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A Note on α_s Determinations from Hadron Collision Data

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Abstract:

Recently, a number of α_s determinations have been presented, based on the ratio of three- to two-jet events at ISR and the Sp$\bar{p}$S Collider. The accuracy of these determinations is probably restricted more by uncertainties in the underlying theory than by experimental errors. We show that, like in e^+e^- annihilation, the use of different fragmentation models may lead to widely different α_s values being reconstructed. For the analysis of ISR data, the spread may be up to a factor of two, which is reduced to roughly 20% at the higher Sp$\bar{p}$S energies.

In the perturbative QCD picture, the lowest order processes are complemented with higher order processes containing extra partons in the (perturbative) final state. The basic processes in hadron collisions are $2\to2$ parton interactions of the types $qq \to qq$, $qg \to qg$, $gg \to gg$, $q\bar{q} \to q'\bar{q}'$, $q\bar{q} \to gg$ and $gg \to q\bar{q}$, of order α_s^2. If the scattered partons obtain large transverse momenta p_T, the resulting event will contain two high-p_T jets, in addition to the "beam remnant" jets. In order α_s^3, $2\to3$ processes like $qq \to qqg$, $qg \to qgg$, $gg \to ggg$, $q\bar{q} \to q'\bar{q}'g$, $q\bar{q} \to ggg$, $gg \to q\bar{q}g$ or $qg \to qq'\bar{q}'$ may give rise to events with three high-p_T jets. In hadron collisions, like in e^+e^- annihilation, the fraction of events that contain three well separated jets is thus directly proportional to α_s.

Recently the first efforts have been presented that use three-jet events to determine α_s in hadron collisions. At ISR energies, studies by the AFS Collaboration yield the value $\alpha_s = 0.18\pm0.03\pm0.04$ [1], with the first the statistical and the second the systematical error. At the higher energies of the Sp$\bar{p}$S Collider, the UA1 Collaboration has published the result $\alpha_s = 0.16\pm0.02\pm0.02$ [2] and the UA2 Collaboration the number $\alpha_s = 0.23\pm0.01\pm0.04$ [3]. One may distinguish three sources of error in α_s determinations. First, uncertainties in the cross sections for different parton configurations. These are mainly related to corrections in higher orders, "K factors", and are generally recognized to be large; the numbers above are actually presented as determinations of $\alpha_s K_3/K_2$, K_2 and K_3 being the two- and three-jet K factors, respectively. Second, limited knowledge of the fragmentation and decay processes that transform the outgoing partons into jets of stable particles. Third, effects of detector imperfections, calorimetric fluctuations, trigger conditions, limited statistics and other experimental issues. It is essentially this latter point that gives the statistical and systematical errors quoted above.

Fragmentation effects are listed among the sources of systematic errors in [1,2,3], but no detailed studies on this issue are presented. It is the intent of the present note to remedy this situation. Naively, one would expect different fragmentation models to yield similar values for α_s, once experimental data has been used to constrain the longitudinal and transverse shape of jets. However, the fact that the cross section is rapidly falling, when moving from two-jetlike to more and more distinctly three-jetlike events, implies that fairly small systematic effects may assume an unproportionate importance. This lesson was learned the hard way in e^+e^- annihilation. Our experiences in this field [4] form the starting point for the present note,

but obviously the situation is more complicated in hadron collisions. By comparing results obtained with different fragmentation models, we will show that uncertainties are very large at ISR energies, and still nonnegligible at Sp$\bar{p}$S energies. An understanding of these effects may possibly help reconcile the α_s values obtained by different experiments.

The basic QCD equations have not yet yielded an exact description of the fragmentation of a parton configuration. Rather, phenomenological models have had to be introduced. Because of the large particle multiplicities involved, only Monte Carlo implementations of such models stand a chance of providing a sufficiently detailed and realistic description. Basically, three different fragmentation schools exist today: string fragmentation (SF), independent fragmentation (IF) and cluster fragmentation (CF). Of these, CF is the youngest and has never found any extensive use in hadron collisions. For the issues that will be discussed here, the general properties of CF closely agree with those of SF, so our omission of CF in the following is probably not a serious limitation. The general formalisms of SF and IF are reviewed e.g. in [5], whereas Monte Carlo details and a minireview on the present experimental status of the models is found in [6]. In e^+e^- annihilation the "string effect", first observed by the JADE Collaboration, is now well established [7], and hence IF models are excluded. In hadron collisions there are no convincing experimental evidence either way. IF is used e.g. in ISAJET [8], COJETS [9], "Fieldjet" [10] and most special-purpose programs constructed within experimental collaborations. Only the Lund program uses SF, but has options that allow several IF alternatives to be emulated. All calculations below are based on the Lund Monte Carlo, using PYTHIA version 4.6 [11] for the hadron physics part and JETSET version 6.2 [6] for the subsequent fragmentation.

The main idea in SF is to use the massless relativistic string, which provides the simplest causal and Lorentz covariant description of a linear force field, to approximate the linearly confining colour flux tube (or vortex line) expected in QCD. Quarks and diquarks are associated with the endpoints of the string, and gluons with energy and momentum carrying kinks on it. In a $q\bar{q}g$ event, the string is stretched from the q via the g to the $\bar{q}$. For hadron collisions, the number of colour charges in the final state is larger, and usually several different ways of connecting these partons with strings are possible [12,11]. The model is Lorentz covariant, and energy, momentum and flavour is conserved at each step of the fragmentation process.

