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On Soft Gluon Emission and the Transverse Momentum Properties of Final State Particles.

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Abstract:

We discuss the infrared problem in a theory with confined quarks and gluons. In QED, the infrared cut-off depends on the adjustable experimental energy resolution. For QCD, the physical observables of the asymptotic final state are the hadron momenta whereas the gluon momenta do not correspond to observable quantities. Thus the energy resolution of a "gluon detector" is determined by the hadronic mass scale. In particular, we study the effect of soft gluon emission on the transverse momentum of the hadrons.

1. Introduction and general considerations

In this paper we will discuss the infrared divergences associated with the emission of soft and collinear gluons. The basic question is: what are the observable effects from the degrees of freedom of a confined force field in which the quanta to not have asymptotically (in space-time) free and independent states?

We feel that the problem related to soft gluon emission must be treated rather differently from the corresponding infrared problem in QED [1]. It is at least in principle possible to observe very low-energetic photons and to distinguish them from a charged particle state, given a sufficiently fine-grained (and distant) detector setup. The observable quantities in connection with gluon emission in a confined theory, however, are not the gluons themselves but rather the final state hadron properties. Therefore the resolution power of a "gluon detector" is related to the phase-space density of hadrons, which in all known situations is rather low, viz. in the mean one to two hadrons per unit rapidity. It is also known that the fluctuations around the mean in phase-space are small and close to Poissonian. In other words, while the infrared cutoff in QED is determined by the adjustable experimental energy resolution, in QCD it is fixed by the masses of the available hadrons. Therefore in order that a description of the strong interaction in terms of quarks and gluons should be meaningful, a necessary requirement is infrared stability. By this we mean that the effects of a soft or collinear gluon emission on the observable hadron momenta should vanish when one approaches the singularity. A branching of a single gluon into several gluons should not be observable in case all the original gluon energy ends up in one final state hadron. In e.g. e^+e^--annihilation events these requirements imply a continuous transition between 3- and 2-jet events (and between 4- and 3-jet events, etc.). The emission of a soft or collinear gluon should result in an event still essentially of quark-antiquark jet character and the transition between $q\bar{q}gg$ and $q\bar{q}g$ events should be continuous when e.g. the invariant mass of the two gluons becomes small.

These requirements are satisfied in the Lund model for soft hadronization [2-5] where the gluons are treated as transverse excitations on a string-like force-field. (In contrast, a model in which jets are assumed to fragment independently and then are joined in the origin in the CM frame is not explicitly infrared stable (or Lorentz invariant).)

Gluon emission will in general lead to a softer z spectrum, with extra hadrons produced around the rapidity of the gluon. A consistent scheme will then require a study of the interrelated modification of both transverse and longitudinal momenta. In the limit of very soft gluons these two problems should however decouple, so that it becomes meaningful to study $p_\perp$ properties separately, as transverse perturbations on an essentially two-jet longitudinal structure.

In this paper we are particularly interested in the transverse momentum properties of the hadrons due to soft gluon emission. If a quark radiates off a soft gluon with a certain transverse momentum and rapidity, it will give some extra $p_\perp$ to those hadrons which are produced with approximately the same rapidity. At the same time the quark will obtain a recoil, thus giving compensating $p_\perp$ to the leading hadrons in the jet.

When the rapidity of a gluon is about the same as, or larger than, the maximum hadron rapidity (a collinear gluon), both the quark and the gluon are forced into the same leading hadron, and the $p_\perp$ effects will disappear. Thus the rapidity range over which we have to study soft gluons is smaller than $\ln(W^2/m^2)$ (with m a typical hadron mass), independently of the $p_\perp$ of the gluon.

While collinear gluons do not influence the transverse momenta of the hadrons, they do give some softening of the longitudinal distributions. This is because they delay the breaking of the colour field, and to produce a large z particle the field has to break early. Our result here is similar to the softening obtained e.g. in jet calculus [6]. We will return to this question in a later publication.

2. Remarks on the soft hadronization model

The final state mesons stem from the breakup of the (constant) force field between a quark (a colour triplet) and an antiquark (a colour antitriplet) by the production of new $q\bar{q}$-pairs from the field energy. The probability for a particular break is assumed proportional to the number of final states available by this break. A causal and relativistically invariant treatment of this condition leads to a model of an iterative cascade type [2,7-10] which is easily implemented in a Monte Carlo generation procedure [11,12].

With regard to the transverse momentum properties we assume that the force field during the soft hadronization process is in the transverse ground state. This assumption does not imply that there are no transverse momenta produced along the field, only that such zero-point fluctuations in the ground state are locally compensated. In a produced $q\bar{q}$-pair, if the q obtains the transverse momentum $\vec{k}_\perp$ the $\bar{q}$ obtains $-\vec{k}_\perp$. The production rate of (heavy) flavour masses μ and transverse momentum can be understood [4,13] as a tunneling phenomenon and we obtain a Gaussian suppression of heavy transverse masses $\mu_\perp = \sqrt{k_\perp^2 + \mu^2}$ governed by κ, the energy per unit length in the field:

$$\simeq \exp\left(-\pi \mu_\perp^2/\kappa\right) \qquad (1)$$

This leads to an in general Gaussian transverse momentum spectrum for the final state particles of width $\simeq 0.35$ GeV/c, if κ is determined e.g. from the linear potential in charm spectroscopy. A relativistically invariant and causal generalization to 3 space-dimensions of a linear force field is given by the massless relativistic string. This system has certain degrees of freedom with properties similar to the gluonic properties expected in QCD. It is possible to localize energy and momentum in a pointlike piece of the string, a "kink", which will move with the velocity of light. It is pulled back by the string with twice the force acting on an end-point quark (in QCD with N colours the expected ratio between the forces acting on a gluon and on a quark is $2/(1 - \frac{1}{N^2})$, i.e. 2 for infinitely many colours).

