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NEW SHOWERS WITH TRANSVERSE-MOMENTUM-ORDERING*

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1. INTRODUCTION

The initial- [1, 2, 3] and final-state [4, 5] showers in the PYTHIA event generator [6, 7] are based on virtuality-ordering, i.e. uses spacelike Q^2 and timelike M^2, respectively, as evolution variables. Other algorithms in common use are the angular-ordered ones in HERWIG [8, 9] and the $p_\perp$-ordered dipole-based ones in ARIADNE/LDC [10, 11]. All three have been comparably successful, in terms of ability to predict or describe data, and therefore have offered useful cross-checks. Some shortcomings of the virtuality-ordering approach, with respect to coherence conditions, have been compensated (especially relative to HERWIG) by a better coverage of phase space and more efficient possibilities to merge smoothly with first-order matrix elements.

Recently, the possibility to combine matrix elements of several orders consistently with showers has been raised [12, 13], e.g. $\mathrm{W} + n$ jets, $n = 0, 1, 2, 3, \ldots$. In such cases, a $p_\perp$-ordering presumably offers the best chance to provide a sensible definition of hardness. It may also tie in better e.g. with the $p_\perp$-ordered approach to multiple interactions [14]. This note therefore is a study of how the existing PYTHIA algorithms can be reformulated in $p_\perp$-ordered terms, while retaining their strong points.

The main trick that will be employed is to pick formal definitions of $p_\perp$, that simply and unambiguously can be translated into the older virtuality variables, e.g. for standard matrix-element merging. These definitions are based on lightcone kinematics, wherein a timelike branching into two massless daughters corresponds to $p_\perp^2 = z(1-z)M^2$ and the branching of a massless mother into a spacelike and a massless daughter to $p_\perp^2 = (1-z)Q^2$. The actual $p_\perp$ of a branching will be different, and e.g. depend on the subsequent shower history, but should normally not deviate by much.

2. TIMELIKE SHOWERS

The new timelike algorithm is a hybrid between the traditional parton-shower and dipole-emission approaches, in the sense that the branching process is associated with the evolution of a single parton, like in a shower, but recoil effects occur inside dipoles. That is, a dipole partner is assigned for each branching, and energy and momentum is 'borrowed' from this partner to give mass to the parton about to branch, while preserving the invariant mass of the dipole. (Thus four-momentum is not preserved locally for each parton branching $a \to bc$. It was in the old algorithm, where the kinematics of a branching was not constructed before the off- or on-shell daughter masses had been found.) Often the two partners are colour-connected, i.e. the colour of one matches the anticolour of the other, as defined by the preceding showering history, but this need not be the case. In particular, intermediate resonances normally have masses that should be preserved by the shower, e.g., in $\mathrm{t} \to \mathrm{bW}^+$ the W^+ takes the recoil when the b radiates a gluon.

The evolution variable is approximately the $p_\perp^2$ of a branching, where $p_\perp$ is the transverse momentum for each of the two daughters with respect to the direction of the mother, in the rest frame of

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the dipole. (The recoiling dipole partner does not obtain any $p_\perp$ kick in this frame; only its longitudinal momentum is affected.) For the simple case of massless radiating partons and small virtualities relative to the kinematically possible ones, and in the limit that recoil effects from further emissions can be neglected, it agrees with the d_{ij} $p_\perp$-clustering distance defined in the PYCLUS algorithm [15].

All emissions are ordered in a single sequence $p_{\perp\mathrm{max}} > p_{\perp 1} > p_{\perp 2} > \ldots > p_{\perp\mathrm{min}}$. That is, each initial parton is evolved from the input $p_{\perp\mathrm{max}}$ scale downwards, and a hypothetical branching $p_\perp$ is thereby found for it. The one with the largest $p_\perp$ is chosen to undergo the first actual branching. Thereafter, all partons now existing are evolved downwards from $p_{\perp 1}$, and a $p_{\perp 2}$ is chosen, and so on, until $p_{\perp\mathrm{min}}$ is reached. (Technically, the $p_\perp$ values for partons not directly or indirectly affected by a branching need not be reselected.) The evolution of a gluon is split in evolution on two separate sides, with half the branching kernel each, but with different kinematical constraints since the two dipoles have different masses. The evolution of a quark is also split, into one $p_\perp$ scale for gluon emission and one for photon one, in general corresponding to different dipoles.

