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A Three-Dimensional Model for Quark and Gluon Jets.

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Abstract

Based on the assumption that a color force-field has a stringlike character (like a stretched-out bag) with no excited degrees of freedom transverse to the field direction (which is strongly supported by the observed polarization of inclusively produced Λ-particles) we derive the probability to produce heavy flavor quark-antiquark pairs and pairs with transverse momentum in the field. We show how to incorporate the results into a soft hadronisation scheme for particle distributions in quark and gluon jets. We point to some non-trivial effects from the finite size of the force-field which result on the one hand in important correlations between the longitudinal scaling variable and the transverse momentum and on the other hand leads to corrections to the simple iterative cascade scheme.

1. Introduction

In this note we will consider the production of heavy quark-antiquark pairs in a confining force-field. We will also consider the production of pairs with momentum components transverse to the direction of the force-field and show that there are large similarities between the two cases.

We have in mind a description of the zero-point fluctuations in a stringlike force-field (a "stretched-out bag") which is in the ground state with respect to the transverse degrees of freedom. We have in an earlier work [1] shown that the existence of a (rather large) polarization for inclusively produced Λ-particles is a strong indication that a force-field obtained by color separation has this property.

The results of this note are meant as a complement to our model for particle production ("soft hadronisation") in quark and gluon jets [2,3]. Based on the dynamics of the massless relativistic string we have in refs [2,3] given a semiclassical treatment of the way the energy and the longitudinal momentum is partitioned among the particles in such jets.

It has been shown by Gottfried and Low [4] that a semiclassical treatment of the longitudinal properties of a jet is justified for high-energy particles. In particular the rapidity and coordinate space position variables can be considered without the need for quantum mechanical corrections due to the uncertainty relations. For the transverse degrees of freedom such a treatment is, however, not justified without great care.

2. The 1+1-dimensional model.

Consider a large energy e^+e^--annihilation into a $q\bar{q}$-pair. A color force field will be created between the quark and the antiquark. This field can be broken by the production of a new $q\bar{q}$-pair which is afterwards pulled apart by the field. Later, more $q\bar{q}$-pairs are produced and the quarks and antiquarks combine to form hadrons [2] as in fig. 1.

In refs [2,3] it is shown that the production of particles in a constant force-field (strength κ corresponding to the tension at rest in the string) is an iterative cascade process [5,6,13]. The probability for a heavy mass system composed of a $q_0\bar{q}_0$-pair and the force-field in between them to break up into a meson $q_0\bar{q}_1$ (of mass m and with energy-momentum fraction z of the $q_0\bar{q}_0$-system) and a (heavy mass) remainder system $q_1\bar{q}_0$ by the production of a $q_1\bar{q}_1$-pair in the field turns out to be independent of z. We note that in the causal and relativistically invariant model the resulting jet of final state particles is of an "inside out" character, i.e. in any particular coordinate frame it is always the slowest particles which first become independent entities. Further, while the mesons are strictly ordered with respect to flavor ("rank" [6]) in the jet: $q_0\bar{q}_1$, $q_1\bar{q}_2$ etc. this ordering only in the mean corresponds to rapidity-ordering.

The model can also be interpreted as a stochastic process (both in energy-momentum and in space-time) and the distribution of production vertices turns out to be centered (however with rather large fluctuations) along a hyperbola with the hyperbolic distance m/κ from the origin of the first $q_0\bar{q}_0$-pair in space-time [2].

3. The Schwinger mechanism.

In the dynamical scheme of ref. [2] the production of a heavy mass flavor $q\bar{q}$-pair in the field does not change the (flat) shape of the z-distribution. Further the relative probabilities to produce different flavors are not determined. In a paper by Schwinger [7] which has lately been reconsidered by different authors [8,9] it is pointed out that in a constant (in space and time) electric field (with κ the electric force) coupled to particles with mass μ the vacuum (the no-particle state) is unstable and decays according to an exponential law $\sim \exp(-P)$. The probability per unit time and (in 1+1 space-dimension) longitudinal space $\frac{dP}{dxdt}$ is essentially given by (for a pedagogical treatment cf. ref. [9])

$$\frac{dP}{dx\,dt} \sim \exp\left(-\frac{\mu^2}{\kappa}\pi\right) \tag{1}$$

and can be understood as the probability for a pair of particles with mass μ to tunnel out through a linear potential barrier [10].

The application of eq.(1) to the relative probability for d- or u-quark pairs (mass $\sim$ o) and s-quark pairs (mass $\sim$ 0.25 GeV) to a stringlike force-field (or a "stretched-out bag") with $\kappa \sim 1$ GeV/f (as obtained from charm spectroscopy and the slope of the Regge trajectories) yields a relative probability:

$$u\bar{u} : d\bar{d} : s\bar{s} = 1 : 1 : 0.37 \qquad (2)$$

This value is close to the observed SU(3)-breaking parameter in $\nu(\bar{\nu})$-production (from the nucleon structure functions) [12] and in the production in quark-jets.

We note that the corresponding value for the production of a $c\bar{c}$-pair (mass ~ 1.4 GeV) would be $\sim 10^{-11}$.

We also note that the result in eq.(1) is obtained in an "infinite-size" electric field and we expect that there will be corrections due to the finiteness of an actual string size.