In figs. 1 and 2 we depict the motion and breakup of such a string system in space-time and the corresponding momentum space density of the final state particles respectively.

In an event like $e^+e^- \to q\bar{q}g$ the colour force field is then stretched from the quark to the antiquark <u>via</u> the gluonic excitation (while in e.g. $\Upsilon \to 3g$ the gluons are at the corners of an expanding triangular closed string). This implies some angular and multiplicity asymmetries for the final state which are in agreement with the observations by the JADE group at PETRA [14].

We will use this hadronization model as a basis to study the influence of soft gluon emission on the transverse momentum properties of the final state particles. In order to determine the state configuration of the string system, the first order matrix element in perturbative QCD for $e^+e^- \to q\bar{q}g$ is used.

3. Remarks on the $p_\perp$ effects of soft gluon emission.

In order to obtain a better understanding of the effects of soft central and collinear gluon emission (a precise partition is given below) we consider the situation in the particular coordinate system where the gluon is moving transversely to the direction of the back-to-back motion of the $q\bar{q}$-pair (fig. 3).

The gluon transverse momentum is during the classical motion of the string system transferred to two string segments which after the gluon is stopped will move one in each direction. It is noteworthy that the transverse momentum carried by a string piece like the one marked out in the figure is independent of the size of $k_\perp$ and only depends on the length of the string segment [3,15], i.e. in particular is proportional to the longitudinal (the projection on the $q\bar{q}$-direction) size. In the example of fig. 3 this proportionality factor is $\frac{\kappa}{\sqrt{2}}$.

As we have remarked upon above, the longitudinal size of a string piece corresponding to a meson m is Lorentz contracted, i.e. proportional to m/E with E the energy in the system where the gluon longitudinal momentum is zero. Therefore a gluonic disturbance will only affect mesons with rapidities y close by the gluon rapidity y_g and we conclude that any such disturbance will fall off like $\exp(-|y-y_g|)$ for $|y-y_g| \gtrsim 1$. We note that a rapidity range of about 2 units is also the typical one in the Lund model for very hard gluon emission. If we e.g. would plot the multiplicity as a function of the angular variable pseudorapidity (defined in a coordinate system like fig. 3), the multiplicity will increase above the general background (related to the q- and $\bar{q}$-jets) inside an angular range of typically 1 unit in pseudorapidity on each side of the gluon rapidity. The central value will increase and the half-width of the "bump" will decrease with increasing gluon energy but "the range of the disturbance" is independent of this energy. A similar structure will appear in e.g. an energy flow investigation.

A useful parametrization of the effect will be given elsewhere. For soft central gluons we obtain essentially that a gluon with transverse momentum $\vec{k}_{\perp g}$ and rapidity y_g will effect mesons in the rapidity range $(y, y+dy)$ like

$$\vec{k}_{\perp g} \frac{N'}{\cosh(y-y_g)} dy \qquad (2)$$

with N' a normalization constant.

The arguments given above and also the fact (derived in ref. [2]) that the median production time at rest for a meson of rest-size ℓ is given by ℓ/c implies that gluons emitted at rapidities more than about a

unit apart will not interfere. If there are several (soft) gluons emitted into a certain angular range, the effect will be very similar, although the straight string segments of fig. 3 will be modified into several broken segments, i.e. for many soft gluons be "rounded off".

There is a further effect on the final state particle distributions which is not noticeable in fig. 3 due to the particular coordinate system chosen. There will be an obvious recoil effect on the q- and $\bar{q}$-particles from the gluon emission. This recoil contribution will be distributed among the fast particles along the q- and $\bar{q}$-directions (cf the discussion above in connection with fig. 1) in accordance with the fractional energy momentum [3], i.e. for the gluon emission discussed above as

$$-\vec{k}_{\perp g}\, dz \approx -\vec{k}_{\perp g}\, dy \exp(|y| - y_m) \qquad (3)$$

with y_m the maximum rapidity of a meson

$$y_m = \ln\left(\frac{W}{m}\right) \qquad (4)$$

Thus the particles in the fragmentation region $|y| \simeq y_m$ are affected by the soft central gluon emission. On the other hand it is obvious that the combined effects from eqs (2) and (3) will imply that the collinear gluon emission i.e. $y_g \simeq y_m$ will give only minor contributions to the meson spectra.

Before we consider the quantitative size of the effects it is necessary to consider the probabilities for gluon emission as well as a few kinematical concepts.

