LU TP 82-7
June 1982

The Lund Monte Carlo for e^+e^- Jet Physics

Torbjörn Sjöstrand

Department of Theoretical Physics
University of Lund
Sölvegatan 14A
S-223 62 LUND
Sweden

Abstract :

We present a Monte Carlo program for e^+e^- annihilation events, based on the Lund Monte Carlo for Jet Fragmentation (described in a companion paper). The main emphasis is on continuum events: flavour production, jet configurations and angular orientation, including weak effects. The decay of heavy onia states (toponium etc.) is briefly discussed. Routines for sphericity, thrust and cluster analysis are also presented. The central part of the paper is a detailed description on how to use the FORTRAN 77 program.

1. Introduction

In the last few years, e^+e^- physics has attracted considerable attention. Due to the simple and well-understood structure of the incoming particles and the equally well-understood electromagnetic annihilation into a virtual photon, the studies of the final state have been far more straightforward than in e.g. ep or pp collisions. In particular this applies to the subject of jets, where the final state itself is complicated enough. In e^+e^- annihilation jets were first observed at SPEAR [1], however only after a careful analysis. With the considerably higher energies attained at PETRA and PEP, two- and three-jet events have become clearly visible [2]. Within the framework of QCD this can readily be explained as the result of the processes $e^+e^- \to \gamma \to q\bar{q}$ or $q\bar{q}g$. At somewhat lower energies, Υ has been shown to give a three-jet structure, naturally explained by the decay $\Upsilon \to ggg$ [2]. Again, however, the energies are so low here that this largely has to rely on statistical analysis. Further studies on this subject are under way at CESR and are planned at DORIS II.

The expansion towards higher energies is continuing. With more RF cavities installed, PETRA will reach approximately 45 GeV CM energy in the search for the top threshold. The LEP project has been approved, and LEP will be used to explore the region around and above the Z^0 pole extensively. A first glimpse of the Z^0 region may be obtained with the SLAC Linear Collider, if approved. Finally, the region a bit below the Z^0 will be studied by TRISTAN.

As we mentioned briefly above, QED and QCD, the candidate theory of strong interactions, can be used perturbatively to describe processes like $e^+e^- \to \gamma \to q\bar{q}$ or $q\bar{q}g$, and data seems to agree very well with the theoretical picture [2]. As examples, the jet axis distribution in $q\bar{q}$ events follows the $1+\cos^2\theta$ behaviour expected for spin 1/2 objects, and the distribution of jets in $q\bar{q}g$ events show that the gluon has spin 1. Although QCD also makes it credible that no quarks or gluons should be

seen free, it is at present not possible within QCD to give a description of the hadronization process which transforms the outgoing partons into jets of hadrons.

A variety of phenomenological models have been proposed to describe the hadronization. In the present paper we will use the so-called Lund model [3]. Due to the complicated final states encountered, it is often advantageous to formulate the whole process, i.e. including the primary interaction, the jet fragmentation, and the subsequent decays of unstable particles, in terms of a Monte Carlo program to be run on a computer. In [4] we present a detailed program simulating the fragmentation and decays given an initial jet configuration - the Lund Monte Carlo for Jet Fragmentation, Lund-JF for short. The purpose of this paper is to present a combination of the Lund-JF with the matrix elements for the initial parton production into a Monte Carlo program for e^+e^- annihilation events.

The probability for producing $q\bar{q}$, $q\bar{q}g$, $q\bar{q}gg$ or $q\bar{q}q'\bar{q}'$ final states will be taken from perturbative QCD. In anticipation of LEP (and the SLC?) we will, in addition to γ exchange, also include the possibility of Z^0 exchange. This will be done according to the standard $SU(2)\times U(1)$ theory of weak and electromagnetic interactions, QFD for short. The details are discussed in section 2.

Another interesting subject for jet studies will be the $t\bar{t}$ toponium states expected but not yet found. Like Υ they are expected to decay predominantly into three gluons, and in the toponium case these gluons will have enough energy to allow realistic comparisons with jet models. This is the subject of section 3.

Due to the large multiplicities encountered in the hadronic final state, a number of collective event shape variables have been suggested. Some of these, like sphericity and thrust, are reasonably well defined, while e.g. the cluster methods appear in a number of variants. In section 4 we discuss some methods, in particular we present a new cluster analysis scheme. Although strictly not of a Monte Carlo nature, this kind of

routines for event analysis is such an important ingredient of contemporary e^+e^- jet physics that some attention to the related problems is justified.

The program elements are introduced in section 5. Here we describe the various subprograms, their purpose and relevant arguments, and the common block variables. We do not discuss the workings of the Lund-JF, however, since the description of this is available separately [4]. Finally, section 6 contains some examples on how to use the program.

2. Annihilation Events in the Continuum

With continuum events we mean reactions of the type $e^+e^- \to \gamma/Z^0 \to q\bar{q}$ etc. without any intermediate state like e.g. J/Ψ, Υ or toponium. The study of these "normal" events will be divided into three parts: production of initial flavour, gluon emission probabilities and angular orientation of the event.

The nature of the boson exchanged, γ or Z^0, and also the polarization of the incoming e^+ and e^-, are important for the first and last of these parts, but the gluon emission probability does not depend on it. We will study two scenarios, one with only QED effects and one also with weak interactions included, QFD [5]. In the standard theory we have the following couplings (here illustrated for the first generation)

$$\begin{aligned} q_\nu &= 0 & v_\nu &= 1 & a_\nu &= 1 \\ q_e &= -1 & v_e &= -1 + 4\sin^2\theta_W & a_e &= -1 \\ q_u &= \tfrac{2}{3} & v_u &= 1 - \tfrac{8}{3}\sin^2\theta_W & a_u &= 1 \\ q_d &= -\tfrac{1}{3} & v_d &= -1 + \tfrac{4}{3}\sin^2\theta_W & a_d &= -1 \end{aligned} \qquad (1)$$

with q the electric charge and v and a the vector and axial couplings to the Z^0. According to the best of present knowledge [5], $\sin^2\theta_W \approx 0.215$. The relative energy dependence of the weak neutral current to the electromagnetic one is given by ($s = Q^2 = W^2$)

$$f(s) = \frac{1}{4\sin^2 2\theta_W} \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \qquad (2)$$

where $M_Z \approx 94$ GeV and $\Gamma_Z \approx 3$ GeV [5].

Although the incoming e^+ and e^- beams normally are unpolarized, it is possible to have machine configurations where they are polarized. This would allow more detailed comparisons with QFD to be made [6]. Therefore we have here included the possibility to have arbitrarily polarized beams, essentially following the formalism presented in [7]. Thus the incoming e^+ and e^- are characterized by polarizations $\bar{P}_\pm$ in the rest frames of the particles

$$\bar{P}_\pm = P_{T\pm}\,\hat{s}_\pm + P_{L\pm}\,\hat{p}_\pm \qquad (3)$$

where $0 \leq P_{T\pm} \leq 1$ and $-1 \leq P_{L\pm} \leq 1$, with the constraint

$$\bar{P}_\pm^2 = P_{T\pm}^2 + P_{L\pm}^2 \leq 1 \qquad (4)$$

Here $\hat{s}_\pm$ are unit vectors perpendicular to the beam directions $\hat{p}_\pm$. To be specific, we choose a right-handed coordinate system with $\hat{p}_\pm = (0,0,\mp 1)$ and standard transverse polarization directions (out of the machine plane for storage rings) $\hat{s}_\pm = (0,\pm 1,0)$, the latter corresponding to azimuthal angles $\phi_\pm = \pm\frac{\pi}{2}$. As free parameters in the program we choose P_{L+}, P_{L-} and

$$P_T = \sqrt{P_{T+}\,P_{T-}} \qquad (5)$$

$$\Delta\phi = \frac{\phi_+ + \phi_-}{2} \qquad (6)$$

2.1. Flavour Production and Total Cross Section

For the QED case, the probability to produce a flavour f is proportional to q_f^2, i.e. up type quarks (u, c, t, ..) are four times as probable as down type ones (d, s, b, ..). In QFD the corresponding probabilities are given by [5,7]

$$\begin{aligned} h_f^{(1)}(s) = {} & q_f^2\,(1 - P_{L+}P_{L-}) \\ & - 2\,q_f v_f\,\mathrm{Re}(f(s))\,(v_e(1 - P_{L+}P_{L-}) - a_e(P_{L-} - P_{L+})) \\ & + (v_f^2 + a_f^2)\,|f(s)|^2\,((v_e^2 + a_e^2)(1 - P_{L+}P_{L-}) - 2v_e a_e(P_{L-} - P_{L+})) \end{aligned} \qquad (7)$$

which depends both on the s value and on the longitudinal polarization of the e^+ and e^- beams in a nontrivial way.

The total cross section in QFD is found to be

$$(\sigma_0)_{QFD} = \frac{4\pi\,\alpha_{em}^2}{s} \sum_f h_f^{(1)}(s) = \frac{86.8\ \text{nb GeV}^2}{s}\, 3 \sum_f h_f^{(1)}(s) \qquad (8)$$

(with the factor 3 coming from colour explicitly written out) which for unpolarized beams at low s reduces to

$$(\sigma_0)_{QED} = \frac{86.8\ \text{nb GeV}^2}{s}\, 3 \sum_f q_f^2 \qquad (9)$$

QCD corrections to this result are small [8],

$$\sigma_1 = \sigma_0 \left(1 + \frac{\alpha_s(Q^2)}{\pi} \right) \qquad (10)$$

in first order and

$$\sigma_2 = \sigma_0 \left(1 + \frac{\alpha_s(Q^2)}{\pi} + (1.986 - 0.115\, n_f)\left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 \right) \qquad (11)$$

in second order, here and henceforth using the $\overline{\text{MS}}$ scheme (formulae for α_s are given below).

So far, quarks have been assumed massless. The problem of the threshold behaviour for new quark flavours is quite nontrivial. Neglecting QCD effects, the corrections are given as functions of $v_q = (1-4m_q^2/s)^{1/2}$, the velocity of a quark with mass m_q, as follows [9]. The vector quark current terms in $h_f^{(1)}$ (proportional to q_f^2, $q_f v_f$ or v_f^2) are multiplied by a threshold factor $v_q(3-v_q^2)/2$ while the axial vector quark current term (proportional to a_f^2) is multiplied by v_q^3. While thus inclusion of quark masses in the QFD formula decreases the cross section, first order QCD corrections tend in the opposite direction [9]. Close to threshold higher order QCD corrections are also expected to be quite significant. Experimentally the overall charm threshold factor seems to be rapidly fluctuating and generally larger than one [2]. It might therefore make sense to skip threshold factors entirely as a first approximation.

