact, as is already allowed for real photons, plus a further 4 where a ‘DIS photon’ from either side interacts with a VMD or GVMD one. With the label resolved used to denote VMD and GVMD, one can write

$$\begin{aligned}
\sigma_{\text{tot}}^{\gamma^*\gamma^*}(W^2, Q_1^2, Q_2^2) &= \sigma_{\text{DIS}\times\text{res}}^{\gamma^*\gamma^*} \exp\left(-\frac{\sigma_{\text{dir}\times\text{res}}^{\gamma^*\gamma^*}}{\sigma_{\text{DIS}\times\text{res}}^{\gamma^*\gamma^*}}\right) + \sigma_{\text{dir}\times\text{res}}^{\gamma^*\gamma^*} \\
&+ \sigma_{\text{res}\times\text{DIS}}^{\gamma^*\gamma^*} \exp\left(-\frac{\sigma_{\text{res}\times\text{dir}}^{\gamma^*\gamma^*}}{\sigma_{\text{res}\times\text{DIS}}^{\gamma^*\gamma^*}}\right) + \sigma_{\text{res}\times\text{dir}}^{\gamma^*\gamma^*} \\
&+ \sigma_{\text{dir}\times\text{dir}}^{\gamma^*\gamma^*} + \left(\frac{W^2}{Q_1^2 + Q_2^2 + W^2}\right)^3 \sigma_{\text{res}\times\text{res}}^{\gamma^*\gamma^*}
\end{aligned} \tag{4}$$

Most of the 13 components in their turn have a complicated internal structure, as we have seen.

An important note is that the  $Q^2$  dependence of the DIS and direct photon interactions is implemented in the matrix element expressions, i.e. in processes such as  $\gamma^*\gamma^* \rightarrow q\bar{q}$  or  $\gamma^*q \rightarrow qg$  the photon virtuality explicitly enters. This is different from VMD/GVMD, where dipole factors are used to reduce the total cross sections and the assumed flux of partons inside a virtual photon relative to those of a real one, but the matrix elements themselves contain no dependence on the virtuality either of the partons or of the photon itself. Typically results are obtained with the SaS 1D PDF’s for the virtual transverse photons [4], since these are well matched to our framework, e.g. allowing a separation of the VMD and GVMD/anomalous components. Parton distributions of virtual longitudinal photons are by default given by some  $Q^2$ -dependent factor times the transverse ones. The new set by Chýla [5] allows more precise modelling here, but first indications are that many studies will not be sensitive to the detailed shape.

Some first studies with this new framework look promising [6], but much further work remains to assess its usefulness.

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