4. Remarks on cross sections and kinematics.

We will assume that the state configuration of the string system is determined from perturbative QCD during a brief space-time period (of the order of $1/\Lambda_{QCD} \sim 0.5$ fm). Gluon emission then corresponds to kink-like excitations on the resulting force field. From then on, during the soft hadronization, there are no more transverse excitations and the force field will stretch in a smooth way and break up as described before. Then we obtain the cross section for $e^+e^- \to q\bar{q}g$ in terms of scaled energy variables in the CM-frame $x_1 = \frac{2E_q}{W}$, $x_2 = \frac{2E_{\bar{q}}}{W}$, $x_3 = \frac{2E_g}{W}$ (with the obvious normalization $\Sigma x_i = 2$) neglecting quark mass corrections [16]:

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{dx_1 dx_2} = \frac{2}{3} \frac{\alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \qquad (5)$$

In [12] it is shown that the situations when there are one or more mesons related to the emitted gluons (from now on true hard gluon emission) can be disentangled by the requirements

$$2\gamma \leq x_i \leq 1 - 2\gamma \qquad i = 1,2 \qquad (6a)$$

$$(1-x_1)(1-x_2) > \gamma\,(x_1 + x_2 - 1) \qquad (6b)$$

with γ defined by

$$\gamma = \left(\frac{M_0}{W}\right)^2 \qquad M_0 \approx 2.5 \text{ GeV} \qquad (7)$$

The requirements in eq.(6a) are usually referred to as "thrust-cuts" on collinear gluons while eq.(6b) defines a situation in which a centrally (in phase-space) emitted gluon is hard enough to permit related particle emission (cf the definition of the variable k below and the discussion after eq.(8)). It should be understood that the requirements in eqs (6) and (7) are "theoretical" conditions, based on the properties of the Lund

model. Many of the events which in that way are classified as 3-jet events are experimentally indistinguishable from 2-jet events. The experimentally observed rate of 3-jet events (which correspond to a scale $M_0 \gtrsim 5$ GeV/c^2) on the 10-20 % level at 30 GeV [14,17] is, however, the same. As both particle production and transverse momentum transfer in connection with gluon emission are localized phenomena in phase-space (connected to a scale of 1-2 units in pseudorapidity), the precise value of the M_0 scale is a matter of convention.

Let us introduce the variables

$$k = W \sqrt{\frac{(1-x_1)(1-x_2)}{x_1 + x_2 - 1}} \qquad (8a)$$

$$y = \frac{1}{2} \ln \left(\frac{1-x_1}{1-x_2}\right) \qquad (8b)$$

We note that k is the transverse momentum of the gluon in the Lorentz frame shown in fig. 3. For soft central gluons (with $x_1 \approx 1$, $x_2 \approx 1$) k and y are also the transverse momentum and rapidity of the gluon in the CM frame with regard to the direction defined by the quark and the antiquark.

Expressed in these variables, eq.(5) takes the form

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{dk^2 dy} = \frac{4}{3} \frac{\alpha_s}{\pi} \frac{1}{k^2} \frac{x_1^2 + x_2^2}{x_1 + x_2} (1 - x_3)$$

$$\approx \frac{4}{3} \frac{\alpha_s}{\pi} \frac{1}{k^2} \qquad x_1 \approx 1,\ x_2 \approx 1 \qquad (9)$$

True hard gluons have $k > M_0$. Collinear gluons correspond to $x_2 > 1 - 2\gamma$ (for $y > 0$) or

$$k^2 < M_0^2 \frac{4\gamma e^{2y}}{1 - 2\gamma - 2\gamma e^{2y}} \qquad (10)$$

The remaining region consists of soft central gluons. This is illustrated in fig. 4. An effective maximum rapidity for soft gluons is given by $k^2 = M_0^2$ in (10). Hence the soft gluon region is approximately described by

$$|y| \lesssim \frac{1}{2} \ln\left(\frac{1-2\gamma}{6\gamma}\right) \approx \frac{1}{2} \ln\left(\frac{W^2}{6M_0^2}\right) \qquad (11a)$$

$$k < M_0 \qquad (11b)$$

The QCD result in eq. (5) for soft and central gluon emission can be formulated as

$$\frac{d\bar{n}}{dk_{\perp g}\, dy_g\, d\phi} = \frac{2\alpha_s}{3\pi^2} \frac{1}{k_{\perp g}} \qquad (12)$$

(with ϕ the azimuthal angle).

The quantity $d\bar{n}$ in eq. (12) corresponds to the mean number of gluons emitted. From now on we will in accordance with ref. [18] assume that the emission of soft central and collinear gluons can be considered as a Poissonian process. Then the probability to obtain e.g. a resulting quantity $\vec{p}_\perp$ from many increments $\vec{k}_\perp$ each determined by a Poissonian emission governed by $d\bar{n}$ is given by [18,19]:

$$\frac{dP}{d^2p_\perp} = \frac{N}{(2\pi)^2} \int d^2b\, \exp(i\vec{p}_\perp \vec{b}) \exp\left\{-\int d\bar{n}\,(1-\exp(i\vec{k}_\perp \vec{b}))\right\} \qquad (13)$$

with N a normalization constant, depending upon the allowed $k_\perp$-range.

After having set up a quantitative formalism, we are ready to give the results on the transverse momentum spectra of the hadrons due to gluon emission.