2.2. Jet Production in First Order QCD

The lowest order process $e^+e^- \to q\bar{q}$ is in first order QCD modified by the probability for the q or the $\bar{q}$ to radiate off a gluon, i.e. by the process $e^+e^- \to q\bar{q}g$. The matrix element for this is conveniently given in terms of scaled energy variables in the CM frame, $x_1 = 2E_q/W$, $x_2 = 2E_{\bar{q}}/W$ and $x_3 = 2E_g/W$, where W is the total energy in the CM frame (i.e. $x_1+x_2+x_3=2$). For massless quarks the matrix element reads [10]

$$\frac{d\sigma_{q\bar{q}g}}{dx_1\,dx_2} = \sigma_0 \frac{\alpha_s(Q^2)}{\pi} \frac{2}{3} \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} \qquad (12)$$

where the kinematically allowed region is $0 < x_i < 1$, i=1,2,3. Here $\alpha_s(Q^2)$ is the running strong coupling constant [8]

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2\,n_f)\ln(Q^2/\Lambda^2)} \qquad (13)$$

where n_f is the number of active flavours, i.e. basically with $2m_q<W$, and Λ is a parameter of the theory. Generally we take the energy scale $Q^2 = W^2$, but this is in principle undetermined in first order.

The cross section in eq. (12) diverges for x_1 or $x_2 \to 1$, but when the first order vertex and propagator corrections also are included, a corresponding singularity with opposite sign appears in the $q\bar{q}$ cross section, so that the total first order cross section becomes finite (eq.(10)). Physically, this cancellation corresponds to a difficulty to distinguish a single quark from a quark accompanied by a soft or collinear gluon. Specifically for QCD, where the experimentally observable entities are not quarks and gluons but hadrons, we expect the (average) properties of a $q\bar{q}g$ event to approach these of a $q\bar{q}$ event when the gluon becomes soft or collinear. This is the case in the Lund model, where the gluon is considered as a kink on the string stretched between the quark and the antiquark [3] (other, more complicated schemes are also possible [11]).

In a Monte Carlo program the problem of divergences may then be solved by imposing cuts, so that events with a hard gluon are generated according to the matrix element in (12), but three-

jet events with a soft or collinear gluon are lumped together with the two-jet events. The cuts we will use to distinguish the "acceptable" three-jet events are the ones presented in section 4.3 in [4]. For essentially massless quarks the cuts take the form

$$x_1, x_2 < 1 - 2\gamma \tag{14}$$

$$\frac{(1-x_1)(1-x_2)}{(1-x_3)} > \gamma \tag{15}$$

in terms of the single relevant parameter

$$\gamma = \frac{8\, m_a^2}{W^2} \tag{16}$$

with m_a = 1 GeV an average transverse mass for hadrons.

The integral of the first order cross section in the three jet region, excluding the factor $\sigma_0 \cdot \alpha_s/\pi$, may be calculated as a function of γ using e.g. Monte Carlo methods, and may easily be parametrized to an accuracy better than 1 % (down to γ=0.0001, which is quite sufficient for W < 200-300 GeV) . For a given W and hence a given γ this parametrization, together with the α_s value, is used to determine the probabilities to have a two- or three-jet event; for the latter events a pair x_1 and x_2 within the region given by eqs. (14) and (15) is then found according to the matrix element in eq. (12). Of course we have to check that the total probability for gluon emission does not exceed unity, and adjust γ upwards if that should happen.

For massive quarks three kinds of corrections appear. Firstly, the kinematically allowed region in phase space is constrained by

$$\frac{(1-x_1)(1-x_2)(1-x_3)}{x_3^2} > \frac{m_q^2}{W^2} \tag{17}$$

with m_q the quark (and antiquark) mass. Secondly, the cut in eq. (14) is replaced by a harder one

$$x_1, x_2 < 1 - \frac{5\, m_a\, (5\, m_a + 4\, m_q)}{2\, W^2} \tag{18}$$

(while modifications to eq. (15) are minor and are neglected here). Thirdly, the matrix element takes the form [12]

$$\frac{d\sigma_{q\bar{q}g}}{dx_1\, dx_2} = \sigma_0 \frac{\alpha_s(Q^2)}{\pi} \frac{2}{3} \{ \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} - \frac{4m_q^2}{W^2} (\frac{1}{1-x_1} + \frac{1}{1-x_2}) - \frac{2m_q^2}{W^2} (\frac{1}{(1-x_1)^2} + \frac{1}{(1-x_2)^2}) - \frac{4m_q^4}{W^4} (\frac{1}{1-x_1} + \frac{1}{1-x_2})^2 \} \tag{19}$$

Since all of these corrections correspond to a reduction of the three-jet rate, they may easily be taken into account: a three-jet event with given values of x_1 and x_2, found as in the massless case, is reassigned to a two-jet event either if it falls in the region outside the cuts in eqs. (17) or (18) or if it fails in a weighting with the correct matrix element in eq. (19) divided by the massless one in eq. (12).

2.3. Jet Production in Second Order QCD

Two new event types are introduced in second order QCD, $e^+e^- \to q\bar{q}gg$ and $e^+e^- \to q\bar{q}q'\bar{q}'$. Virtual corrections and "degenerate" four-jet events, i.e. events where two of the jets lie so close that they from a fragmentation point of view should be considered as one, modify the three-jet rate. The expression for α_s is also modified to [8]

$$\alpha_s(Q^2) = \frac{12\,\pi}{(33 - 2n_f)\, \ln(\frac{Q^2}{\Lambda^2}) + 6\, \frac{153 - 19n_f}{33 - 2n_f}\, \ln(\ln(\frac{Q^2}{\Lambda^2}))} \tag{20}$$

The four-jet $q\bar{q}gg$ cross section has been calculated by several groups [13-15], which agree on the result. We will use the formulae given in [15] (due to their length, they are not reproduced here), in which the angular orientation has been integrated out, so that five internal kinematical variables remain. These may be related to the ten variables $y_{ij} = (p_i+p_j)^2/W^2$ and $y_{ijk} = (p_i+p_j+p_k)^2/W^2$ (where p_i are the four-momenta), in terms of which the matrix elements are given. As in the $q\bar{q}g$ case divergences appear when some of the $y_{ij} \to 0$, which are removed by cuts on the y_{ij}. Also here we assume that a string is stretched between the partons, going from the q via each of the two gluons to the $\bar{q}$, and corresponding to a colour ordering of the partons. For an event with the colour flow

$q(p_1)g(p_3)g(p_4)\bar{q}(p_2)$ the requirements on acceptable four-jet events then take the form

$$y_{13},\ y_{24} > 2\gamma \tag{21}$$

$$y_{34} > 8\gamma \tag{22}$$

$$\frac{y_{13}\,y_{34}}{y_{14}},\ \frac{y_{24}\,y_{34}}{y_{23}} > \gamma \tag{23}$$

with the same γ as in eq. (16).

Whereas the way the colour flux tube is stretched is uniquely given in $q\bar{q}g$ events, for $q\bar{q}gg$ events there are two different possibilities: $qg_1g_2\bar{q}$ or $qg_2g_1\bar{q}$. However, a knowledge of the quark and gluon colours, obtained by perturbation theory, will uniquely specify the stretching of the string, so long as the two gluons do not have the same colour. The probability for the latter is down in magnitude by a factor $1/N_C^2$, where $N_C = 3$ is the number of colours. Now, it is possible that topological properties of QCD, not exhibited in perturbation theory, play a fundamental rôle [16]. If e.g. two gluons always can be distinguished by the topology of the connecting colour field, there should be no interference terms. Following [17], we will choose to neglect these terms, the corrections of 10-20% (for a given kinematical setup, less for the total four-jet rate) coming from this being tolerable compared to the many other uncertainties present anyhow.

This means that the matrix elements given in [15] are modified as follows: the planar QED type graphs with group weight C_F^2 (the term A in Appendix B in [15]) are unchanged, the non-planar (interference) QED type graphs with group weight $C_F(C_F-\frac{1}{2}N_C)$ (the term B) are neglected, and the QCD graphs involving the three-gluon vertex with group weight C_FN_C (the term C) are multiplied by $(N_C^2-1)/N_C^2 = 8/9$. Further, in [15] only one fourth of the total cross section is written out, the remaining parts being obtained by the independent interchange of momenta $q \leftrightarrow \bar{q}$ and $g_1 \leftrightarrow g_2$. If the two gluons are assumed distinguishable, only the simultaneous interchange is acceptable [17], but a compensating factor of 2 is gained from the change of phase space.

The four-jet cross section $e^+e^- \to q\bar{q}q'\bar{q}'$ has also been included. Since the cross section for this process is down by more than an order of magnitude compared to $q\bar{q}gg$ it will be of relatively minor importance. Although the cuts in eqs. (22)-(23) no longer are necessary from a hadronization point of wiew, we must still include them to be consistent, since $g \to gg$ and $g \to q'\bar{q}'$ are competing processes (in jet calculus [18] language, the probability for a g of given virtuality to "decay" into $q'\bar{q}'$ is reduced by the fact that it could "already" have decayed into gg at a higher virtuality). Since we wish to simulate $q\bar{q}q'\bar{q}'$ and $q\bar{q}gg$ events in the same program, specifically to use the cuts in eqs. (21) - (23) for both processes, we choose to label the momenta $q(p_1)\bar{q}'(p_3)q'(p_4)\bar{q}(p_2)$ where $q'\bar{q}'$ is the pair coming from the gluon. Similarly to the $q\bar{q}gg$ case, an assumed knowledge of the colour flow means that we may neglect the graphs with group weight $C_F(C_F-\frac{1}{2}N_C)$ (the term E) and only keep those with group weight C_FT_R (the term D), a minor modification of the total cross section. Properly the terms with group weight C_F (the term F) should also be included when the flavours $q=q'$, but they give no net contribution to the jet cross section, only a minor correction to the flavour assignment of the jets, and have here been neglected.

