Abstract
There are experimental evidence for the occurrence of colour reconnection, but the mechanisms involved are far from understood. Previous reconnection studies are briefly summarized, and some potential implications for LHC physics are outlined.
1 Introduction

LHC events have a complicated structure, which involves many physics components, the main ones being hard-process matrix elements, parton distribution functions, multiple parton interactions (MPIs), initial-state radiation (ISR), final-state radiation (FSR), beam remnants, hadronization and decays. All of these contain challenges, but are still understood individually, to some extent. When combined, additional sources of uncertainty appear, however. Foremost among these, colour reconnection (CR) represents the uncertainty induced by the high density of colour charges, that may interact in a nontrivial nonlinear manner.

To put numbers on the challenge, about ten charged particles are produced per unit of rapidity for LHC events at around $y = 0$. These come from around ten primary hadrons, which in their turn come from ten colour strings [1] crossing $y = 0$, according to Pythia [2] simulations. The distributions are very widely spread around this average, so much higher densities are common. The string density is largely driven by the MPI component, where each gluon–gluon scattering may lead to two strings crossing $y = 0$, but it also receives contributions from ISR and FSR. The string width is the same as that of a hadron, the two being dictated by the same confinement physics, and most of the strings are produced and evolve within the transverse area of the original proton–proton collision. Therefore many strings overlap in space and time, potentially leading to nonlinear effects. Furthermore, the small number of colours, $N_C = 3$, inherently leads to ambiguities which partons belong together in separate colour singlets.

One approach to this issue would be modify or abandon existing hadronization models, colour ropes [3] being an example of the former and quark–gluon plasma of the latter. Less dramatic is the CR road, where hadronization as such is unmodified and the nonlinear effects are introduced via models that “only” reassign colours among partons. In the following we will study such models and some of their consequences.

2 Historical overview

The idea that colour assignments provided by perturbation theory could be modified by nonperturbative effects was around already soon after the birth of QCD. The colour octet production mechanism $g^* \rightarrow c\bar{c} \rightarrow J/\psi$ [4] is an early example. Such colour rearrangement effects were studied more systematically for $B$ decay [5], and the sequence $B \rightarrow J/\psi \rightarrow \ell^+\ell^-$ was proposed as an especially convenient test [6]. Indeed the $B \rightarrow J/\psi$ branching ratio suggests a non-negligible but not dominant fraction of the $b \rightarrow cW^- \rightarrow c\bar{s}s$ rate, kinematical restrictions taken into account [7].

Colour reconnection in minimum-bias hadronic physics was first introduced [8] to explain the rising trend of $\langle p_\perp(n_{ch}) \rangle$ observed by UA1 [9]. The starting point here is that large charged-particle multiplicities predominantly come from having a large MPI activity, rather than from high-$p_\perp$ jets, say. If each such MPI produces particles more-or-less independently of each other, then the $\langle p_\perp \rangle$ should be independent of the number of MPIs, and hence of $n_{ch}$. The alternative is that each further MPI brings less and less additional $n_{ch}$, while still providing an equally big $p_\perp$ kick from the (semi-)hard interaction itself, to be shared among the produced hadrons. This is possible in scenarios with CR, if reconnections tend
to reduce the total string length $\lambda$ [10],

$$\lambda \approx \sum_{i,j} \ln \left( \frac{m_{ij}^2}{m_0^2} \right),$$

(1)

where $i, j$ runs over all colour-string-connected parton pairs and $m_0 \approx 1$ GeV is a reference scale of a typical hadronic mass.

As an aside, other aspects (well modeled in generators) drive the rise of $\langle p_\perp \rangle(n_{ch})$ at small $n_{ch}$. Furthermore, the absolute normalization of $\langle p_\perp \rangle$ in this region comes straight from tunes of hadronization to $e^+e^-$ data, supporting the notion that beam-reminant hadronization is no different from that of jets so long as the string density is low.

$W$ pair production at LEP 2 was expected to offer an interesting test bed for such concepts, i.e. whether the $q\bar{q}$ pair produced in each $W$ decay would hadronize separately or whether e.g. the $q$ from one $W$ could hadronize together with the $\bar{q}$ from the other. Notably, this could mess up $W$ mass determinations. Unfortunately, results were not conclusive.

- Perturbative effects are suppressed for a number of reasons, notably that hard-gluon exchanges would force the $W$ propagators off-shell, giving a negligible uncertainty $\langle \delta M_W \rangle \leq 5$ MeV [11].
- Several nonperturbative CR models predicted large effects and could promptly be ruled out. More conservative ones [11] could not be excluded, but were not favoured [12], and gave $\langle \delta M_W \rangle \sim 40$ MeV.
- Additionally Bose-Einstein effects, i.e. that the wave function of identical integer-spin hadrons should be symmetrized, could affect the separate identities of the $W^+$ and $W^-$ decay products. Effects on $\langle \delta M_W \rangle$ could be as large as 100 MeV, but again more likely around 40 MeV [13]. An effect of the latter magnitude is disfavoured by data, but again not fully ruled out [14].