With the choices above, the evolution factorizes. That is, a set of successive calls, where the $p_{\perp\mathrm{min}}$ of one call becomes the $p_{\perp\mathrm{max}}$ of the next, gives the same result (on the average) as one single call for the full $p_\perp$ range. This is the key element to allow Sudakovs to be conveniently obtained from trial showers [13], and to veto emissions above some $p_\perp$ scale, as required to combine different n-parton configurations efficiently.

The formal $p_\perp$ definition is $p_{\perp\mathrm{evol}}^2 = z(1-z)(M^2 - m_0^2)$, where $p_{\perp\mathrm{evol}}$ is the evolution variable, z gives the energy sharing in the branching, as selected from the branching kernels, M is the off-shell mass of the branching parton and m_0 its on-shell value. This $p_{\perp\mathrm{evol}}$ is also used as α_s scale.

When a $p_{\perp\mathrm{evol}}$ has been selected, this is translated to a $M^2 = m_0^2 + p_{\perp\mathrm{evol}}^2/(z(1-z))$. Note that the Jacobian factor is trivial: $\mathrm{d}M^2/(M^2 - m_0^2)\,\mathrm{d}z = \mathrm{d}p_{\perp\mathrm{evol}}^2/p_{\perp\mathrm{evol}}^2\,\mathrm{d}z$. From there on, the three-body kinematics of a branching is constructed as in the old routine. This includes the detailed interpretation of z and the related handling of nonzero on-shell masses for branching and recoiling partons, which leads to the physical $p_\perp$ not agreeing with the $p_{\perp\mathrm{evol}}$ defined here. In this sense, $p_{\perp\mathrm{evol}}$ becomes a formal variable, while M really is a well-defined mass of a parton.

Also the corrections to $b \to b\mathrm{g}$ branchings (b being a generic coloured particle) by merging with first-order $a \to bc\mathrm{g}$ matrix elements closely follows the existing machinery [5], once the $p_{\perp\mathrm{evol}}$ has been converted to a mass of the branching parton. In general, the other parton c used to define the matrix element need not be the same as the recoiling partner. To illustrate, consider a $\mathrm{Z}^0 \to \mathrm{q}\overline{\mathrm{q}}$ decay. Say the q branches first, $\mathrm{q} \to \mathrm{q}\mathrm{g}_1$. Obviously the $\overline{\mathrm{q}}$ then takes the recoil, and the new q, g_1 and $\overline{\mathrm{q}}$ momenta are used to match to the $\mathrm{Z}^0 \to \mathrm{q}\overline{\mathrm{q}}\mathrm{g}$ matrix element. The next time q branches, $\mathrm{q} \to \mathrm{q}\mathrm{g}_2$, the recoil is taken by the colour-connected g_1 gluon, but the matrix element corrections are based on the newly created q and g_2 momenta together with the $\overline{\mathrm{q}}$ (not the g_1!) momentum. That way one may expect to achieve the most realistic description of mass effects in the collinear and soft regions.

The shower inherits some further elements from the old algorithm, such as azimuthal anisotropies in gluon branchings from polarization effects.

The relevant parameters will have to be retuned, since the shower is quite different from the old mass-ordered one. In particular, it appears that the five-flavour Λ_{QCD} value has to be reduced relative to the current default, roughly by a factor of two (from 0.29 to 0.14 GeV).

3. SPACELIKE SHOWERS

Initial-state showers are constructed by backwards evolution [1], starting at the hard interaction and successively reconstructing preceding branchings. To simplify the merging with first-order matrix elements, z is defined by the ratio of $\hat{s}$ before and after an emission. For a massless parton branching into one spacelike with virtuality Q^2 and one with mass m, this gives $p_\perp^2 = Q^2 - z(\hat{s} + Q^2)(Q^2 + m^2)/\hat{s}$, or $p_\perp^2 = (1-z)Q^2 - zQ^4/\hat{s}$ for $m = 0$. Here $\hat{s}$ is the squared invariant mass after the emission, i.e.

excluding the emitted on-mass-shell parton.