As mentioned above, second order corrections also influence the three-jet rate. The size of these corrections is still a matter of controversy. Two groups [15,19] claim that the corrections are large and positive, the third [20] that they are small and negative. Since different quantities are calculated, both results may be mathematically correct, but it is not clear how they should be related to physically observable quantities [21]. Specifically, four-jet events that fail the cuts in eqs. (21)-(23) should, in some not quite well-defined way, be projected onto three-jet events. The singularities obtained when a $y_{ij} \to 0$ in the four-jet cross section will then cancel against the divergences in the second order virtual graphs to give a finite three-jet cross section for $x_1,\ x_2 \neq 1$. Formally we may write this as

$$\left(\frac{d\sigma_{q\bar{q}g}}{dx_1\,dx_2}\right)^{(2)} = \left(\frac{d\sigma_{q\bar{q}g}}{dx_1\,dx_2}\right)^{(1)} \left(1 + \frac{\alpha_s(Q^2)}{\pi}\, g(\gamma,x_1,x_2)\right) \qquad (24)$$

where $(d\sigma_{q\bar{q}g}/dx_1dx_2)^{(1)}$ is the first order cross section given in eq. (12), with the exception that the second order formula (eq. (20)) is used for α_s. Which function g to use then depends on which approach [15,19,20,21] is the physically most relevant one (however, in neither case can the published formulae be used as they stand; for an example of some of the necessary modifications to take into account the exact boundaries used between four-, three- and two-jet events see [21]). In the present paper we have put g identically zero, but the user may insert the formula of his choice (this is particularly simple if g can be approximated by a constant, which may well be the case even if the second order corrections are large [15,19]). It should be noted that, while many interesting jet studies can be performed without explicitly introducing g, a correct determination of α_s to second order will require that g is known.

To choose jet configuration (with g given), a straightforward generalization of the first order method is used. The four-jet cross section, excluding the trivial factor $\sigma_0\cdot(\alpha_s/\pi)^2$, is parametrized as a function of γ. An upper limit g_{max} to g (i.e. $g(\gamma,x_1,x_2) < g_{max}$ for all x_1, x_2 fulfilling eqs. (14) - (15)) is assumed known and a preliminary three-jet cross section is found by multiplying the first order expression by $(1+\alpha_s g_{max}/\pi)$. If the total probability for three or four jets becomes larger than unity, γ is increased. For a three- or four-jet event the internal kinematical variables are chosen according to the matrix elements. A three-jet event is then allowed to reduce to a two-jet one if a weighting with $(1+\alpha_s g(\gamma,x_1,x_2)/\pi)\ /\ (1+\alpha_s g_{max}/\pi)$ fails.

The quark mass corrections to the four-jet cross sections have been calculated [13] but are quite tedious and have never been published, whereas the methods used to calculate the second order corrections to the three-jet cross section precludes the insertion of quark masses. We will therefore not include mass corrections to the second order matrix elements. (Corrections to phase space and to cuts on divergences must be included as

usual, however.) The application of the variables y_{ij} and y_{ijk}, obtained from the massless matrix element, to a situation with nonzero quark masses is thus not unique. For definiteness we have chosen to assume that $y_{ijk} = ((p_i+p_j+p_k)^2-p_l^2)/W^2$ so that $y_{ijk} = 1-x_l$ (i, j, k, l a permutation of 1, 2, 3, 4) and $y_{ij} = ((p_i+p_j)^2-(p_1^2+p_2^2+p_3^2+p_4^2)/3)/W^2$ so that $y_{ijk} = y_{ij}+y_{ik}+y_{jk}$.

2.4. Angular Orientation of the Events

The angular orientation of a three- or four-jet event may be described in terms of three angles χ, θ and ϕ, for two-jet events only θ and ϕ are necessary. From a standard orientation, with the q along the +z axis and the $\bar{q}$ in the xz plane with $x > 0$, an arbitrary orientation may be reached by rotations $+\chi$ in azimuthal angle, $+\theta$ in polar angle and $+\phi$ in azimuthal angle (in that order). Cross sections, including QFD effects and arbitrary beam polarizations have been given for two- and three-jet events e.g. in [7,22]. We will here use the formalism presented in [7] (however with $\chi \to \pi-\chi$ and $\phi \to -(\phi+\frac{\pi}{2})$). Since the resulting formulae are rather tedious, but straightforward to apply once the internal jet configuration is known, we will not reproduce them here.

The angular orientation of four-jet events has only been studied in the QED case [13]. Rather than reproducing those formulae here, we have chosen to approximate four-jet events with similar three-jet ones for a determination of the differential cross sections to use. With this scheme some angular asymmetries [13] are disregarded, but a simulation of QFD effects becomes straightforward. Many possibilities exist to project four-jet onto three-jet events, we have chosen the simple one of adding the gluon momenta in a $q\bar{q}gg$ event (the $q'\bar{q}'$ momenta in a $q\bar{q}q'\bar{q}'$ event) vectorially to give the gluon momentum in the $q\bar{q}g$ event, and then rescale energies so that $x_1+x_2+x_3 = 2$. (For a $qg_1g_2\bar{q}$ event generated according to the QED-like graphs it could be more realistic to count the combination qg_1, g_1g_2 or $g_2\bar{q}$ with the smallest invariant mass as one jet. We have checked that this would make a negligible difference.)

The expressions presented in [7] refer to the massless case. For the QED $q\bar{q}$ and $q\bar{q}g$ cases we may also optionally include the correct mass dependence [23], but in general these corrections are minor.

2.5. Remarks on Multijet Events and Radiative Corrections

In the sections above we have discussed the production of two-, three- and four-jet events. At present energies (below 40 GeV) these are probably the only jet situations that can be studied in any detail, and they will dominate even at considerably larger energies, but eventually more complicated geometries will become observable, geometries for which the present program is not adequate. The explicit calculation of five-jet etc. cross sections probably being impractical, one possibility could be to combine the program with some leading log jet evolution (see e.g. [24]). Then the formalism presented above would be taken to describe the breakup of quarks and gluons at high virtuality (and also to give the angular orientation of the event), to be supplemented with leading log formulae for breakups at lower virtualities. If the colour flow is kept track of, the final jet configuration could then be allowed to fragment in the standard fashion [4]. Although no provisions have been made for such an extension of the present program, this would be quite feasible.

So far we have only discussed purely hadronic final states. However, in experimental situations radiative corrections become important. The emission of hard photons leads to events with a smaller hadronic energy and hence a different distribution of flavours (due to thresholds and QFD effects) and different gluon emission probabilities. Radiative corrections to the total cross section may also be quite significant, in particular close to the Z^0 peak. The recoil of the hadronic system will influence angular distributions, e.g. the two jets in a $q\bar{q}$ event will no longer be back-to-back. Also, the forward-backward flavour asymmetries expected in QFD will be superimposed on asymmetries coming just from higher

order QED.

We have not explicitly included any of these corrections in the present program. One reason for this is that detailed Monte Carlo programs for radiative corrections, also including the possibility of Z^0 exchange, already are available [25]. However, provisions are made to include an initial state radiative photon, obtained from an external program such as [25], into the event generation chain (the possibility of ending up on a resonance, as e.g. Υ, is not included). This possibility is in principle also open for final state radiation, although this process may be less dangerous to neglect, since it is suppressed by a factor q_f^2 compared to initial state radiation and since the associated event structure is more similar to nonradiative events, with the radiative photon normally residing within a jet.

Conflicts with the rest of the program may arise on the question of beam polarization, since photon emission will change the polarization properties. A reasonable approximation may be to take the quantities defined in eqs. (3) - (6) to refer to the mean properties after initial state radiation.

3. Jets from Onia Decays

For heavy $J^{PC} = 1^{--}$ resonances, specifically toponium, the dominant decay modes are expected to be three-gluon and photon-gluon-gluon decays. Decays via a virtual photon into e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$, $q\bar{q}$, $q\bar{q}g$ etc. are also important, but since these decays are of the same character as seen off-resonance, no new model is necessary in this case.

The matrix element for $q\bar{q} \to ggg$ is (in lowest order) [26]

$$\frac{1}{\sigma_{ggg}} \frac{d\sigma_{ggg}}{dx_1\, dx_2} = \frac{1}{\pi^2 - 9} \left\{ \left(\frac{1-x_1}{x_2 x_3}\right)^2 + \left(\frac{1-x_2}{x_1 x_3}\right)^2 + \left(\frac{1-x_3}{x_1 x_2}\right)^2 \right\} \quad (25)$$

where the $x_i = 2E_i/W$ in the CM frame. This is a well-defined expression, without the kind of singularities encountered in e.g. the $q\bar{q}g$ matrix element.

From the fragmentation point of view, cuts on the matrix elements are still necessary, however. In ggg decays we expect the string pieces between the gluons to form an expanding triangle with the gluons at the corners [3,4]. Hence, we encounter cuts of the same kind as in eq. (22), i.e.

$$x_1, x_2, x_3 < 1 - 8\gamma = 1 - \left(\frac{8\, m_a}{W}\right)^2 \qquad (26)$$

Cuts of the form presented in eq. (15) also appear but are here less significant than in the $q\bar{q}g$ case, since the cuts in eq. (26) are harder than the ones in eq. (14) and since we here have no double divergence to contend with.

In case the cut requirements are not satisfied, there still is a definite three-gluon colour structure, but two gluons lie so close that they effectively give a single, somewhat broader jet. Hence we can allow the ggg system to fragment as were it a two-gluon jet system. Unfortunately this procedure breaks down for Υ where, with these cuts, all events are reduced to gg events, in clear disagreement with experimental data. Although some ad hoc improvements could be made, it is difficult to construct a realistic three-jet Monte Carlo for Υ within the present formalism. The program presented here is thus only concerned with toponium (and beyond).

Another process is $q\bar{q} \to \gamma gg$, obtained by replacing a gluon in $q\bar{q} \to ggg$ by a photon, which has the same normalized differential cross section as in eq. (25) above if e.g. x_1 is taken to refer to the photon. The relative rate is [26]

$$\frac{\sigma_{\gamma gg}}{\sigma_{ggg}} = \frac{36}{5}\, q_f^2\, \frac{\alpha_{em}}{\alpha_s(Q^2)} \qquad (27)$$

where the electromagnetic coupling constant $\alpha_{em} \approx 1/137$. For masses $M_{gg} > 8m_a \approx 8$ GeV the two-gluon jet model may be used to describe the fragmentation of the system recoiling from the photon, while a phase space model is used for $2m_a < M_{gg} < 8m_a$. Below $2m_a \approx 2$ GeV we do not attempt at a description. This region is interesting for glueball searches and studies, but will obviously not require any jet Monte Carlo.