4. Corrections for the finite field size.

If the quarks are massless, the q and the $\bar{q}$ can classically be produced in a single space-time point with zero energy and then be pulled apart by the field. However, if the quarks have a mass (or a transverse mass) they classically, in order to conserve energy and momentum, have to be produced at a certain distance, so that the field energy between them can be transformed into (transverse) mass energy.

Classically they will move along the two branches of a hyperbola (cf. fig. 2). The asymptotes of the hyperbola are the (light-like) trajectories in the corresponding massless case. In the

Λ-polarization model of ref. [1] we have shown that this feature implies a correlation between the transverse momentum of a $q\bar{q}$-pair produced in a constant force-field (with no excited transverse degrees of freedom) and the spin polarization of the pair.

In this note we remark upon another consequence of the finite separation between q and $\bar{q}$ for massive quarks. In order to produce such a pair at a certain distance from each other, the field must be sufficiently long, i.e. we have to wait until the original $q\bar{q}$-pair has come sufficiently far apart. On the other hand, in order to produce a very energetic meson in the jet (having z close to 1) the field must break very early. Therefore we obtain a correlation between z and transverse momentum which is much stronger than the simple energy-momentum conservation constraint.

The typical field length in an e^+e^--annihilation event as obtained from the model treated as a stochastic process is $2m/\kappa$ for the case of massless $q\bar{q}$-pair production.

In fig. 3 we show a situation where we expect corrections to our earlier results. In this case the heavy mass (μ) $q\bar{q}$-pair cannot (classically) be produced at rest inside the field. In particular the proper time τ for the production vertex is too small as compared to the quantity μ/κ . The Lorentz-frame has been chosen such that the difference vector OV (corresponding to the proper time τ) has no space-component. It is obvious that the pair cannot be produced at rest in any other frames.

5. A simple model.

In order to investigate the quantitative consequences of the effect we consider a simple quantum mechanical model. We start by constructing the wave function $\psi_q(x,t)(\psi_{\bar{q}}(x,t))$ for the $q(\bar{q})$-particle when it is moving attached to a flat string in accordance with fig. 2. The Hamiltonian H is given by

$$H = -\kappa x + \sqrt{p^2 + \mu^2} \tag{3}$$

and we obtain for the energy E=0 the classical trajectory for the q-particle which is at rest at the time t=0

$$p(t) = \kappa t \tag{4}$$

$$x(t) = \frac{1}{\kappa} \sqrt{(\kappa t)^2 + \mu^2} \tag{5}$$

For the corresponding $\bar{q}$-particle we obtain the same result with $\kappa \to -\kappa$. The coordinate-space trajectories are then the branches of the hyperbola H_q in fig. 2.

The quantum mechanical wave function for the q-particle in momentum space with energy eigenvalue E is

$$\Psi_q(p) = A \exp\left(i \frac{Ep}{\kappa}\right) \varphi_q(p) \tag{6}$$

$$\varphi_q(p) = \exp\left\{-\frac{i}{2\kappa}\left[p\sqrt{p^2+\mu^2} + \mu^2 \ln\left(\frac{p+\sqrt{p^2+\mu^2}}{\mu}\right)\right]\right\} \tag{7}$$

To localize the particle in time we study a wave packet by letting A depend on E . The motion in time is then given by

$$\Psi_q(p,t) = \tilde{A}_q\left(\frac{p}{\kappa} - t\right) \varphi_q(p) \tag{8}$$

with $\tilde{A}$ the Fourier transform of A. The localization of the wave packet is as usual the better, the wider the energy packet is; in particular for a constant A the wave function is $\delta(p - \kappa t)$. We expect that the width of the wave packet should be of the order $\frac{m}{\kappa}$ so that the wave function of the produced $q\bar{q}$-pair should be localizable inside a final state meson. The wave function $\psi_{\bar{q}}(p,t)$ is again obtained by changing $\kappa \to -\kappa$ in eq.(8). The wave functions $\psi_q(x,t)(\psi_{\bar{q}}(x,t))$ in coordinate space:

$$\Psi_q(x,t) = \int \tilde{A}_q\left(\frac{p}{\kappa} - t\right) dp \quad \exp(ipx)\, \mathcal{S}_q(p) \qquad (9)$$

can in general not be expressed in terms of elementary functions.

As a model for the production matrix element $M(\mu,S)$ for a pair with the vertex situated in the origin we take the overlap between the product wave functions $(\psi_q(x,t)(\psi_{\bar{q}}(x,t))$ and a constant corresponding to the flat field integrated over the space-time region S where the field is nonvanishing:

$$\mathcal{M}(\mu, S) = \int_S dx\,dt \quad \Psi_q(x,t) \cdot \Psi_{\bar{q}}(x,t) \cdot 1 \qquad (10)$$

It is then easily shown from eqs.(6)-(9) above that for the case when S corresponds to all of space-time

$$\mathcal{M}(\mu,\infty) = \exp\left(-\frac{\mu^2 \pi}{2\kappa}\right) \; f\left(\frac{\mu^2}{2\kappa}\right) \times$$

$$\times\, 2\pi \int_{-\infty}^{+\infty} dz\; \tilde{A}_{\bar{q}}(z)\, \tilde{A}_q(-z) \qquad (11)$$

with f a slowly varying function of its arguments

$$f(z) = \mathrm{Im} \int_1^\infty d\lambda \; \exp\left(i 4 z \int_1^\lambda d\lambda' \sqrt{\lambda'^2 - 1} \right) \qquad (12)$$

With $\tilde{A}_{\bar{q}}(z) = \tilde{A}_q^*(-z)$ and proper normalization of the wave functions this essentially coincides with the Schwinger result in eq.(1).