5. Quantitative results.

A. True hard gluon emission.

This effect which seems to be experimentally observable on the 10÷20 % level at the higher PETRA energies [14,17] corresponds to a different topology for the final state hadronic distributions. It can be disentangled by means of e.g. thrust-cuts on the events. The angular energy and multiplicity distributions seem to follow the predictions from lowest order QCD combined with a simple hadronization model [14], and we are justified to state that the gluonic degrees of freedom are revealed on an energy and transverse-momentum scale larger than 5 GeV/c^2 but hardly on a scale smaller than $M_0 \approx 2$ GeV/c^2 in eq.(7).

B. Collinear gluons.

In section 3 we noted that the combined effect of collinear gluon emission and the corresponding recoil of the quark should be compensating. A Monte Carlo study shows that the combined effect is actually so small that it can be completely neglected given the general zero-point Gaussian fluctuations in eq. (1).

C. Soft and central gluon emission.

The available phase-space is defined by eq. (11) while the effects on the final state particles is twofold. According to eqs (2) and (3) we expect a $p_\perp$-bump in the rapidity neighbourhood of the gluon and a corresponding recoil in the fragmentation region with negligible interference. According to the discussion in section 3, neighbouring soft gluons will give a combined effect which, given the size of the rapidity neighbourhood, can be computed from eq. (13).

Our results for the integral in eq. (13) differ, however, rather essentially from those of refs [18,19] due to our phase-space considerations above. In general, earlier authors have performed the integral in eq. (13) over the whole (pseudo)rapidity range of the gluons similarly as in connection with the infrared problems in QED to obtain results like

$$\frac{d\bar{n}}{d^2 k_{\perp g}} = \frac{2\alpha_s}{3\pi^2} \frac{1}{k_{\perp g}^2} \ln\left(\frac{W^2}{k_{\perp g}^2}\right) \tag{14}$$

As we have shown above the size of the rapidity phase-space, inside which gluon emission gives transverse momentum effects noticeable from the zero-point fluctuations, is

$$\Delta y_{max} \approx \ln\left(\frac{W^2}{6 M_0^2}\right) \tag{15}$$

independently of $k_{\perp g} \leq M_0^2$.

We obtain for the mean number of soft gluons emitted inside the (sub)range Δy:

$$\frac{d\bar{n}}{d^2 k_{\perp g}} = \frac{2\alpha_s}{3\pi^2} \Delta y \frac{1}{k_{\perp g}^2} \theta(M_0^2 - k_{\perp g}^2) \tag{16}$$

The upper limits on the $k_{\perp g}$-integrals imply that a good approximation for the combined effect according to the integral in eq. (13) is

$$\frac{dP}{d^2 p_\perp} = \frac{1}{2\pi} \frac{a}{p_\perp^{2-a} M^a} \theta(M^2 - p_\perp^2) \tag{17}$$

$$a = \frac{4}{3\pi} \bar{\alpha}_s \cdot \Delta y \tag{18}$$

The quantity M is a mass with the size of order M_0, representing the boundary for soft central gluons, at which the cross section should join smoothly to the hard gluon cross section. The normalization

$$\int_0^M \frac{dP}{d^2 p_\perp} d^2 p_\perp = 1 \tag{19}$$

implies that the soft gluons emitted within a (pseudo-)rapidity range Δy of the order of 1 unit have been summed up to one effective gluon with a combined $\vec{p}_\perp$ such that $p_\perp^2 < M^2$. The exact value used for M is not critical: Since a is close to zero (typically $a \approx 0.1$) the form in (17) (neglecting the θ function) deviates but little from a $1/p_\perp^2$ behaviour over a wide $p_\perp$ range around M. A change in M will then essentially only correspond to a reshuffling between gluons considered as hard and as soft.

We have then assumed that the Q^2-dependent coupling constant α_s can be taken as a constant with an effective value $\bar{\alpha}_s$. Unless there is an essential singularity in the expression for $d\bar{n}$ for values of $k_{\perp g} \simeq 0$ this assumption will have little influence because of the appearance of the factor $(1 - \exp(i\vec{k}_\perp \vec{b}))$ in the exponential integral in eq. (13) [18].

The validity of the approximation (17) to the expression (13) has been checked numerically. In fig. 5 we show the behaviour of (13) normalized to (17) without the θ function.

For relevant values $\alpha_s \cdot \Delta y \approx 0.2$ a deviation from (17) is noticeable only for $0.9 < \frac{p_\perp}{M} < 1.1$.

If we combine n gluons from adjacent Δy bins, each gluon with a $\vec{p}_\perp$ chosen according to (17), the $\vec{p}_\perp$ of the resulting gluon

$$\vec{p}_\perp = \sum_{j=1}^{n} \vec{p}_{\perp j} \tag{20}$$

is distributed according to

$$\frac{dP}{d^2p_\perp} = \frac{1}{2\pi} \frac{na}{p_\perp^{2-na} M^{na}} f_n\left(\frac{p_\perp}{M}, a\right) \qquad (21)$$

where, for na small (na < 1) we expect

$$f_n\left(\frac{p_\perp}{M}, a\right) \approx \theta(M^2 - p_\perp^2) \qquad (22)$$

The exact behaviour of f_n is shown for some cases in fig. 6. Combined with the previous figure, this shows the numerical stability in the choice of a Δy size of approximately 1 .