As for continuum events, higher order QCD corrections will give rise to events with more complicated structure, e.g. gggg, $gggq\bar{q}$, γggg and $\gamma gq\bar{q}$ [27], but this is not studied in the present paper.

In the present implementation the angular orientation of the ggg and γgg events is given for the $e^+e^- \to \gamma \to$ onium case [26] (optionally with beam polarization effects as for continuum events), i.e. weak effects [22] have not been included so far.

4. Event Analysis

To describe the complicated geometries encountered in e^+e^- events, a number of collective variables have been introduced. Many of these are of a rather specialized nature, but some have become standard measures, in particular the sphericity and thrust families. While these two give some information on the event shape, they do not explicitly reconstruct any jet axes (except for two-jet events). For such reconstructions a number of different cluster methods have been developed; we will here present one.

Neither of the methods presented here are Lorentz invariant. Although they in principle can be used (and useful) in any frame of reference, specifically the lab frame, it is understood that quantities intended for comparisons between different experiments should be calculated in the CM frame of the hadronic system. If possible, both charged and neutral particles should be included in the analysis. In any case, a specification of which particles have been used should always be given.

For the rest of this section we will assume an event to be characterized by the particle momenta vectors $\vec{p}_i$, with i = 1,2,.....,n an index running over the particles in the event. Cartesian components are designated $p_{i\alpha}$, α = 1,2,3 and the absolute momenta p_i (while in other sections this has been used to designate four-vectors).

4.1. The Sphericity Family

The sphericity tensor is defined as [28]

$$S_{\alpha\beta} = \frac{\sum_i p_{i\alpha}\, p_{i\beta}}{\sum_i p_i^2} \tag{28}$$

By standard diagonalization of $S_{\alpha\beta}$ three eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$ (with $\lambda_1+\lambda_2+\lambda_3=1$) and corresponding eigenvectors $\bar{v}_1$, $\bar{v}_2$ and $\bar{v}_3$ may be found. The sphericity S is then defined as

$$S = \frac{3}{2}(\lambda_2 + \lambda_3) \tag{29}$$

so that $0 \leq S \leq 1$. Sphericity is essentially a measure of p_t^2 with respect to the jet axis; two back-to-back jets with no transverse momentum spread corresponds to S=0 and a perfectly isotropic event to S=1. Similarly the aplanarity $A = \frac{3}{2}\lambda_3$, $0 \leq A \leq \frac{1}{2}$, is defined to describe the momentum component out of the event plane.

Since the sphericity tensor (or, rather, the dividend and divisor of it) is quadratic in momentum, sphericity is sensitive to fragmentation effects and is not an infrared safe quantity in QCD perturbation theory. A useful generalization of the sphericity tensor is

$$S^r_{\alpha\beta} = \frac{\sum_i p_i^{r-2}\, p_{i\alpha}\, p_{i\beta}}{\sum_i p_i^r} \tag{30}$$

where r is the power of the momentum dependence. While r=2 thus corresponds to sphericity , r=1 corresponds to linear measures calculable in perturbation theory [29]. Quantities that have been defined in literature for the r=1 case are [15,30]

$$C = 1 - H_2 = 3(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3) \tag{31}$$

$$D = 27\,\lambda_1\lambda_2\lambda_3 \tag{32}$$

Noninteger r values may obviously also be used, and corresponding generalized sphericity S^r and generalized aplanarity A^r calculated. (Thus, the average angle between the true and reconstructed jet axis in two-jet events at PETRA/PEP energies has a minimum for $r \approx 1.5$, even if the gain in accuracy compared to r=1 or r=2 is rather modest.)

4.2. The Thrust Family

We define the quantity thrust T by [31]

$$T = \max_{|\bar{n}|=1} \frac{\sum_i |\bar{n}\cdot\bar{p}_i|}{\sum_i p_i} \tag{33}$$

and the thrust axis $\bar{v}_1$ is given by the $\bar{n}$ vector for which maximum is attained. The allowed range is $\frac{1}{2} \leq T \leq 1$ with "perfect" back-to-back jets corresponding to T=1 and perfectly isotropic events to $T=\frac{1}{2}$. In passing we note that the definition above is not the only one found in literature. For events studied in the CM frame and where all particles are detected the definitions agree, whereas e.g. a definition according to the formula

$$T = 2 \max_{|\bar{n}|=1} \frac{|\sum_i \theta(\bar{n}\cdot\bar{p}_i)\,\bar{p}_i|}{\sum_i p_i} = 2 \max_{\theta_i=0,1} \frac{|\sum_i \theta_i\,\bar{p}_i|}{\sum_i p_i} \tag{34}$$

($\theta(x)=1$ if $x>0$, else 0) would give different results if e.g. only charged particles are detected. It would e.g. be possible to have $T>1$; to avoid such problems an extra particle, balancing the total momentum, is often introduced [32].

Eq. (33) may be rewritten as

$$T = \max_{\varepsilon_i=\pm 1} \frac{|\sum_i \varepsilon_i\,\bar{p}_i|}{\sum_i p_i} \tag{35}$$

(this may also be viewed as applying eq. (34) to an event with 2n particles, n carrying the momenta $\bar{p}_i$ and n the momenta $-\bar{p}_i$, thus automatically balancing the momentum). To find the thrust value and axis this way, 2^{n-1} different possibilities would have to be tested (the reduction by a factor 2 comes from T being unchanged when all $\varepsilon_i \to -\varepsilon_i$). This rapidly becomes prohibitive.

An alternative method is to iterate the thrust axis from some starting direction $\bar{n}^{(0)}$ according to

$$\bar{n}^{(j+1)} = \frac{\sum_i \varepsilon(\bar{n}^{(j)}\cdot\bar{p}_i)\,\bar{p}_i}{\left|\sum_i \varepsilon(\bar{n}^{(j)}\cdot\bar{p}_i)\,\bar{p}_i\right|} \qquad (36)$$

($\varepsilon(x)=+1$ if $x>0$, $=-1$ if $x<0$). It is easy to show that the related thrust value will never decrease, $T^{(j+1)} \geq T^{(j)}$. In fact, the method normally converges in 2-4 iterations. Unfortunately, this convergence need not be towards the correct thrust axis but is occasionally only towards a local maximum of the thrust function [32]. We know of no foolproof way around this complication, but the danger of an error may be lowered if several different starting axes $n^{(0)}$ are tried and found to agree. These $n^{(0)}$ are suitably constructed from the n' ($n'\simeq 4$) particles with largest momenta in the event, and the $2^{n'-1}$ starting directions $\Sigma\,\varepsilon_i\bar{p}_i$ constructed from these are tried in falling order of the corresponding absolute momentum values. When a predetermined number ($\simeq$2-3) of these starting axes have given convergence towards the same (best) thrust axis this is accepted.

In the plane perpendicular to the thrust axis, a major [33] axis and value may be defined in just the same fashion as thrust, i.e.

$$\text{Major} = \max_{|\bar{n}|=1,\ \bar{n}\cdot\bar{v}_1=0} \frac{\sum_i |\bar{n}\cdot\bar{p}_i|}{\sum_i p_i} \qquad (37)$$

In a plane more efficient methods can be used to find an axis than in three dimensions [34], but for simplicity we will use the same method as above. Finally, a third axis, the minor axis, is defined perpendicular to the thrust and major ones, and a minor value is calculated. The difference between major and minor is called oblateness O or O_B. The upper limit on oblateness depends on the thrust value (for $T > 1/\sqrt{2}$ it is given by $O_B^2 < 1 - T^2$, while for $T < 1/\sqrt{2}$ the exact boundary is more complicated, but obviously with $O_B < T$). In general $O_B\simeq 0$ corresponds to events symmetrical around the thrust axis and high O_B to planar events.

As in the case of sphericity, a generalization to arbitrary momentum dependence may easily be obtained, here by replacing

the $\bar{p}_i$ in the formulae above by $p_i^{r-1}\bar{p}_i$ and the p_i by p_i^r. We have included this possibility, although so far it has not found any experimental use.

4.3. A Cluster Algorithm

The objective of cluster algorithms is to determine the number of jets present in an event and to reconstruct the corresponding jet axes. A number of such algorithms have been proposed [35], often based on various theoretical considerations in clustering theory. We will here present a more "nuts and bolts" approach based on our physical picture of jet properties. In actual practice, different methods may well be roughly comparable, since the performance is mostly dictated by smearing in the jet fragmentation and particle decays that no algorithm could hope to disentangle, e.g from particles in the central region where the jets overlap. We will here only make use of particle momenta; the problem of including information from charge, strangeness etc. is a far more subtle one.

The characteristic feature of a jet is that the particles in it have limited transverse momenta with respect to the common jet axis and hence also with respect to each other. A distance measure d between two particles with momenta $\bar{p}_i$ and $\bar{p}_j$ should thus not depend critically on the longitudinal momenta but only on the relative transverse momentum. We have chosen the distance measure

$$d_{ij}^2 = \tfrac{1}{2}(p_i p_j - \bar{p}_i\bar{p}_j)\frac{4\,p_i p_j}{(p_i+p_j)^2} = \frac{4\,p_i^2\,p_j^2\sin^2(\theta_{ij}/2)}{(p_i+p_j)^2} \qquad (38)$$

For small relative angles θ_{ij} this reduces to

$$d_{ij} \simeq \frac{|\bar{p}_i\times\bar{p}_j|}{|\bar{p}_i+\bar{p}_j|} \qquad (39)$$

which has a simple physical interpretation as the transverse momentum of either particle with respect to the direction given by the sum of the two particle momenta. Unlike the approximate expression, however, d does not vanish for two particles with

opposite momenta but is then more related to the invariant mass of them.

The basic scheme goes as follows. Initially each particle is assumed to be a cluster by itself. Then the two clusters with smallest relative distance d is found and, if $d < d_{join}$, with d_{join} some predetermined distance, the two clusters are joined to one, i.e. their momenta are added vectorially to give the momentum of the new cluster. This is repeated until the distance between any two clusters is $> d_{join}$. The number and momenta of these final clusters then represent our reconstruction of the initial jet situation, and each particle is assigned to one of the clusters.