It is also rather simple to convince oneself that, almost independently of the properties of the wave packets $\tilde{A}_q$ and $\tilde{A}_{\bar{q}}$ the eq.(10) obtains its principal contributions from the space-time region

$$|x| \lesssim \frac{\mu}{\kappa}$$
$$|t| \lesssim \frac{\mu}{\kappa} \qquad (13)$$

and that the overlap-wave function $\psi_q \psi_{\bar{q}}$ is exponentially decreasing in all directions outside this region. We therefore expect the Schwinger result to be essentially unchanged for the production-matrix element $M(\mu, S)$ as long as the space-time region S includes the region in eq.(13).

In the situation described by fig. 3 the region S is, however, small compared to the region in eq.(13), in particular the parameter $\epsilon = \frac{\kappa\tau}{\mu}$ is small.

A close investigation of the matrix element $M(\mu, S) \equiv M(\mu, \tau)$ reveals that the ratio

$$g(\mu, \epsilon) = \frac{\mathcal{M}(\mu, \tau)}{\mathcal{M}(\mu, \infty)} \qquad (14)$$

1. is small for small ϵ
2. rapidly approaches 1 for $\epsilon \gg 1$
3. is smoothly varying in between.

The precise behaviour is, however, not independent of the details of the model (the behaviour of $\tilde{A}$, i.e. the boundary conditions etc.). Fortunately, for the applications we have in mind the detailed properties of the ratio g are irrelevant. As a matter of fact, <u>we find that different functions g with the properties 1-3 give almost identical results when used for the production of $q\bar{q}$-pairs in quark and gluon jets</u>.

Before we go into these applications we add the following comments to the discussion above.

A. We have considered a model for the matrix element M without taking into account the spin effects etc. This can be done and a treatment of e.g. spin 1/2-particles will lead to a very similar result.

B. We note that besides ϵ there is another dimensionless parameter $\kappa\tau^2$ which may be of importance, in particular in connection with the boundary conditions for the wave functions ψ_q and $\psi_{\bar{q}}$ in the investigation of the matrix element M. We have looked for such a dependence (note that it is independent of the flavor of the produced pair) but we have not been able to find any observable effects.

C. The basic assumption of the investigation has been that there are no excited (transverse) degrees of freedom in the force-field where the production of $q\bar{q}$-pairs occurs. We have in particular completely neglected the influence on the (soft) hadronisation of the emission of hard gluons, which should be possible on a short-time basis (characteristically less than $\sim$ 1/2 fm) in a color force field.

Hard gluon emission and the possible properties of gluon jets inside the dynamics of a stringlike force-field have been treated in ref. [13] and we will return to a detailed discussion applicable for e^+e^--annihilation in a forthcoming paper [14].

D. The extension of the above-mentioned considerations to the production of transverse mass in the field is straightforward. Our assumption of no excited transverse degrees of freedom for the force-field implies that a $q\bar{q}$-pair is formed with total transverse momentum equal to zero, i.e. with transverse components $\vec{k}_\perp$ and $-\vec{k}_\perp$, respectively. The probability to form such a pair inside the field is given by eq. (5) with $\mu \rightarrow \mu_\perp$, i.e. the transverse mass $\mu_\perp = \sqrt{\mu^2 + k_\perp^2}$. (Although the classical trajectory for an endpoint of a string carrying transverse momentum with respect to the string direction is not a hyperbola (for a discussion cf. refs. [3,15]), the hyperbola is the locus for the "effective" longitudinal coordinate describing the endpoint and a small adjoining piece of the string (the part which is "bent").)

6. Applications to the soft hadronisation scheme.

We note at this point that our results provide a justification for the method of generating transverse momentum in quark-jets that was first suggested by Feynman and Field (FF) as an easily implemented phenomenological recipe. Then the quark pairs are assumed to be produced [6,16] with compensating transverse momentum

$\vec{k}_\perp$ weighted by the distribution $\sim d^2k_\perp \exp(-\vec{k}_\perp^2/(0.35\ \mathrm{GeV/c})^2)$. There are further no correlations between the transverse and longitudinal dimensions.

The inclusion of the ratio $|g|^2$ as a weight function for the production vertices in the particle generation will, however, modify the cascade character of the model in refs [2,3]. In particular the fragmentation functions will no longer satisfy integral equations of the kind obtained in iterative cascade models [5].

To investigate the consequences we have used the Lund Monte Carlo jet program [16] with several different parametrizations of the vertex weighting factors $|g|^2$ including the properties 1-3 above e.g.

$$|g_1|^2 = \frac{\epsilon^2}{\epsilon^2+1} = \frac{(\kappa\tau)^2}{(\kappa\tau)^2+\mu^2} \tag{15}$$

$$|g_2|^2 = \int_{\frac{\mu}{2\sqrt{\kappa}}\left(\epsilon-\frac{1}{\epsilon}\right)}^{\infty} d\lambda\ \exp(-\lambda^2) \tag{16}$$

In figs 4 and 5 we show a few results, in particular the single particle distribution of π and ρ for the two cases. There are obviously very small differences despite the large difference in parametrizations, and we have not found any significant differences elsewhere either. Due to this fact, we feel that there is no reason to bother about the precise properties of the ration g in eq. (14) and we will for convenience use e.g. the parametrization $|g_1|^2$ in eq. (15)

It turns out that in general the differences to the simple 1+1 dimensional model derived in ref. [2] and used phenomenologically with surprisingly successful result in ref. [13] are not so large except for certain parts of phase space and certain particular correlations.