We have used the Lund Monte Carlo [12] to study the effects of soft gluon emission on the event structure in e^+e^- annihilation, according to the principles outlined above. For central hadrons the main result is an increase in the effective Gaussian width in (1) from $\simeq 0.35$ to $\simeq 0.42$ GeV/c. The non-Gaussian nature of the soft gluon contribution is effectively masked by $p_\perp$ from the tunneling process, from particle decays and from true hard gluons.

The combined recoil effect on the fragmentation region hadrons corresponds to the two jets not being back-to-back. Since the original jet axis is not known in $e^+e^- \to q\bar{q}$, it is not meaningful to specify exactly how the recoil is shared between the two sides.

From (17) one can calculate the size of the recoil from an interval Δy

$$\langle p_\perp^2 \rangle = \frac{a}{2+a} M^2 \qquad (23)$$

Naively it would seem that the size of the recoil depends very critically on M . This is not so, since e.g. an increase in M would correspond

to a decrease in the number of hard gluons. The overall effects on the event shape in varying M are then very small and predominantly due to different approximations used in the hadronization scheme for soft and hard gluon events.

The total recoil from soft gluons is approximately 800 MeV/c. The shape of the recoil $p_\perp$ distribution will be of an almost Gaussian character, with significant deviations only at small $p_\perp$. In passing we note that the resulting change in event structure is not all that different from the one obtained with the Field-Feynman recipe [10] of giving also the primary quarks a $p_\perp$ with respect to the jet direction.

In leptoproduction the direction of the outgoing, struck quark is known. It is then meaningful to separate what is traditionally called primordial $k_\perp$ into two pieces [20]. One is a true primordial $k_\perp$ due to Fermi motion inside the proton. This should give a $k_\perp$ of the order of 500 MeV/c compensated in the target fragmentation region. The other contribution is due to soft gluon emission from the struck quark, which is compensated rather centrally. The size of the latter $k_\perp$ contribution depends on the cuts used for hard gluons, but it will generally be larger than the true primordial $k_\perp$.

The sizes of these effects depend on the value of the effective coupling constant $\bar{\alpha}_s$ in eq.(20). We have, remembering our assumption in section 4 that the state configuration is determined during a brief space-time period, used the value $\bar{\alpha} \approx 0.2$ which seems relevant to $Q^2 = W^2$ values in the high energy PETRA range. In case we used instead a value of $\bar{\alpha}_s$ relevant to $Q^2 = M^2$ we obtain effects above of about double the size.

6. Concluding remarks.

True hard gluon emission reveals gluonic degrees of freedom consistent with perturbative QCD and a simple soft hadronization model on an energy-scale of 2-5 GeV and above. Soft central gluonic emission, on the other hand, distorts the transverse momentum properties of the final state particles in a more subtle way. Compared to the assumption that the force field is in a transverse ground state with an energy density per unit length as determined from e.g. charm spectroscopy, we obtain a generally broader $p_\perp$-spectrum stemming from the superposition of several $p_\perp$-bumps each of a general width 1-2 units in rapidity in the central region of phase-space. Further, the general topology of 2-jet events will not be "back-to-back" jets but the events will be "bent", due to the recoil effects from the soft gluon emission. Both effects are, however, rather small for e^+e^--events, where there are no dynamical directions given by the production mechanism. For leptoproduction events the effects will be more pronounced if referred to the momentum transfer (the virtual probe, $\vec{Q}$-) direction.

In Drell-Yan production, finally, the emission of gluons is related to the annihilating $q\bar{q}$-pair and it seems reasonable to expect that the effects are much more confined to the central region. Therefore a combined investigation of leptoproduction and the crossed process Drell-Yan is a promising tool for disentangling the properties of soft gluon emission as compared to primordial $k_\perp$-effects.

Finally, we note that the transverse momentum effects of collinear gluon emission are negligible. This fact does not mean that there are no collinear degrees of freedom. There are e.g. some (rather small) effects related to

the z-spectrum. It is, however, essential to take into account, not only the well-known results from OED in designing a "collinear-gluon-detector" but also the soft hadronization scale, which in the case of transverse momentum observables is essentially larger than the present values of Λ_{QCD} .