To make this scheme workable, two further ingredients are necessary, however. Firstly, when joining two clusters, particles belonging to either of these may, after the joining, end up lying closer to another cluster. Hence, after each joining all particles are reassigned to the closest of the remaining clusters, and afterwards the cluster momenta are recalculated. To save time, this assignment procedure is not iterated until a stable maximum is reached (except after all joinings have already been made) but, since all particles are reassigned at each step, such an iteration is effectively taking place in parallel with the cluster joining. For these assignments the same distance measure d as above may be used, but here it is also possible to use the "multicity" [32] measure, i.e. assign a particle to the cluster with respect to which it has the largest projected momentum (i.e. again mimimum p_t, but in this case using an asymmetric transverse momentum measure). In practice the difference is minor, but by stopping the joining at a predetermined number of clusters n this may be used to give a, probably quite accurate, estimate of the corresponding n-ticity.

Secondly, the large multiplicities normally encountered means that, if each particle initially is to be treated as a separate cluster, the program will become extremely slow. Therefore a smaller number (typically ≈8) of clusters, from which the iteration above starts, is constructed as follows. The particle

with highest momentum is found and, together with all particles within a distance $d < d_{init}$ ($d_{init} << d_{join}$) from it, it forms one cluster. For the remaining particles, not assigned to this cluster, the procedure is iterated, until all particles have been used up. Particles in the central region, $p < 2 \cdot d_{init}$, are treated separately; if their vectorial momentum sum is $> 2 \cdot d_{init}$ they are allowed to form one cluster, otherwise they are left unassigned in the initial configuration. The value of d_{init}, so long as reasonably small, has no physical importance, in that the same final cluster configuration will be found as if each particle initially is assumed to be a cluster by itself.

Thus the jet reconstruction depends on one single parameter, d_{join}, with a clearcut physical meaning as a transverse momentum "jet resolution power" (neglecting smearing from fragmentation, d between two clusters of equal energy corresponds to half the invariant mass of the corresponding original partons). If we only wish to reconstruct very distinct jets, a large d_{join} should be chosen, while a small d_{join} would allow the separation of close jets, at the cost of sometimes artificially dividing a single jet into two. In particular, b quark jets may here often be a nuisance. The value of d_{join} to use is in principle independent of the hadronic energy W, in practice a d_{join} rising slightly with W may be necessary to avoid being swamped by hadronization effects. Measured in angular resolution, however, there is a rapid improvement with higher W.

5. The Program Components

The program presented here consists of a number of subroutines to be called by the user, some subprograms used internally, and a common block with switches and parameters. The main program is supplied by the user.

Jet fragmentation and particle decays are taken care of by the Lund-JF. The main point of contact between the two programs is the common block LUJETS, in which the event record is stored.

Hence the user must be familiar with the content of section 6.1 in [4], where the organization of the event record is discussed. Some knowledge of the functions KLU and PLU and of the subroutines LULIST and LUEDIT may also be useful. Sensible default values are provided for all parameters in the Lund-JF, but obviously one may wish to change some of them for detailed studies.

The program is written in FORTRAN 77. As in the Lund-JF we assume the existence of a random number generator RANF. Also as in the Lund-JF, subroutine, function and common block names have been standardized so as to begin with LU or, for real functions, UL.

5.1. Routines for Continuum Events

The generation of a jet event in the continuum is administrated by the subroutine LUEEVT, which calls some other subroutines and functions for specific tasks. A user might wish to replace the dummy LURADK and ULX3CO routines, the rest he will normally never see (possibly except LUXTOT). In addition, several of the parameters in the LUDATE common block may be used to modify the behaviour of the program. The right-handed coordinate system is oriented so that the incoming e^- moves in the +z direction and the incoming e^+ in the -z direction, with equal and opposite momenta.

SUBROUTINE LUEEVT(IFL,ECM)

Purpose : to generate a complete $e^+e^- \to \gamma/Z^0 \to q\bar{q}$, $q\bar{q}g$, $q\bar{q}gg$ or $q\bar{q}q'\bar{q}'$ event.

IFL : the q quark flavour according to IFL code (1=u, 2=d, 3=s, 4=c, 5=b, 6=t, 7=l, 8=h). If IFL=0, a flavour will be chosen at random according to the relevant probabilities.

ECM : the total energy of the system in the CM frame.

SUBROUTINE LUXTOT(ECM,XTOT)

Purpose : to calculate the total hadronic cross section including quark threshold , weak, beam polarization and QCD effects (see MSTE(1) - MSTE(3) and PARE(11) - PARE(13)).

Radiative corrections are not included.

ECM : the total energy of the system.

XTOT : the total cross section in nb.

Remark : the subroutine is called from LUEEVT according to specification in MSTE(12), with results saved in PARE(22) and PARE(23) (=XTOT), but may also be called directly by the user.

SUBROUTINE LURADK(ECM,MK,PAK,THEK,PHIK,ALPK)

Purpose : to describe radiative corrections, specifically the initial state emission of a photon. This routine must be supplied by the user, e.g. following the formalism presented in [25]. The routine is called from LUEEVT if MSTE(7)=1, with ECM as input value.

ECM : the total energy of the system.

MK : 0 if no photon emitted, 1 if photon emitted.

PAK : photon momentum $k = |\bar{k}|$ (if MK=1).

THEK : photon polar angle θ_k (if MK=1).

PHIK : photon azimuthal angle ϕ_k (if MK=1).

ALPK : the polar angle α_k that either the e^+ or e^- direction, whichever relevant, is rotated in a boost from the overall CM frame to the hadronic CM frame, cf. [25] (if MK=1).

Note : the transformation from the hadronic CM frame, with the same event orientation there as in the CM frame of an event with no radiative photon, to the overall CM frame is given by 1) a rotation $-\phi_k$ in azimuthal angle, 2) a rotation $+\alpha_k$ in polar angle, 3) a boost $\bar{\beta} = -k(\sin\theta_k, 0, \cos\theta_k)/(W-k)$, and 4) a rotation $+\phi_k$ in azimuthal angle (the first rotation is to ensure that no net azimuthal rotation of the event takes place in the limit $k \to 0$).

SUBROUTINE LUXIFL(ECM,IFL)

Purpose : to generate the quark flavour of an event when the flavour is not specified in the LUEEVT call (IFL=0). This may be done according either to QED or QFD, optionally including mass (threshold) and beam polarization effects.

ECM : CM energy of the hadronic system, i.e. excluding a radiative photon.

IFL : the flavour generated.

SUBROUTINE LUXJET(ECM,NJET,CUT)

Purpose : to determine the number of jets (2, 3 or 4) to be generated within the kinematically allowed region, which has to be chosen so that no problems are encountered in the hadronization and so that the total probability for 3 or 4 jets is smaller than 1.

ECM : CM energy of the hadronic system, i.e. excluding a radiative photon.

NJET : number of jets to be generated. If 3 or 4 additional cuts may be made in LUX3JT or LUX4JT which lead to two-jet events.

CUT : the single parameter γ which determines the shape of the allowed region in phase space for 3- or 4-jet events, see eqs. (14) - (16) and (21) - (23).

SUBROUTINE LUX3JT(NJET,CUT,IFL,ECM,X1,X2)

Purpose : to generate the internal momentum variables for a three-jet $q\bar{q}g$ event. Also to take into account mass effects or second order corrections to the three-jet cross section.

NJET : number of jets, may be changed to 2 in connection with mass effects or second order corrections to cross section.

CUT : parameter describing the allowed region in phase space, determined in LUXJET.

IFL : quark flavour, needed for mass-dependent corrections.

ECM : (= W) CM energy of the hadronic system.

X1, X2 : $x_i = 2E_i/W$, 1=q, 2=$\bar{q}$, 3=g, i.e. twice the energy fractions taken by the quark and the antiquark.

Remark : in LUEEVT the latter variables are called X1 and X3, as they refer to the first and third jet following the colour flow $qg\bar{q}$.

FUNCTION ULX3CO(CUT,X1,X2)

Purpose : to describe the second order corrections to the three-jet cross section, i.e. the function $g(\gamma,x_1,x_2)$ of eq. (24). This routine is a dummy, to be replaced by the user following e.g. one of [15,19,20]. The maximum value (or at least an upper limit) of ULX3CO within the allowed region should be stored in PARE(6).

CUT, X1, X2 : as in LUX3JT.

Remark : For the case that the second order corrections are assumed to give the same event shape distribution as the lowest-order cross section, the routine can be used as it stands by putting PARE(6) as the (constant) value of g in eq. (24).

SUBROUTINE LUX4JT(NJET,CUT,IFL,ECM,IFLN,X1,X2,X4,X12,X14)

Purpose : to generate the internal momentum variables for a four-jet $q\bar{q}gg$ or $q\bar{q}q'\bar{q}'$ event, however without taking into account mass corrections to the matrix element.

NJET : number of jets, may be changed to 2 due to harder cuts on events with heavy quarks.

CUT : parameter describing the allowed region in phase space, determined in LUXJET.

IFL : quark flavour, needed for mass-dependent cuts.

ECM : (= W) CM energy of the hadronic system.

IFLN : q' quark flavour in $q\bar{q}q'\bar{q}'$ events, 0 in $q\bar{q}gg$ events.

X1, X2, X4 : $x_i = 2E_i/W$, i.e. twice the energy fraction taken by the partons. In a four-jet event with the colour flow in the order $qg_1g_2\bar{q}$ we have 1=q, 2=g_1, 3=g_2 and 4=$\bar{q}$ while for $q\bar{q}'q'\bar{q}$ we have 1=q, 2=$\bar{q}'$, 3=q' and 4=$\bar{q}$.

X12, X14 : $x_{ij} = 2p_ip_j/W^2$, i.e. twice the four-vector product of two parton momenta, properly normalized, with the same parton numbering as for the x_i.

Remark : internally the kinematical variables y_{ij} and y_{ijk} are used, where $y_{ij} = (p_i+p_j)^2/W^2 = x_{ij}$ and $y_{ijk} = (p_i+p_j+p_k)^2/W^2 = 1 - x_l$ for massless partons (i, j, k, l a permutation of the indices 1, 2, 3, 4). To use the formulae of e.g. [15] we have in this case either 1=q, 2=$\bar{q}$, 3=g_1, 4=g_2 or 1=$\bar{q}$, 2=q, 3=g_2, 4=g_1 with equal probability for $qg_1g_2\bar{q}$ events and either 1=q, 2=$\bar{q}$, 3=$\bar{q}'$, 4=q' or 1=$\bar{q}$, 2=q, 3=q', 4=$\bar{q}'$ with equal probability for $q\bar{q}'q'\bar{q}$ events.