Given the clean LEP environment, it was feasible to trace the space–time evolution of the strings [11], and use that to decide if and where a reconnection would occur. Two alternative scenarios were inspired by Type II and Type I superconductors. In the former, narrow vortex lines at the core of the strings carry the topological information, and so it was assumed that strings could reconnect only if and where these cores crossed. In the latter, strings are viewed as elongated bags with no marked internal structure, and therefore the reconnection probability was related to the integrated space–time overlap of these bags. In both cases reconnections that reduced the total string length could be favoured.

A future high-luminosity $e^+e^-$ collider for the study of Higgs production would, as a by-product, provide much larger $W^+W^-$ samples and thereby allow more precise tests. Assuming an effect is found, its energy and angular-orientation dependence could constrain the range of allowed models [15].

The observation of diffractive event topologies in Deeply Inelastic Scattering at HERA has also been interpreted as a consequence of CR [16]. This offers an alternative to the Ingelman–Schlein picture [17] of scattering on a Pomeron (or glueball, in modern language) component inside the proton. Both approaches can be tuned to give comparable phenomenology, so there is no clear winner at HERA. Nevertheless, HERA, Tevatron and LHC diffractive data can provide significant constraints on any universal model of colour reconnection. This also includes topics such as diffractive jet, $W$ and Higgs production. Diffraction and models for diffraction is such a major topic in its own right [18] that it is impossible to cover it here.
Figure 1: $\langle p_{\perp} \rangle(n_{\text{ch}})$ with the default PYTHIA 8 Tune 4C [21], and the same with CR switched off, compared with ATLAS data [22].

It is also plausible that both CR and Pomeron mechanisms contribute to the appearance of rapidity gaps. To exemplify, a rapidity gap between two high-$p_{\perp}$ jets likely is dominated by reconnection, whereas small-mass diffraction comes more naturally in a traditional Pomeron language.

3 Status at the LHC

While most of the basic ideas for MPI modelling existed a long time ago [8], gradually models have become more sophisticated. One key example is the handling of beam remnants [19]. As a starting point, the colour flow in each separate MPI, including its associated ISR and FSR, is traced in the $N_C \rightarrow \infty$ limit [20]. (This limit gives a well-defined colour topology, as needed for the string hadronization model.) But any colour coming into an MPI must be compensated by a corresponding anticolour left behind in a remnant, which for $N_C \rightarrow \infty$ leads to a remnant momentum to be shared between a multitude of string endpoints. Such a scenario is not ruled out, since essentially no data exists on how the remnant structure changes as a function of the central multiplicity, and since a modelling could introduce many free tuning parameters, but neither is it plausible.

Instead it is likely that the $N_c = 3$ reality leads to a smaller remnant colour charge, as the initial colour of one MPI often compensates the anticolour of another, thereby correlating the colour flow of these two MPIs right up to the final state. Such correlations means that fewer strings need to be drawn out to the beam remnants for high MPI multiplicities, offering a mechanism for a rising $\langle p_{\perp} \rangle(n_{\text{ch}})$, but nowhere near enough. Thus, also with modern models, LHC data reconfirm the need for a further mechanism, such as CR.
is illustrated in Fig. 1.

The almost perfect agreement in Fig. 1 is fortuitous, and it looks less impressive with other selection criteria [22, 23], even if the qualitative features still are reproduced. So there is room for improvements of the CR modelling, or for other physics mechanisms.

Over the years, Pythia 6.4 has come to contain a dozen of CR scenarios, many closely related. Unlike the above-mentioned $e^+e^-$ scenarios there is no attempt to trace a space–time evolution. Instead the guiding principle is to reduce the total string length, as defined by the $\lambda$ measure of eq. (1) or, alternatively, by the $\sum_{i,j} m_{ij}^2$ (GAL, Generalized Area Law [24]). Typically an algorithm may go something like [25]

- Calculate a reconnection probability $P_{\text{rec}} = 1 - (1 - \chi)^{n_{\text{MPI}}}$, where $n_{\text{MPI}}$ is the number of MPIs in the current event and $\chi$ is a free reconnection strength parameter.
- Each string piece is chosen to be a candidate for reconnection with a probability $P_{\text{rec}}$.
- Use a simulated annealing algorithm to perform reconnections between the candidates picked in the previous step, favouring a reduced $\lambda$.

By contrast, currently Pythia 8.1 only contains one scenario, where either all or none of the final-state partons of a MPI system are attached to the string pieces of a higher-$p_\perp$ system, in a way so as to keep $\lambda$ minimal. The lower the $p_\perp$ scale of an MPI, and the larger the number of other MPIs, the more likely it is to be disassembled by CR.