References

1. T. Kinochita, J. Math. Physics 3 (1962) 650
   T.D. Lee, M. Nauenberg, Phys. Rev. 133 (1964) B1549
   P. Bloch, A. Nordsieck, Phys. Rev. 52 (1937) 54
2. B. Andersson, G. Gustafson, C. Peterson, Z. Physik C1 (1979) 105
3. B. Andersson, G. Gustafson, Z. Physik C3 (1980) 22
4. B. Andersson, G. Gustafson, T. Sjöstrand, Z. Physik C6 (1980) 235
5. B. Andersson, G. Gustafson, T. Sjöstrand, Phys. Lett. 94B (1980) 211
6. D. Gross, Phys. Rev. Lett. 32 (1974) 1071
   K. Konishi, A. Ukawa, G. Veneziano, Nucl. Phys. B157 (1979) 45
7. A. Krzywicki, B. Petersson, Phys. Rev. D6 (1972) 924
8. F. Niedermayer, Nucl. Phys. B79 (1974) 355
9. B. Andersson, G. Gustafson, C. Peterson, Nucl. Phys. B135 (1978) 273
10. R.D. Field, R.P. Feynman, Nucl. Phys. B136 (1978)1
11. T. Sjöstrand, B. Söderberg, LUTP 78-18
12. T. Sjöstrand, LUTP 80-3
13. A. Casher, H. Neuberger, S. Nussinov, Phys. Rev. D20 (1979) 179
14. A. Petersen, Moriond Proceedings 1980
    JADE Collaboration, Phys. Lett. 101B (1981) 129
15. X. Artru, Orsay preprint LPTHE 78/25
16. J. Ellis, M.K. Gaillard, G.G. Ross, Nucl. Phys. B111 (1976) 253
17. P. Söding: Rapporteur talk at the EPS International conference on high energy interactions, Geneva 1979
    B. Wiik: Rapporteur talk at the Madison Conference 1980
18. G. Parisi, R. Petronzio, Nucl. Phys. B154 (1979) 427
19. Yu. Dokshitzer, D. Dyakonov, S. Troyan, Proceedings of the 13th Winter School, Leningrad (1978)
20. B. Andersson, G. Gustafson, G. Ingelman, T. Sjöstrand, LUTP 80-6 (to be published in Z. Physik C).

Figure captions

Fig. 1. The space-time development of a quark-antiquark-gluon event. The quark and antiquark move along the directions marked q and $\bar{q}$ and are at the endpoints of a string field. The gluon is a pointlike energy-momentum carrying piece of the string moving along the direction g, thereby causing a triangular shape of the outmoving string field. The field breaks by the production of $q\bar{q}$-pairs and the directions of the final state mesons are marked by arrows when they become independent entities. (Note that the slowest mesons in the cms are the first ones to emerge, and also take the larges pieces of the string.)

Fig. 2. The momentum space distribution of the final state particles which appear in the mean along two hyperbolae. The size of the hatches indicates the size of the transverse momentum fluctuations in a string field without excited transverse degrees of freedom. a) is for a general case while b) illustrates the case of almost collinear gluon and quark momenta and c) the case of a soft gluon.

Fig. 3 The space-time motion of a soft kink-gluon, which is stopped before the string breaks up, in a Lorentz frame where its momentum is transverse to the (back-to-back) motion of the endpoint q- and $\bar{q}$-particles. The transverse momentum of the kink-gluon is transferred to two straight string segments, which after the stopping moves in opposite directions. The transverse momentum carried by a string piece, like the one marked out in the figure, is proportional to its length.

Fig. 4 The phase-space boundary curves separating true hard gluon emission (I), collinear gluon emission (II) and soft central gluon emission (III). The exponential-like curve is given by eq. (10), with an asymptote represented by the dotted line.

Fig. 5 The $p_\perp$ spectrum obtained in one Δy interval from eq. (13) with upper $k_\perp$ cutoff M_0, normalized to the expression in eq. (17) (neglecting the θ-function). The resulting functions are shown for three values of $\alpha_s \cdot \Delta y$: 0.2 (full), 0.5 (dashed) and 1 (dotted).

Fig. 6 Three examples of the functions f_n defined in eq. (21): f_2 (full), f_3 (dashed) and f_5 (dotted), all for $\alpha_s \cdot \Delta y = 0.2$. The deviation from a θ function represents the error made in combining the $p_\perp$ from n adjacent Δy bins into one single $p_\perp$.