SUBROUTINE LUXDIF(NC,NJET,IFL,ECM,CHI,THE,PHI)

Purpose : to describe the angular orientation of the jets. For 2- and 3-jet events complete formulae are given according to QED or QFD, optionally with beam polarization and, for QED, mass effects included. 4-jet events are projected onto 3-jet events by adding the momenta of the two gluons vectorially.

NC : jets are stored in lines NC+1 onwards in the common block

LUJETS (used to read out the kinematical variables).
NJET : number of jets.
IFL : quark flavour.
ECM : CM energy of the hadronic system.
CHI, THE, PHI : setup of angles used to characterize rotation from standard orientation of event, with q along +z axis and $\bar{q}$ in the xz-plane with x>0, as follows: 1) rotation $+\chi$ in azimuthal angle, 2) rotation $+\theta$ in polar angle, 3) rotation $+\phi$ in azimuthal angle.

FUNCTION ULALPS(Q2)
Purpose : to calculate the running strong coupling constant $\alpha_s(Q^2)$ in first or second order QCD, taking into account the number of active flavours n_f.
Q2 : the energy scale Q^2 of the problem, in e^+e^- annihilation taken to be the W^2 of the hadronic system.
Remark : the relevent scale parameter, Λ, is stored in PARE(1) for the first order and in PARE(2) for the second order expression. In a call, the α_s value calculated is also stored in PARE(20) and the value for $1.986 - 0.115 \cdot n_f$ (cf. eq. (11)) in PARE(21).

5.2. A Routine for "onium" Decays

In LUONIA we have implemented the decays of heavy "onium" resonances, specifically toponium, into ggg or γgg. Unfortunately the energies involved in τ decays are so low that, with the cuts used in the Lund model, no realistic representation can be given of a three-jet final state. In the present implementation the angular orientation of the event is according to QED only, and higher order QCD corrections to jet production are not included.

SUBROUTINE LUONIA(IFL,ECM)
Purpose : to simulate the process $e^+e^- \to \gamma \to 1^{--}$ "onium" resonance (toponium and above) $\to$ ggg or γgg.
IFL : the flavour of the quark which gives rise to the "onium" resonance (6=t, 7=1, 8=h), used to determine the γgg/ggg ratio ($\propto q_f^2$), if IFL=0 only ggg events are simulated.

ECM : the mass of the resonance.
Remark : to determine the γgg/ggg ratio a call is made to ULALPS, using the lowest order QCD expression (i.e. the default value MSTE(1)=1 should be used).

5.3. Routines for Event Analysis

The three subroutines LUSPHE, LUTHRU and LUCLUS contain some tools for event analysis, ranging from the reasonably standardized sphericity and thrust families to a new algorithm for cluster analysis. All particles (or partons) present in the LUJETS common block which have K(I,1) < 2000, i.e. which have not decayed (fragmented), are used for the event analysis (this e.g. also includes neutrinos!). To alter this, one may e.g. use LUEDIT to throw away unwanted particles, or raise the K(I,1) value of these particles above 2000, say by adding 2000 temporarily. Experimental data could of course also be placed in LUJETS and analyzed the same way. Information about the jet axes found is stored after the event proper in the LUJETS common block. This information is overwritten if later another of the three routines is called. Also, for all particles used in the cluster analysis, the particle history information is overwritten by cluster assignment information. After a call to one of the three routines has produced a set of axes, LUORIE can be used to bring the event into a standard orientation by suitable rotations.

SUBROUTINE LUSPHE(SPH,APL)
Purpose : to diagonalize a momentum tensor of the kind in eq. (30), i.e. find eigenvalues $\lambda_1 > \lambda_2 > \lambda_3$ and corresponding eigenvectors $\bar{v}_1$, $\bar{v}_2$ and $\bar{v}_3$. The dependence on the momenta is determined by PARE(30), r of eq. (30), initially set =2 to correspond to the sphericity family. To analyze an event at least two particles are required, otherwise we just put SPH=APL=-1.
SPH : $3(\lambda_2 + \lambda_3)/2$, i.e. sphericity (for r = 2).
APL : $3\lambda_3/2$, i.e. aplanarity (for r = 2).
Remark : the lines N+1 through N+3 in the common block LUJETS will, after a call, contain the following information :

K(N+i,1) = i, the axis number for i=1,2,3,
K(N+i,2) = 0,
P(N+i,1) - P(N+i,3) = $\bar{v}_i$, the i:th eigenvector,
P(N+i,4) = λ_i, the corresponding eigenvalue, and
P(N+i,5) = 0.

SUBROUTINE LUTHRU(THR,OBL)

Purpose : to find the thrust, major and minor axes and corresponding projected momentum quantities, in particular thrust and oblateness. The number of different starting configurations used in the thrust analysis is determined by MSTE(15). The r value, corresponding to particle vectors $\bar{p}_i$ being given a weight p_i^{r-1}, is determined by PARE(31), initially set =1 to correspond to the thrust family. To analyze an event at least two particles are required, otherwise we just put THR=OBL=-1.

THR : thrust (for r = 1).

OBL : oblateness = major - minor (for r = 1).

Remark : the lines N+1 through N+3 in the common block LUJETS will, after a call, contain the following information :
K(N+i,1) = i, the axis number for i=1,2,3,
K(N+i,2) = 0,
P(N+i,1) - P(N+i,3) = $\bar{v}_i$, the thrust, major or minor axis,
P(N+i,4) = thrust, major or minor, and
P(N+i,5) = 0.

SUBROUTINE LUCLUS(NJET,TGEN,DMIN)

Purpose : to reconstruct an arbitrary number of jets using a cluster analysis method based on particle momenta. The distance scale d_{join}, above which two clusters may not be joined, is given by PARE(33). The default value, 2.5, has been chosen under the assumption that all particles are used in the analysis. With only charged particles used, a more reasonable value may be 1.5. In general, PARE(33) may be varied to describe different "jet resolution powers". A smaller distance d_{init}, given by PARE(32), is used to construct the starting clusters around fast particles. MSTE(16) contains the minimum number of clusters to be reconstructed; by making PARE(33) very large a predetermined number of jets may be reconstructed. To analyze an event at least 2·MSTE(16)+1 particles, i.e. normally 3, are required, otherwise we just put NJET=-1 and TGEN=DMIN=-1. MSTE(16) may also be used to start the iteration from given initial clusters, and MSTE(17) may be used to introduce a "multicity" measure for the assignment of particles to clusters.

NJET : the number of jets reconstructed.

TGEN : generalized thrust, the sum of (absolute values of) cluster momenta divided by the sum of particle momenta (≈ "multicity" if jet reconstruction was successful).

DMIN : the minimum distance d between two jets in the final jet configuration (0 if NJET=1).

Remark : the lines N+1 through N+NJET in the common block LUJETS will, after a call, contain the following information :
K(N+i,1) = i, the jet number $1 \leq i \leq$ NJET, with the jets arranged in falling order of momentum,
K(N+i,2) is the number of particles assigned to jet i,
P(N+i,1) - P(N+i,5) is momentum, energy and invariant mass of jet i, using the correct particle masses to define the latter two quantities.
Also, for each particle I, $1 \leq I \leq N$, which has been used in the analysis, K(I,1) = i, where i is the number of the jet the particle has been assigned to. Particles/partons not used in the analysis, characterized by K(I,1) > 2000, will not be affected by this.

SUBROUTINE LUORIE(MORI)

Purpose : to define a standard orientation of events following the analysis in either of the three preceding routines.

MORI= : level of orientation.

=1 : rotate largest axis along z axis and second largest into xz plane. For LUCLUS it can be further specified to +z axis and xz plane with x>0, respectively.

=2 : mainly intended for LUSPHE and LUTHRU, this gives a further alignment of the event, following the one implied by MORI=1. The "slim" jet, defined as the side (z>0 or z<0) with the smallest $(\sum_i |\bar{p}_{ti}|)/\sqrt{n}$, n the number of particles on either side, is rotated into the +z hemisphere. In the opposite hemisphere (now z<0), the side x>0 or x<0 which

has the largest $\sum_i |p_z|$ is rotated into the z<0, x>0 quadrant.

5.4. Information in the Common block

The common block LUDATE contains a number of parameters and switches for alternative modes of operation related to e^+e^- physics. All are given sensible default values in the BLOCK DATA LUDATE subprogram, below indicated by (D=...). These values may be changed by the user to modify the behaviour of the program. Also some values related to the event generated are stored in the common block.

COMMON /LUDATE/ MSTE(20),PARE(40)

MSTE(1)= : (D=1) gives the order in α_s of the matrix elements used for gluon emission in continuum events.
=0 : only $q\bar{q}$ events are generated.
=1 : $q\bar{q}$ + $q\bar{q}g$ events are generated according to first order QCD.
=2 : $q\bar{q}$ + $q\bar{q}g$ + $q\bar{q}gg$ + $q\bar{q}q'\bar{q}'$ events are generated according to second order QCD.
=-1 : only $q\bar{q}g$ events are generated (using standard cuts given by γ), however, since the change in flavour composition from mass cuts etc. is not taken into account, this option can not be used for quantitative studies.
=-2 : only $q\bar{q}gg$ + $q\bar{q}q'\bar{q}'$ events are generated (with the same comment as for MSTE(1)=-1).

MSTE(2)= : (D=1) inclusion of weak effects (Z^0 exchange) for flavour production and angular orientation in continuum events.
=1 : QED, i.e. no weak effects included.
=2 : QFD, i.e. including weak effects.

MSTE(3) : (D=7) mass effects in matrix elements, in the form MSTE(3) = $M_1 + 2 \cdot M_2 + 4 \cdot M_3$ where M_1 = 0 if no mass effects and M_1 = 1 if mass effects should be included. Here
M_1 : threshold factor for new flavour production according to QFD result (see sect. 2.1).
M_2 : gluon emission probability (only applies if |MSTE(1)| < 1, otherwise no mass effects anyhow).

M_3 : angular orientation of event (only applies if |MSTE(1)| < 1 and MSTE(2)=1, otherwise no mass effects anyhow).

MSTE(4) : (D=5) number of allowed flavours, i.e. flavours that can be produced in a continuum event (e.g. LUEEVT with IFL=0) if the energy is big enough. A change to 6 makes top production allowed above the threshold, etc. Also used when checking the number of flavours n_f to use in α_s.

MSTE(5)= : (D=1) fragmentation and decay in LUEEVT or LUONIA calls.
=0 : no LUEXEC calls, i.e. only matrix element treatment.
=1 : LUEXEC calls are made to generate fragmentation and decay chain.