The major result is that the factor in eq.(15) corresponding to the situation in fig. 3 implies a suppression of the production of mesons with z close to 1. In particular for values of z inside the region $1 > z \gtrsim 1 - \langle \frac{\mu_\perp^2}{m_\perp^2} \rangle$ ($m_\perp$ is here the transverse mass of the meson) the earlier constant (in z) probability from ref. [2] is changed to an effective behaviour $\sim (1 - z)$. This is also observed in the DECO data (cf. fig. 4). The $(z, p_\perp)$ correlation implies further that in an e^+e^--annihilation into a two-jet-situation two mesons moving with large z in opposite directions tend to be more "back-to-back" than in e.g. the FF model which does not contain a suppression of $p_\perp$ for large z.

As a consequence of this suppression for $z \sim 1$ we find that the single particle spectra for π- and K-particles for $z > 0.1$ turn out to be rather insensitive to the ratio of vectors to pseudoscalars produced in the jet. This is a necessary input parameter in all models of this kind, and in our simple model in ref. [13] we assumed that this ratio was 3:1 from simple spin-counting ideas. Comparison of the Monte Carlo model to the data from the DESY-Cornell streamer chamber group (DECO) [17] on

ρ^0- production implies that this ratio is closer to 1:1 (cf. fig 5). This value of the ratio is also used by FF.

Another situation, in which one may at first sight expect similar mass effects is shown in fig. 6. Here one production vertex for a heavy (transverse) mass flavor pair is very "late" compared to the adjacent vertices so that one meson is followed in rank by another one having much more energy. This is a possible situation (the probability is calculated for the model in ref. [2] in eq.(27)) but the occurrence is rare due to the competition from the many available "earlier" vertex points in the stochastic process.

In a classical description the production of the pairs at the vertices V_1 and V_2 in fig. 6 implies that the field vanishes outside the light-cone segments V_1O' and V_2O'. Further the massive $q\bar{q}$-pair must classically be produced before they meet their "partners" at the space-time-points A and B. Thus even if there are several similarities between the situations in fig. 3 and in fig. 6, we expect that it is necessary to understand the quantum mechanics of the final state interactions in order to compute a realistic suppression factor for the vertex V'.

The properties of the vertex V' can be studied in strange-quark-production resulting in the appearance of a K^+K^--pair. In fig. 7 we show one effect of introducing an extra suppression factor $(g_1)^2(\epsilon')$ like in eq. (15) with $\epsilon' = \frac{K\tau'}{\mu_\perp}$ (τ' is the proper time-difference $O'V'$ in fig. 6) for the production of K^+K^--pairs in a u-quark jet. We note that the K^--spectrum becomes much softer (an extra power of $(1-z)$ for $z \sim 1$). The suppression is also reflected in the average rapidity difference between K^+ and K^- , which is increased from 0.71 to 0.83 . (In the FF-model, where the density of particles is larger, this value is 0.48.)

The corresponding effect on the π^--spectrum in a u-quark-jet (which should have some similarities to the K^--situation) is not very noticeable since the π^- often stems from vector-meson decay.

We conclude that we have with very few dynamical assumptions, essentially that a color force-field during the hadronisation process has a stringlike character with no excited transverse degrees of freedom, obtained a model of a stochastic character which is very stable against changes in the detailed behaviour of the vertex production matrix elements. There is, however, a particular setup of observables, related to the production of K^+K^--pairs in a quark jet, which may give more precise dynamical information.

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17 The DESY-Cornell Streamer Chamber Group DESY 78/12, DESY 78/34

Figure captions

1. The decay of an excited string system into ground state "yo-yo-modes", used as models for final states mesons in refs [2,3]. The shaded areas correspond to non-vanishing force-fields.

2. The production of a heavy mass (μ) $q\bar{q}$ pair in a string field. The q- and $\bar{q}$-particles will classically move along the branches of the hyperbola H_q with hyperbolic distance $\frac{\mu}{\kappa}$ from the production vertex.

3. The production of a heavy mass (μ) $q\bar{q}$ pair in the force-field from the original $(q_o\bar{q}_o)$ pair which start out in the space-time point O. The invariant space-time distance τ from O to the production vertex V is smaller than $\frac{\mu}{\kappa}$, and the $q\bar{q}$ pair cannot classically be produced at rest inside the field.

4. The fragmentation distributions for a. π^+, b. π^- in a u-quark jet. The curves correspond to the suppression factor $|g_1|^2$ (—), $|g_2|^2$ (---) in eq. (15) and to the parametrization in ref. [6] (-·-·). The experimental data points correspond to ref.[17] (full dots) and ref.[12](open dots), νH_2 at BEBC. The data from ref. [12] include all positive and negative particles (i.e. around 10-20% kaons).