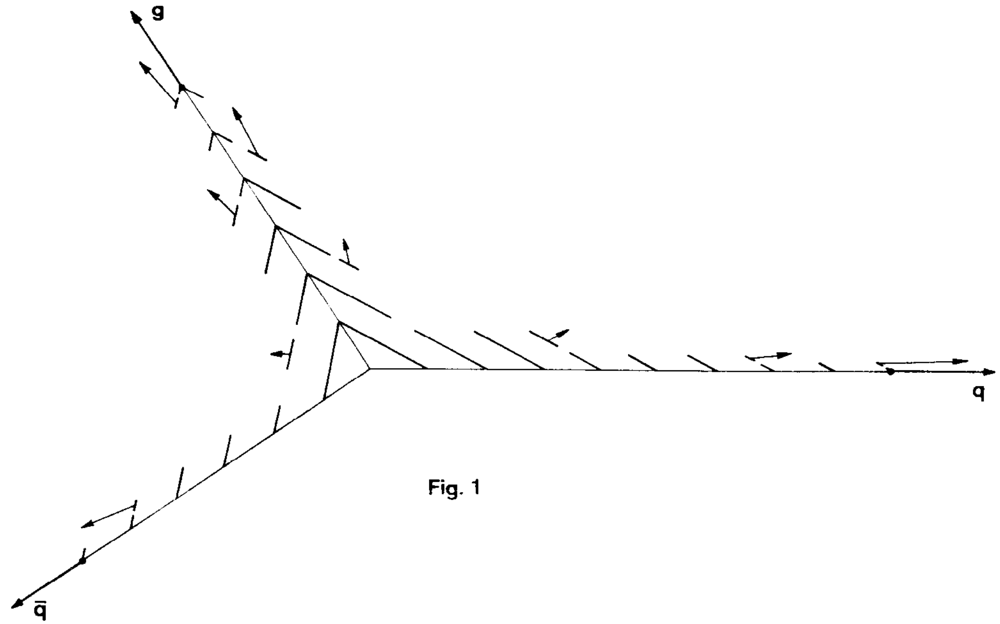

Fig. 1

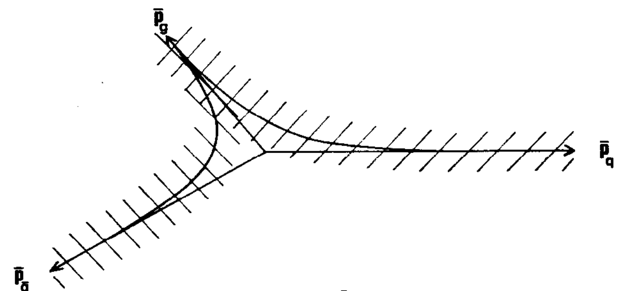

Fig. 2a

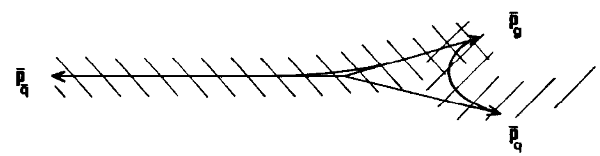

Fig. 2b

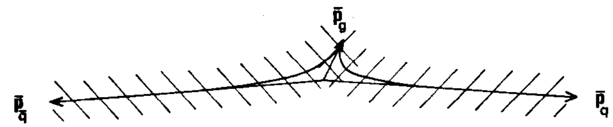

Fig. 2c