MSTE(6)= : (D=1) angular orientation in LUEEVT or LUONIA calls.
=0 : standard orientation of events, i.e. q along +z axis and $\bar{q}$ along -z axis or in xz plane with x>0 for continuum events, and $g_1g_2g_3$ or $\gamma g_2 g_3$ in xz plane with g_1 or γ along the +z axis for onium decays.
=1 : orientation according to matrix element.

MSTE(7)= : (D=0) radiative corrections to continuum events.
=0 : no radiative corrections.
=1 : radiative corrections as specified in the user-supplied routine LURADK.

MSTE(8) - MSTE(9) : unused at present.

MSTE(10)= : type of event generated in continuum.
=1 : $q\bar{q}$.
=2 : $q\bar{q}g$.
=3 : $q\bar{q}gg$ from abelian (QED-like) graphs in matrix elements.
=4 : $q\bar{q}gg$ from non-abelian (i.e. containing three-gluon coupling) graphs in matrix element.
=5 : $q\bar{q}q'\bar{q}'$.

MSTE(11)= : (D=1) documentation of continuum or onium events (in increasing order of completeness).
=0 : only the fragmenting partons and the generated hadronic system are stored in the LUJETS common block.
=1 : also a radiative photon is stored (for continuum events).
=2 : also the original e^+e^- are stored (with K(I,1)/1000 = 4).
=3 : also the γ or γ/Z^0 exchanged for continuum events or

the onium state for resonance events is stored (with K(I,1)/1000 = 5).

MSTE(12)= : (D=1) calculation of total cross section for continuum events in LUEEVT calls.

=0 : never.

=1 : whenever |ECM - PARE(25)| > PARE(26), after which we put PARE(25) = ECM.

=2 : always (PARE(25) = ECM is also updated).

MSTE(13) - MSTE(14) : unused at present.

MSTE(15) : (D=42) regulates the behaviour of thrust analysis, MSTE(15) = $10 \cdot M_1 + M_2$, where

M_1 : ($M_1 > 1$) is the number of the fastest (i.e. with largest momentum) particles used to construct the (at most) 10 most promising starting configurations for the iteration.

M_2 : ($1 < M_2 < 9$) number of different starting configurations above which have to converge to the same (best) value before this is accepted.

MSTE(16) : (D=1) |MSTE(16)| is the minimum number of clusters to be reconstructed by LUCLUS. If MSTE(16) negative then the cluster directions already stored in LUJETS, in lines N+1 through N+NJET, are used as starting values for the iteration.

MSTE(17)= : (D=1) distance measure used for assigning particles to clusters in LUCLUS.

=1 : the same as the distance between clusters.

=2 : assign particles to the cluster with respect to which they have the largest longitudinal momentum, i.e. as in "multicity".

MSTE(18) - MSTE(20) : unused at present.

PARE(1) : (D=0.5 GeV) Λ in first order QCD.

PARE(2) : (D=0.5 GeV) Λ in second order QCD.

PARE(3) : (D=0.215) $\sin^2\theta_W$ in QFD.

PARE(4) : (D=1/137) α_{em}.

PARE(5) : (D=1. GeV) the "average transverse hadron mass" m_a used for matrix element cuts.

PARE(6) : (D=0.) maximum second order corrections to the three-jet (continuum) cross section, normalized to the first-order cross section and excluding a factor of α_s/π (i.e. maximum of $g(\gamma,x_1,x_2)$ in eq. (24)).

PARE(7) : (D=1.) additional thrust cut on jet energies, so that $x_i = 2E_i/W <$ PARE(7) for all jets in $q\bar{q}g$, $q\bar{q}gg$, $q\bar{q}q'\bar{q}'$, ggg and γgg events (with γ here considered as a separate jet). The continuum events that fail these cuts are reassigned to two-jet events (if MSTE(1) > 0), while for onium events a new kinematical setup is chosen.

PARE(8) : (D=0.) additional cut on minimum jet energies, so that all $x_i >$ PARE(8), with comments as for PARE(7).

PARE(9) : (D=0.) additional cut on minimum four-vector products, so that $x_{ij} = 2p_ip_j/W^2 >$ PARE(9) for all jet pairs i, j in $q\bar{q}gg$ and $q\bar{q}q'\bar{q}'$ events, with comments as for PARE(7).

PARE(10) : unused at present.

PARE(11) - PARE(12) : (D=2*0.) longitudinal polarizations P_{L+} and P_{L-} of incoming e^+ and e^-.

PARE(13) : (D=0.) transverse polarization $P_T = \sqrt{P_{T+}P_{T-}}$, with P_{T+} and P_{T-} transverse polarizations of incoming e^+ and e^-.

PARE(14) : (D=0.) mean of transverse polarization directions of the incoming e^+ and e^-, $\Delta\phi = (\phi_+ + \phi_-)/2$, with ϕ azimuthal angle of polarization, leading to a shift in the ϕ distribution of jets by $\Delta\phi$.

PARE(15) - PARE(19) : unused at present.

PARE(20) : value of α_s in current event.

PARE(21) : value of $1.986 - 0.115 \cdot n_f$ in current event.

PARE(22) : value of R, excluding QCD or radiative corrections, in last LUXTOT call (see MSTE(12)).

PARE(23) : cross section in nb, including QCD corrections but not radiative corrections, in last LUXTOT call (see MSTE(12)).

PARE(24) : unused at present.

PARE(25) : W = ECM stored in last LUXTOT call (see MSTE(12)).

PARE(26) : (D=0.2) maximum deviation of W from PARE(25), above which a new call to LUXTOT is made if MSTE(12)=1.

PARE(27) : calculated for four-jet events only, it represents quotient of total unmodified four-jet cross section in [15] to the (by colour factors) modified cross section [17] used in the program, evaluated for the jet configuration actually chosen. This means that, if PARE(27) were to be considered as a weight for the four-jet events, the unmodified cross section would be recovered.

PARE(28) - PARE(29) : unused at present.

PARE(30) : (D=2.) power of momentum-dependence in LUSPHE (r in eq. (30)), default corresponds to sphericity, PARE(30) = 1. to linear event measures (C, D, H_2).

PARE(31) : (D=1.) power of momentum-dependence (r) in LUTHRU, default corresponds to thrust.

PARE(32) : (D=0.25 GeV) maximum distance d_{init} allowed in LUCLUS when forming starting clusters used to speed up reconstruction.

PARE(33) : (D=2.5 GeV) maximum distance d_{join}, below which it is allowed to join two clusters into one in LUCLUS.

PARE(34) : (D=0.0001) convergence criterion for thrust (in LUTHRU) or generalized thrust (in LUCLUS), i.e. when the value changes by less than this amount between two iterations the process is stopped.

PARE(35) - PARE(40) : unused at present.

6. Examples on How To Use the Program

The Monte Carlo is built as a slave system, i.e. the user supplies the main program, and from this the Monte Carlo subroutines are called on to execute specific tasks, after which control is returned to the main program. Below we give some examples of what type of statements could appear in the main program.

An ordinary annihilation event in the continuum, at a CM energy of 40 GeV may be generated with

```
CALL LUEEVT(0,40.)
```

In this case the flavour and orientation of a $q\bar{q}$ or $q\bar{q}g$ event is generated according to lowest order QED and first order QCD. Before a call to LUEEVT, however,a number of default values may be changed to modify the behaviour of the program, e.g.

```
COMMON/LUDAT2/KTYP(120),PMAS(120),PWID(60),KFR(80),CFR(40)
COMMON /LUDATE/ MSTE(20),PARE(40)
MSTE(1)=2          (allow qq̄gg and qq̄q´q̄´ events as well)
PARE(2)=0.4              (modify second order Λ value)
MSTE(2)=2                      (include weak effects)
PARE(3)=0.22                        (modify sin²θ_W)
```



```
PMAS(2)=92.                              (modify Z⁰ mass)
PWID(1)=2.5                              (modify Z⁰ width)
MSTE(4)=6                        (allow t quark production)
PMAS(106)=30.                          (modify t quark mass)
PARE(13)=0.85      (incoming e⁺e⁻ transversely polarized)
CALL LUEEVT(0,90.)
```

Obviously this is just a sample, in particular a wealth of parameters to control jet fragmentation and particle decays are available in the Lund-JF [4].

A toponium event at 50 GeV may be generated with

```
CALL LUONIA(6,50.)
```

which will result in the simulation of a ggg or γgg event (whereas decays onia → γ → $q\bar{q}$ or $q\bar{q}g$ may be generated as continuum events).

In the generation, the event is stored in the common block LUJETS, as discussed in [4]. All tools to study the event presented there are thus at the disposal of the user. In particular we repeat that

```
CALL LULIST(1)
```

will give a printout of the event. This should be used for a few events in the beginning of a run to check that the generation is working as intended.

The routines for sphericity, thrust or cluster analysis may be called directly after event generation

```
CALL LUONIA(7,100.)
CALL LUSPHE(SPH,APL)
```

and then all stable final state particles (including neutrinos!) are used in the analysis. With the use of LUEDIT it is possible to throw away unwanted particles before the analysis. If one still wants to keep the original jets in the record, this may be achieved e.g. as follows

```
COMMON /LUJETS/ N,K(250,2),P(250,5)
COMMON /LUDAT1/ MST(20),PAR(20),FPAR(40)
COMMON /LUDATE/ MSTE(20),PARE(40)
MSTE(5)=0          (inhibit automatic jet fragmentation)
CALL LUEEVT(0,40.)          (generate partons in event)
NSAV=N                        (save number of partons)
```

```
    CALL LUEXEC       (jet fragmentation and particle decays)
    MST(1)=NSAV+1              (limit range of LUEDIT action)
    CALL LUEDIT(3)      (keep only charged, stable particles)
    MST(1)=0               (restore range, for listing below)
    PARE(33)=1.8             (modify "jet resolution power")
    CALL LUCLUS(NJET,TGEN,DMIN)           (cluster analysis)
    CALL LULIST(2)    (list event and reconstructed clusters)
```

This technique may also be used to try different fragmentation schemes on one setup of parton momenta, e.g. by adding

```
    N=NSAV                              (restore number of jets)
    DO 100 I=1,N
100 IF(K(I,1)/1000.EQ.2.OR.K(I,1)/1000.EQ.3) K(I,1)=
   &K(I,1)-2000                  (mark jets as not fragmented)
    MST(5)=2              (one alternative: independent jets)
    CALL LUEXEC      (generate new fragmentation/decay chain)
```

Acknowledgements :

It is a pleasure to thank Bo Andersson and Gösta Gustafson for many useful discussions on the subjects covered in this paper. Also, many people have contributed with valuable comments on previous versions of the program. Needless to say, the responsibility for any remaining errors in physics or programming rests solely with the author.