Also the other standard LHC generators face similar issues. The inclusion of CR into Herwig/Herwig++ [26] is of fairly recent date [27]. CR is necessary not only to to describe $\langle p_\perp \rangle(n_{\text{ch}})$ but also e.g. the $\text{d}n_{\text{ch}}/\text{d}\eta$ distribution. Again a simulated annealing approach is used to reduce $\sum m^2$, where the sum runs over all clusters, akin to the GAL above. Sherpa [28] currently has an MPI model based on the Pythia one, but without any colour reconnection. Therefore it also fails to describe the $\langle p_\perp \rangle(n_{\text{ch}})$ distribution. A new model for minimum-bias and underlying events is in preparation [29] that should address it.

4 The mass of unstable coloured particles

Confinement leads to ambiguous masses for coloured particles, since they cannot be studied in isolation. Short-lived coloured particles, like the top, do not even form hadrons with well-defined masses. For the kinematics of production and decay, an event generator therefore have to use its own mass definition, that is close to but not necessarily identical with the pole mass. This inherently leads to ambiguities in a translation of a generator-assisted top mass measurement into a corresponding $\overline{\text{MS}}$ mass.

Furthermore the top quark, as well as the $W$ and $Z$ gauge bosons, travel a distance $c\tau \approx 0.1$ fm before they decay, i.e. significantly less than a proton radius. Therefore their decays take place right in the middle of the showering/hadronization region, and so quarks (and gluons) produced in the decays are subject to the CR issues already discussed. That is, in a decay $t \rightarrow b u d\bar{d}$ the $b$ for sure is colour-connected somewhere else, giving mass ambiguities, but additionally the $u\bar{d}$ system may or may not remain as a separate singlet, further contributing to the uncertainty.

Studies with Pythia 6.4 for the Tevatron suggested a total uncertainty approaching 1 GeV [25] when comparing different tunes. Of this a large part comes from the description of the perturbative stage, i.e. ISR and FSR uncertainties, which should have shrunk con-
siderably since, with the advent of more sophisticated matching/merging techniques. But up to 0.5 GeV remains as a potential error related to CR issues. To put this in context, current top mass measurements at the Tevatron and the LHC now have statistical errors of the order 0.5 GeV, and quote systematic errors below 1 GeV [30].

Clearly this issue needs to be studied further, to try to constrain the possible magnitude of effects from data itself. CR effects should depend on the event kinematics, which would allow to test and constrain models. Such studies have already begun in CMS [31], although statistics does not yet allow any conclusions to be drawn.

As already mentioned, PYTHIA 8.1 does not yet have a range of CR scenarios to contrast, but CR on or off gives a shift of $\approx 0.15$ GeV. Unfortunately this difference does not vary dramatically as a function of some obvious kinematical variables, but further studies are planned.

In top decays to leptons, $t \rightarrow b\ell^+\nu_\ell$, the lepton $p_\perp$ spectrum offers a CR-independent observable, that may allow an alternative route. It will face other challenges, however.

5 Summary and outlook

Colour reconnection as such is well established, e.g. from $B \rightarrow J/\psi$. Given the high string and particle densities involved in a high-energy $pp$ collision, it is hard to imagine that it would not play a prominent role also there.

This does not mean that what we today ascribe to CR could not be a much richer mixture of high-density effects, such as colour ropes or collective flow. The particle composition as a function of $p_\perp$ is one example of LHC distributions not well described by PYTHIA simulations, and where thus some further mechanism may be at play. There is a twist to this story, however, in that CR in $pp$ events can give some of the observed effects similar to the collective flow of heavy-ion collisions [32], by a combination of two factors. Firstly, a string piece moving with some transverse velocity tends to transfer that velocity to the particles produced from it, albeit with large fluctuations, thereby giving larger transverse momenta to heavier hadrons. Secondly, a string piece has a larger transverse velocity the closer to each other the two endpoint partons are moving, which is precisely what is favoured by CR scenarios intended to reduce the string length.

In the near future, the intention is to implement new CR models for $pp$ collisions into PYTHIA 8, partly to offer a broader spectrum of possibilities, partly to add further physics aspects, such as the space–time and colour structure, to provide more realistic scenarios. Other generator authors will also offer their schemes. When systematically confronted with a broad spectrum of data the hope is to see a pattern emerge, where some approaches are more favoured than others. It would be foolish to promise that a unique answer will be found, however; we will have to live with CR uncertainties in many precision measurements. The top mass is the obvious example, but others are likely to emerge as LHC exploration continues.

In the far future, a high-luminosity $e^+e^-$ Higgs factory would offer a second chance to study CR and related effects in $W^+W^-$ events.
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