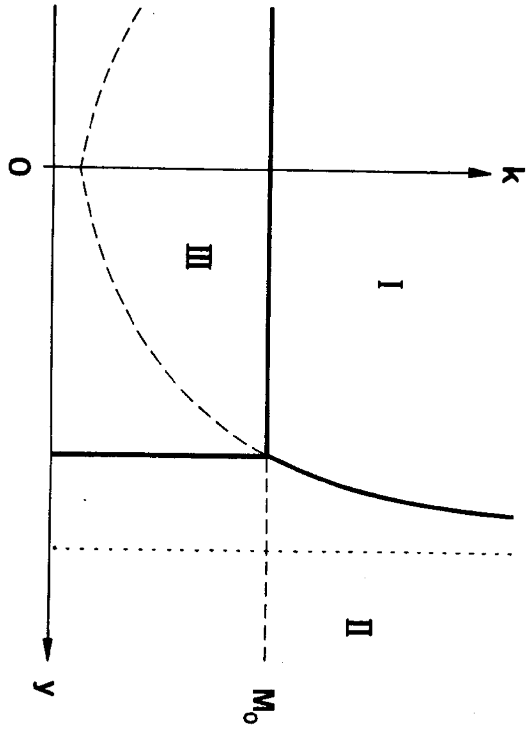

Fig. 4

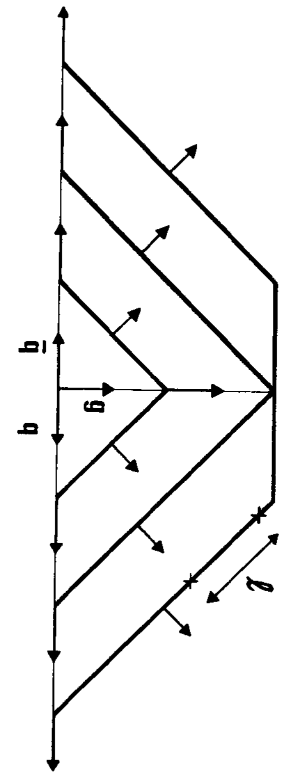

Fig. 3

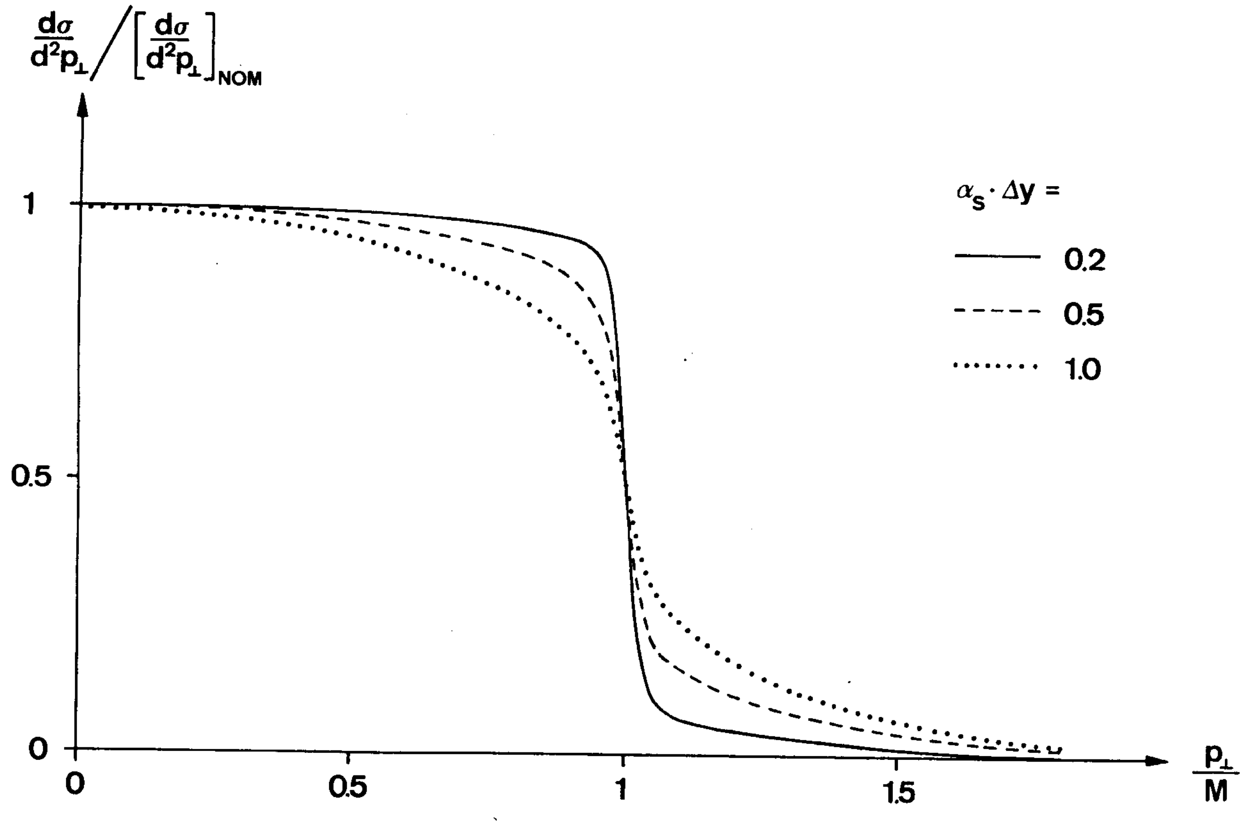

Fig. 5

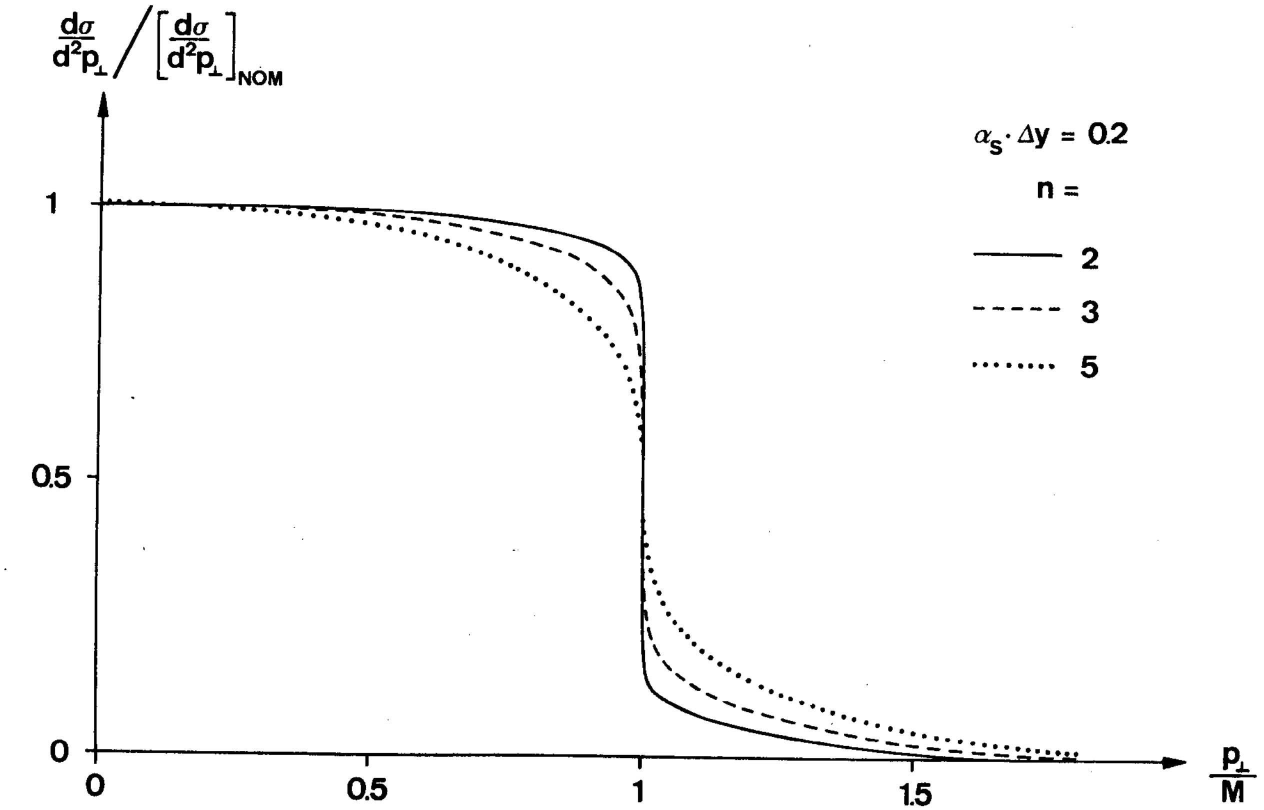

Fig. 6