References :

1. Mark I Collaboration, G. Hanson et al.
Phys. Rev. Lett. 35 (1975) 1609

2. G. Wolf
DESY 80/13

B. H. Wiik
DESY 80/129

P. Söding, G. Wolf
DESY 81-013

3. B. Andersson, G. Gustafson, C. Peterson
Z. Physik C1 (1979) 105

B. Andersson, G. Gustafson
Z. Physik C3 (1980) 233

B. Andersson, G. Gustafson, T. Sjöstrand
Z. Physik C6 (1980) 235

B. Andersson, G. Gustafson, T. Sjöstrand
Phys. Lett. 94B (1980) 211

B. Andersson, G. Gustafson, T. Sjöstrand
Nucl. Phys. B197 (1982) 45

4. T. Sjöstrand
Lund preprint LU TP 82-3, to be published in Computer Physics Communications

5. H. Fritzsch, P. Minkowski
Physics Reports 73 (1981) 67

J. Ellis
Lectures at Les Houches 1981

6. K. Koller, H. G. Sander, T. F. Walsh, P. M. Zerwas
Z. Physik C6 (1980) 131

J. K. Bienlein
ed. C. Joseph, J. Soffer, High-Energy Physics with Polarized Beams and Polarized Targets (Birkhäuser Verlag, Basel, 1981), p.60

7. H. A. Olsen, P. Osland, I. Överbö
Nucl. Phys. B171 (1980) 209

8. E. Reya
Physics Reports 69 (1981) 195

G. Altarelli
Physics Reports 81 (1982) 1

9. J. Jersák, E. Laermann, P. M. Zerwas
Phys. Lett. 98B (1981) 363

10. J. Ellis, M. K. Gaillard, G. G. Ross
Nucl. Phys. B111 (1976) 253

11. I. Montvay
Phys. Lett. 84B (1979) 331

12. B. L. Ioffe
Phys. Lett. 78B (1978) 277

13. A. Ali, J. G. Körner, Z. Kunszt, E. Pietarinen, G. Kramer, G. Schierholz, J. Willrodt
Nucl. Phys. B167 (1980) 454

14. K. J. F. Gaemers, J. A. M. Vermaseren
Z. Physik C7 (1980) 81

15. R. K. Ellis, D. A. Ross, A. E. Terrano
Nucl. Phys. B178 (1981) 421

16. G. 't Hooft
Nucl. Phys. B190 [FS3] (1981) 455

17. G. Gustafson
Lund preprint LU TP 82-2

18. K. Konishi, A. Ukawa, G. Veneziano
Nucl. Phys. B157 (1979) 45

19. J. A. M. Vermaseren, K. J. F. Gaemers, S. J. Oldham
Nucl. Phys. B187 (1981) 301

20. K. Fabricius, G. Kramer, G. Schierholz, I. Schmitt
Z. Physik C11 (1982) 315

21. Z. Kunszt
Phys. Lett. 99B (1981) 429

Z. Kunszt
CERN Report TH 3141 (1981)

T. D. Gottschalk
Phys. Lett. 109B (1982) 331

G. Kramer
DESY 82-029

22. G. Schierholz, D. H. Schiller
DESY 80/88

J. G. Körner, D. H. Schiller
DESY 81-043

23. G. Kramer, G. Schierholz, J. Willrodt
Z. Physik C4 (1980) 149

24. R. Odorico
Nucl. Phys. B172 (1980) 157

K. Kajantie, E. Pietarinen
Phys. Lett. 93B (1980) 269

G. C. Fox, S. Wolfram
Nucl. Phys. B168 (1980) 285

25. F. A. Berends, R. Kleiss
Nucl. Phys. B178 (1981) 141

F. A. Berends, R. Kleiss, S, Jadach
preprint Instituut-Lorentz, Leiden (1982)

26. K. Koller, T. F. Walsh
Nucl. Phys. B140 (1978) 449

27. K. Koller, K. H. Streng, T. F. Walsh, P. M. Zerwas
SLAC-PUB-2858

28. J. D. Bjorken, S. J. Brodsky
Phys. Rev. D1 (1970) 1416

29. G. Parisi
Phys. Lett. 74B (1978) 65

J. F. Donoghue, F. E. Low, S. Y. Pi
Phys. Rev. D20 (1979) 2759

30. G. C. Fox, S. Wolfram
Nucl. Phys. B149 (1979) 413

31. S. Brandt, Ch. Peyrou, R. Sosnowski, A. Wroblewski
Phys. Lett. 12 (1964) 57

E. Fahri
Phys. Rev. Lett. 39 (1977) 1587

32. S. Brandt, H. D. Dahmen
Z. Physik C1 (1979) 61

33. MARK J Collaboration, D. P. Barber et al.
Phys. Rev. Lett. 43 (1979) 830

34. S. L. Wu, G. Zobernig
Z. Physik C2 (1979) 107

35. J. Dorfan
Z. Physik C7 (1981) 349

H. J. Daum, H. Meyer, J. Bürger
Z. Physik C8 (1981) 167

K. Lanius, H. E. Roloff, H. Schiller
Z. Physik C8 (1981) 251

M. C. Goddard
Rutherford preprint RL-81-069

A. Bäcker
Z. Physik C12 (1982) 161

Appendix 1 : A Vocabulary of Variable Names

As in [4] variable names are partly standardized. Many of the variables listed in Appendix 2 in [4] are also used in the present program, e.g. CHI, ECM, I., IFL., J., KF., L., M., N., P., PAR, PHI, R., THE and X. Below we give a list of some frequently used name elements specific to this program. Dots before and/or after indicates a preponderant use as suffix, prefix or anywhere in the name, with examples given in parantheses.

A. axial couplings to Z^0 (AE, AF).
ALSPI $\alpha_s(Q^2)/\pi$.
CUT cut γ used to separate two-, three- and four-jet events.
D. related to distance measure in cluster analysis (D2, DMIN).
.E coupling constants to electron (AE, VE).
.F properties related to a given quark flavour (QF, AF).
HF. factors used in cross sections, containing the dependence on flavour, energy and polarization (HF1, HF2, HF1W).
.K related to radiative photons (MK, PAK).
NF number of active flavours.
NJET number of jets in event.
POL combinations of polarization variables (POLL).
Q. electric charge (QF).
QME $4m_q^2/W^2$.
R. cross section normalized to the pointlike $\mu^+\mu^-$ QED cross section (RF, RTOT).
S. variables in sphericity analysis (SM, SPH).
SF. variables related to Z^0 propagator (SFF, SFW).
SIG. differential cross sections for angular dependence (SIG, SIGU).
T. variables in thrust analysis (TDI, THR).
V. vector couplings to Z^0 (VE, VF).
VQ speed of quark (with c=1).
X. total cross sections for event types (XTOT, X3JET).
Y. internal variables used to describe four-jet events (Y12, Y134, YR3).

Appendix 2 : Example of an Event Listing

Example of an event generated with CALL LUEEVT(0,60.), having MSTE(2)=2, MSTE(11)=3 and (to suppress π^0 decays) IDB(23)=0. The event was analyzed with CALL LUCLUS(NJET,TGEN,DMIN) and listed with CALL LULIST(2). (ORI gives cluster assignment for particles used in the analysis,i.e. those with last character in PART/JET blank).

EVENT LISTING

I	ORI	PART/JET	PX	PY	PZ	E	M
1	0	E - B	0.000	0.000	30.000	30.000	0.001
2	0	E B+ B	0.000	0.000	-30.000	30.000	0.001
3	1	GA/Z V	0.000	0.000	0.000	60.000	60.000
4	3	S JETF	-17.607	10.190	-10.114	22.724	0.500
5	3	G JETF	17.155	-11.890	-5.476	21.579	0.000
6	3	SA JETF	0.452	1.701	15.590	15.697	0.500
7	2	K +	7.368	-4.652	-1.858	8.923	0.494
8	4	LAM 0 D	-14.730	8.526	-8.334	18.983	1.116
9	5	RHO B- D	6.133	-3.662	-2.308	7.545	0.766
10	1	PI +	-2.508	1.250	-1.369	3.122	0.140
11	1	P B-	-0.158	-0.183	-0.289	1.011	0.938
12	5	K* B D	0.041	-0.417	-0.176	1.006	0.898
13	5	RHO B- D	1.369	-1.794	0.227	2.394	0.766
14	6	ETAP D	0.193	0.581	4.400	4.544	0.958
15	5	RHO + D	1.081	0.363	0.821	1.600	0.766
16	2	PI0	1.046	-0.679	-0.321	1.295	0.135
17	6	K 0 D	0.164	0.880	9.261	9.317	0.498
18	2	PI0	0.001	-0.214	-0.054	0.258	0.135
19	1	P +	-11.512	6.666	-6.436	14.808	0.938
20	1	PI B-	-3.217	1.860	-1.897	4.175	0.140
21	2	PI B-	5.125	-2.749	-1.857	6.107	0.140
22	2	PI0	1.008	-0.912	-0.451	1.439	0.135
23	12	K B D	0.232	-0.075	0.006	0.554	0.498
24	1	PI0	-0.190	-0.341	-0.182	0.452	0.135
25	2	PI B-	0.598	-0.636	0.427	0.982	0.140
26	2	PI0	0.771	-1.159	-0.200	1.413	0.135
27	3	GAMM	0.174	0.221	1.296	1.326	0.001
28	14	RHO0 D	0.019	0.360	3.104	3.218	0.770
29	2	PI +	1.027	0.277	0.398	1.145	0.140
30	3	PI0	0.053	0.085	0.423	0.455	0.135
31	17	K0S D	0.164	0.880	9.261	9.317	0.498
32	2	K0L	0.232	-0.075	0.006	0.554	0.498
33	3	PI +	0.293	0.058	0.676	0.753	0.140
34	3	PI B-	-0.274	0.302	2.427	2.465	0.140
35	3	PI0	-0.024	0.366	5.572	5.586	0.135
36	3	PI0	0.188	0.514	3.689	3.731	0.135
37		1 5	-17.587	9.252	-10.173	23.568	7.553
38		2 9	17.176	-10.799	-3.910	22.115	7.884
39		3 6	0.410	1.546	14.084	14.317	2.016