5. The fragmentation distributions for ρ^0 in a u-quark jet with the same notations as in fig. 4. Data are from ref.[17]

6. The production of a heavy mass (μ) $q\bar{q}$ pair such that the meson $(\bar{q}_2q)$ is much slower than the next-rank meson $(\bar{q}q_1)$. The invariant space-time distance τ' between the production vertex V' and the classical endpoint O' of the field obtained by the production of the $q_2\bar{q}_2$ pair at the vertex V_2 and the $q_1\bar{q}_1$ pair at the vertex V_1 is small compared to $\frac{\mu}{\kappa}$

7. The ratio of the fragmentation functions for K^+ and K^- from a u-quark jet with (---) and without (—) the suppression factor $g_1^2(\epsilon')$ from eq. (15).

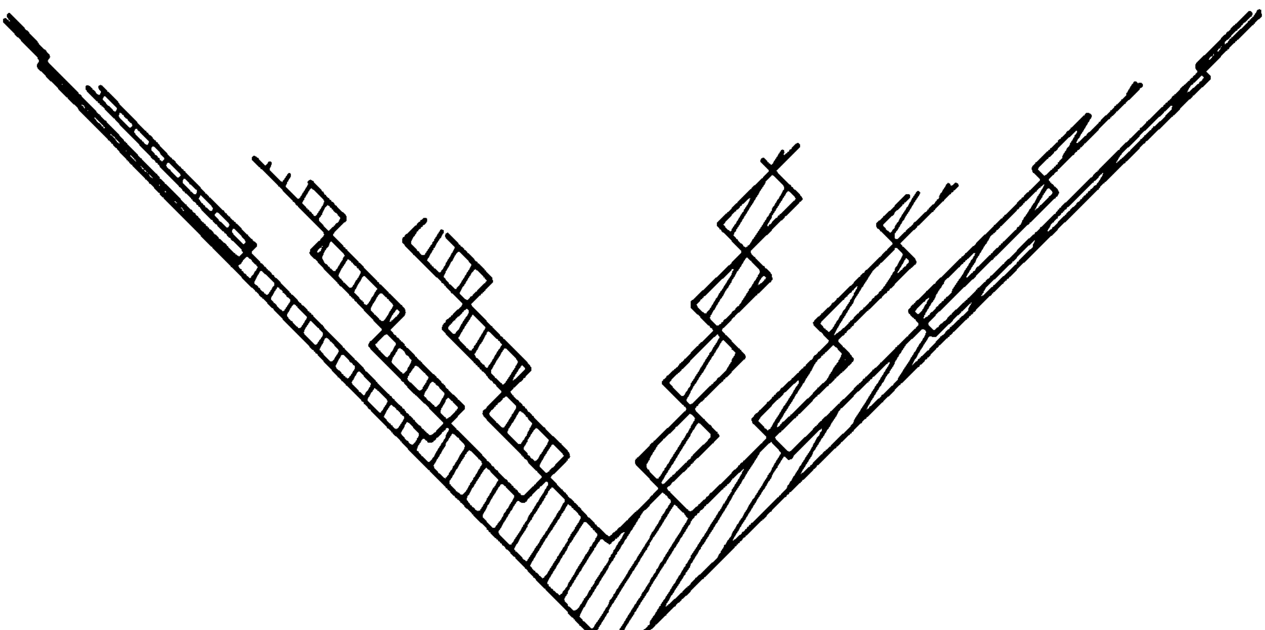

Fig 1

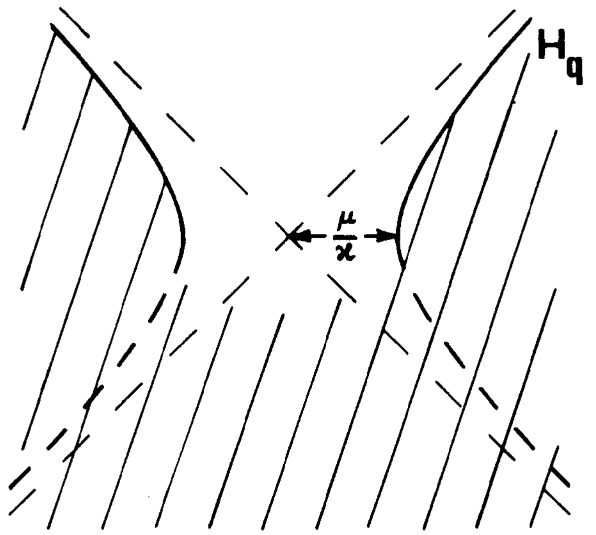