The last term, $zQ^4/\hat{s}$, while normally expected to be small, gives a nontrivial relationship between $p_{\perp}^2$ and Q^2, e.g. with two possible Q^2 solutions for a given $p_{\perp}^2$. To avoid the resulting technical problems, the evolution variable is picked to be $p_{\perp\mathrm{evol}}^2 = (1-z)Q^2$. Also here $p_{\perp\mathrm{evol}}$ sets the scale for the running α_{s}. Once selected, the $p_{\perp\mathrm{evol}}^2$ is translated into an actual Q^2 by the inverse relation $Q^2 = p_{\perp\mathrm{evol}}^2/(1-z)$, with trivial Jacobian: $\mathrm{d}Q^2/Q^2 \; \mathrm{d}z = \mathrm{d}p_{\perp\mathrm{evol}}^2/p_{\perp\mathrm{evol}}^2 \; \mathrm{d}z$. From Q^2 the correct $p_{\perp}^2$, including the $zQ^4/\hat{s}$ term, can be constructed.

Emissions on the two incoming sides are interspersed to form a single falling $p_{\perp}$ sequence, $p_{\perp\mathrm{max}} > p_{\perp 1} > p_{\perp 2} > \ldots > p_{\perp\mathrm{min}}$. That is, the $p_{\perp}$ of the latest branching considered sets the starting scale of the downwards evolution on both sides, with the next branching occurring at the side that gives the largest such evolved $p_{\perp}$.

In a branching $a \to bc$, the newly reconstructed mother a is assumed to have vanishing mass — a heavy quark would have to be virtual to exist inside a proton, so it makes no sense to put it on mass shell. The previous mother b, which used to be massless, now acquires the spacelike virtuality Q^2 and the correct $p_{\perp}$ previously mentioned, and kinematics has to be adjusted accordingly.

In the old algorithm, the b kinematics was not constructed until its spacelike virtuality had been set, and so four-momentum was explicitly conserved at each shower branching. In the new algorithm, this is no longer the case. (A corresponding change occurs between the old and new timelike showers, as noted above.) Instead it is the set of partons produced by this mother b and the current mother d on the other side of the event that collectively acquire the $p_{\perp}$ of the new $a \to bc$ branching. Explicitly, when the b is pushed off-shell, the d four-momentum is modified accordingly, such that their invariant mass is retained. Thereafter a set of rotations and boosts of the whole $b+d$-produced system bring them to the frame where b has the desired $p_{\perp}$ and d is restored to its correct four-momentum.

Matrix-element corrections can be applied to the first, i.e. hardest in $p_{\perp}$, branching on both sides of the event, to improve the accuracy of the high-$p_{\perp}$ description. Also several other aspects are directly inherited from the old algorithm.

Work on the algorithm is ongoing. In particular, an optimal description of kinematics for massive quarks in the shower, i.e. c and b quarks, remains to be worked out.

Some first tests of the algorithm are reported elsewhere [16]. In general, its behaviour appears rather similar to that of the old algorithm.

4. OUTLOOK

The algorithms introduced above are still in a development stage. In particular, it remains to combine the two. One possibility would be to construct the spacelike shower first, thereby providing a list of emitted partons with their respective emission $p_{\perp}$ scales. This list would then be used as input for the timelike shower, where each emission $p_{\perp}$ sets the upper evolution scale of the respective parton. This is straightforward, but does not allow a fully factorized evolution, i.e. it is not feasible to stop the evolution at some $p_{\perp}$ value and continue downwards from there in a subsequent call. The alternative would be to intersperse spacelike and timelike branchings, in one common $p_{\perp}$-ordered sequence.

Obviously the finished algorithms have to be compared with data, to understand how well they do. One should not expect any major upheavals, since checks show that they perform similarly to the old ones at current energies, but the hope is for a somewhat improved and more consistent description. The step thereafter would be to study specific processes, such as $\mathrm{W}+n$ jets, to find how good a matching can be obtained between the different n-jet multiplicities, when initial parton configurations are classified by their $p_{\perp}$-clustering properties. The PYCLUS algorithm here needs to be extended to cluster also beam jets. Since one cannot expect a perfect match between generated and clustering-reconstructed shower histories, it may become necessary to allow trial showers and vetoed showers over some $p_{\perp}$ matching range, but hopefully then a rather small one. If successful, one may expect these new algorithms to become standard tools for LHC physics studies in the years to come.

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