Fig. 2

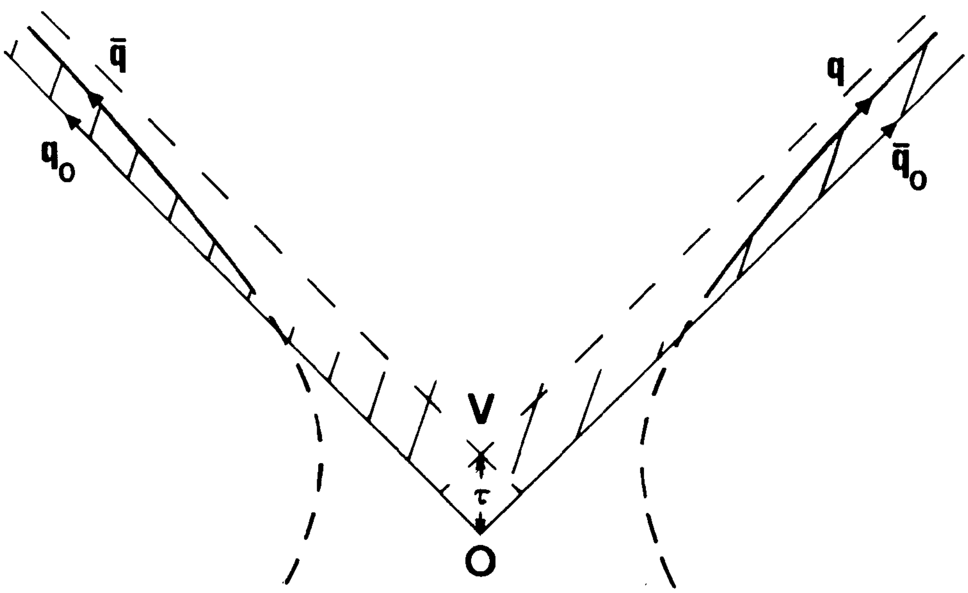

Fig. 3

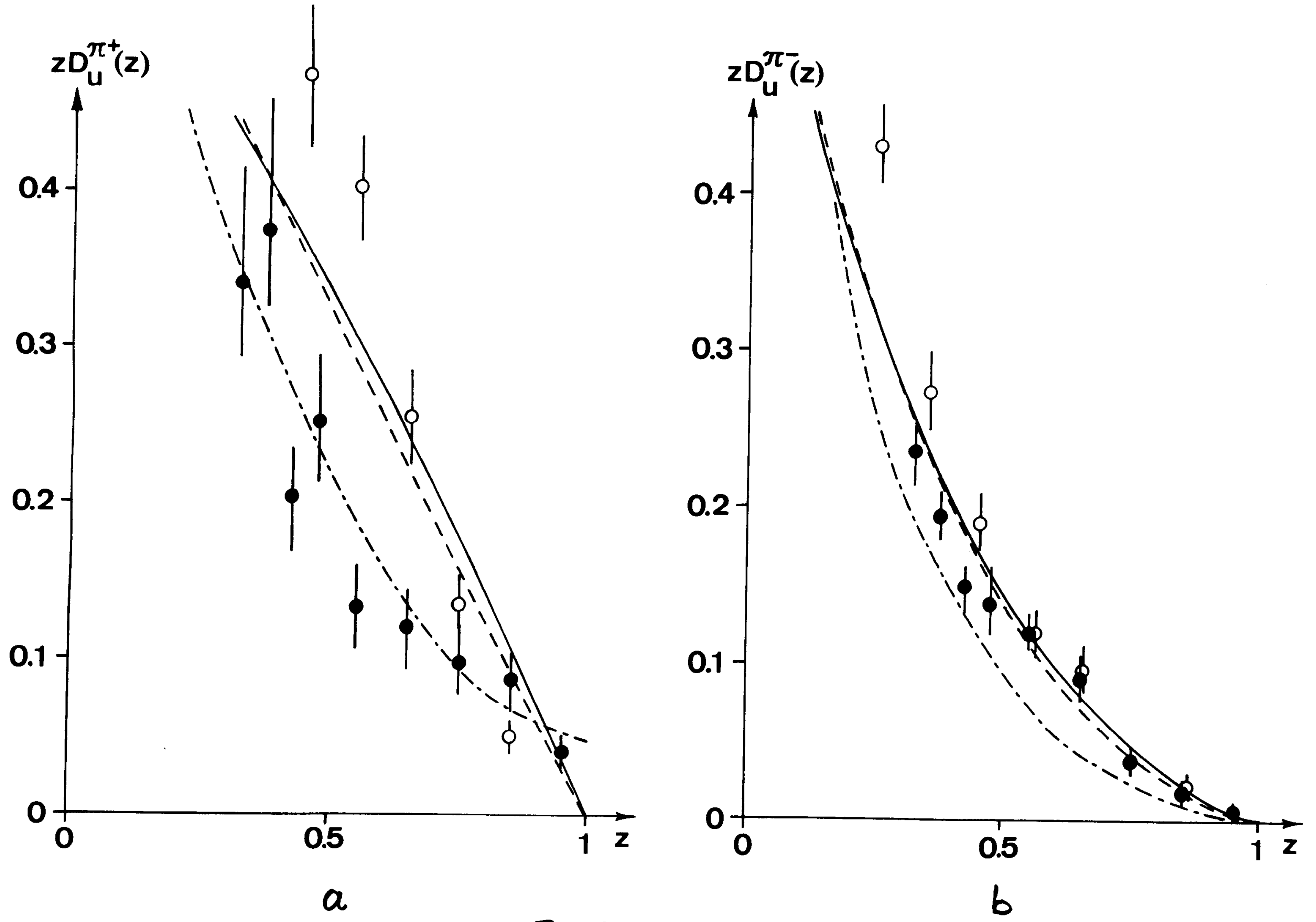

Fig 4

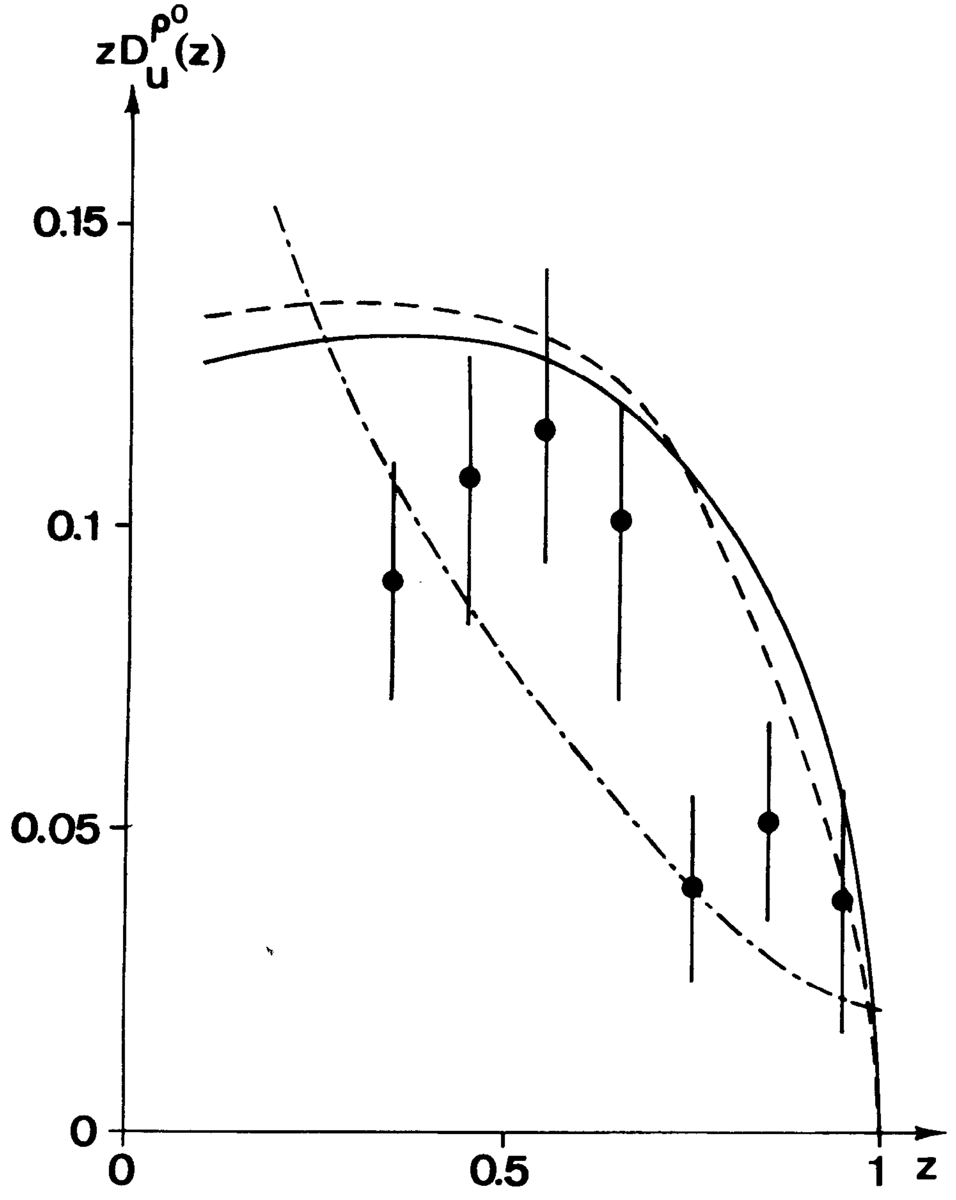

Fig 5

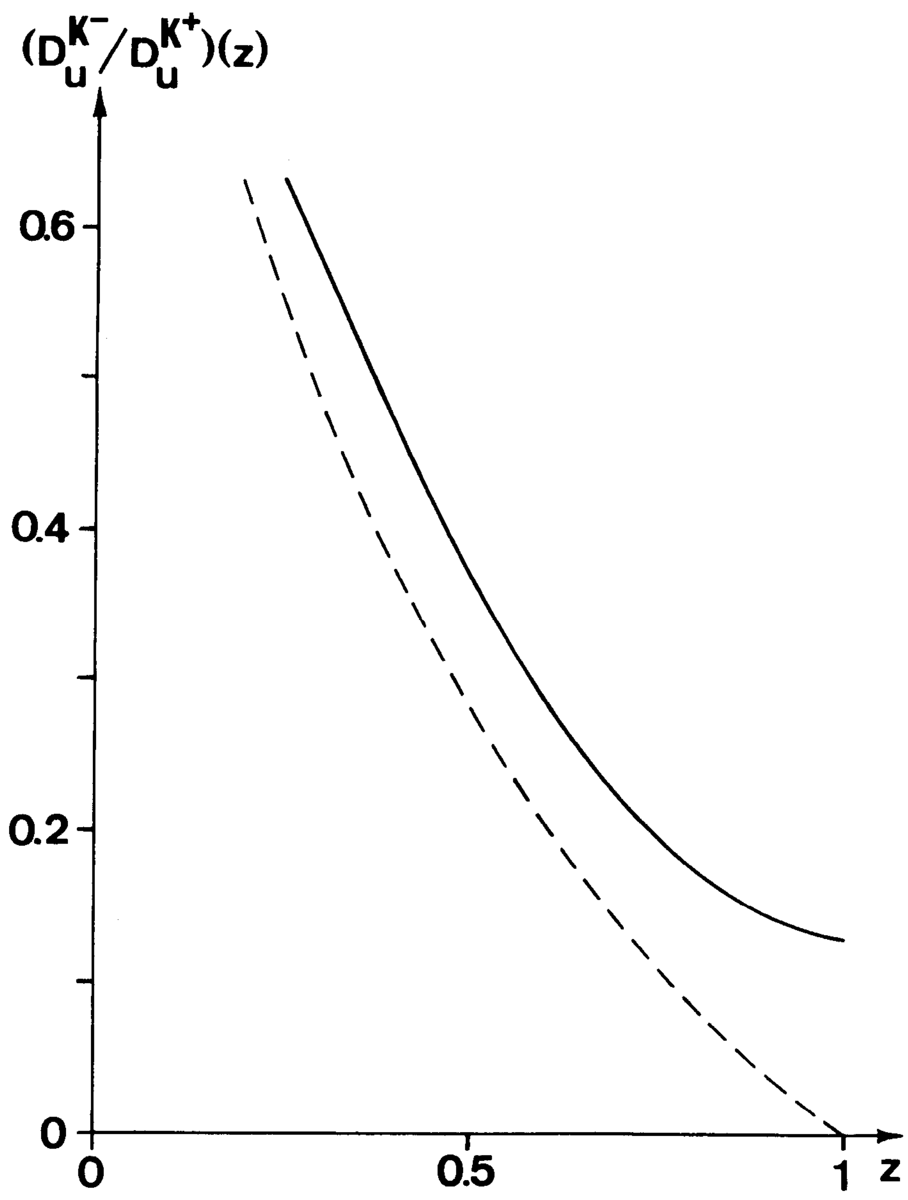

Fig 7

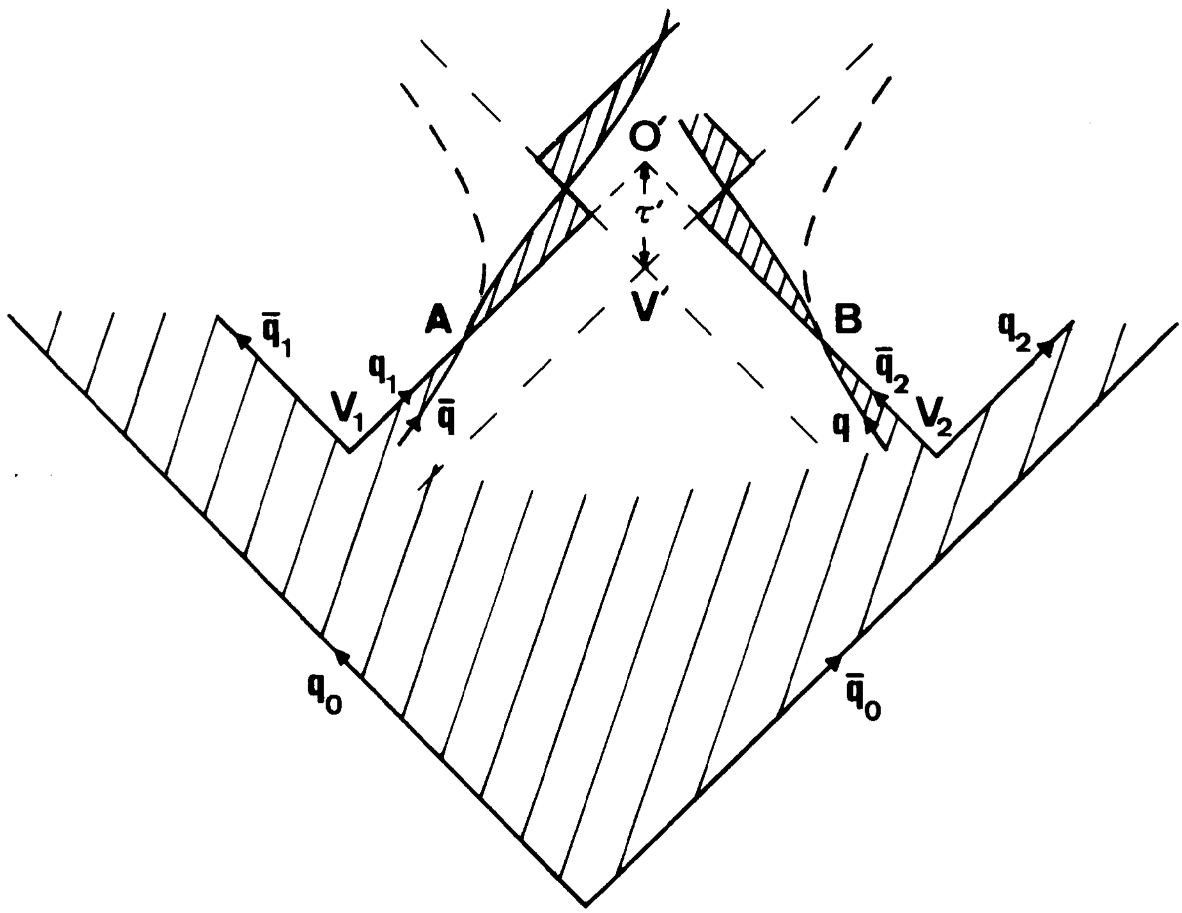

Fig. 6