The main assumption in IF is that the fragmentation of each outgoing parton can be considered independently of that of the others. Since the fragmentation process is to transform a single coloured and essentially massless parton into a massive jet of colourless hadrons, neither colour, flavour, energy nor momentum can be conserved exactly. It is the lack of momentum conservation that will be crucial for the continued discussion. In the Field-Feynman type of algorithm for quark jets [13] the energy of the final jet on the average agrees with that of the original parton, whereas the jet longitudinal momentum is smaller than the parton one by roughly 1.2 GeV on the average [4]. If a parton configuration is fragmented in its CM frame, the sum of final particle momenta is therefore not vanishing. One possibility is to let the imbalance be, this will be denoted pc=n for no momentum conservation. In the method developed for the Hoyer at al. e^+e^- Monte Carlo [14], the rescaling method pc=r, transverse momentum is conserved locally within each jet. The longitudinal momenta of particles may then be rescaled separately for each jet, such that the ratio of rescaled jet to initial parton momentum is the same for all the jets in the event. A completely different approach, the boost method pc=b, was chosen in the Ali et al. e^+e^- Monte Carlo [15]. Given the imbalance $\bar{p}_{imbal}$ and the total energy E_{tot}, a boost vector $\bar{\beta} = -\bar{p}_{imbal}/E_{tot}$ is defined, such that the Lorentz boosted event has vanishing total momentum. Within the IF framework, the fragmentation properties of quark and of gluon jets may be defined separately. We will below compare two different possibilities. In the first, denoted g=q, a gluon is assumed to fragment like a u, d or s quark or antiquark chosen at random. In the second, $g=q\bar{q}$, the gluon is split up into a parallel quark and antiquark, sharing the energy according to the Altarelli-Parisi splitting function for $g\to q\bar{q}$. Experimental evidence is in favour of gluons fragmenting softer than quarks [16].

It was first noted by the CELLO Collaboration [17] that, applying these different fragmentation models to the $q\bar{q}g$ three-parton configurations in e^+e^-, different α_s values were needed to reproduce the experimental three-jet rate. This comes about as follows [4]. In the fragmentation of a three-jet event $q\bar{q}g$, with the g=q variety of IF, the average momentum of each parton is reduced by the same amount. Since the gluon jet usually starts out with a smaller momentum, its relative loss will be larger, so that the final state net momentum vector $\bar{p}_{imbal}$ is typically pointing oppositely to the gluon direction (Fig. 1). This effect will be even larger for the $g=q\bar{q}$ option, since then the gluon momentum is reduced even more during fragmentation. For the rescaling method of momentum conservation, the momentum of the gluon jet is then scaled up whereas that of the quark and antiquark are scaled down. Since

the parton and jet energies agreed well to start with, the gluon jet energy is systematically larger than that of the gluon parton. From an energy sharing point of view, the event is thus more three-jetlike, whereas opening angles between jets are unchanged. If the boost method is used, the boost tends to be along the gluon jet direction, such that the q and $\bar{q}$ jets become more back-to-back. In angular correlations, the shift is then towards more two-jetlike events, whereas the gluon energy gain is fairly modest. In the SF model, the systematic momentum imbalance obtained in IF is avoided by the string effect: the distribution of low-momentum particles is shifted towards the gluon jet. The net effect is to make the event look less three-jetlike (the reconstructed q and $\bar{q}$ jet axes tend to be more back-to-back than the parton directions).

Since in one case (the rescaling method) the average event becomes more three-jetlike and in the others (the boost method, SF) more two-jetlike, the α_s value needed in the former case is smaller than that in the latter two. At first, it may seem surprising that an average momentum imbalance of roughly 1 GeV in 30 GeV events can lead to α_s values almost a factor two apart for the extreme cases. The event distribution in the various measures constructed to gauge three-jetness is rapidly falling when moving away from the two-jet region, however, so small systematic shifts near the border between what is experimentally classified as three-jet and as two-jet events may change the three-jet rate significantly. The spread in α_s values depends on the measure used, but is never vanishing. In particular, the problem can not be solved by restricting the event sample to more and more perfectly symmetric three-jet events: for a small deviation from three-jet symmetry the average systematic momentum imbalance is directly proportional to this deviation, so that the three-jet region is uniformly expanded or contracted, depending on momentum conservation scheme. Only by going to higher energies, and better separation between the jets, can the problem be removed.

Momentum conservation effects in hadron collisions may be subdivided into two (known) kinds. The first kind is related to the momentum balance of the event as a whole. For this purpose one may consider an event consisting of two high-p_T jets and two beam jets. The momentum of the four original "partons" (including diquarks etc. for the beam remnants) is, on the average, reduced by the same amount by the fragmentation. If the momentum of the two high-p_T partons was smaller than that of the beam remnants to begin with, the rescaling IF scheme will tend to shuffle energy from the beam jets to the high-p_T jets, whereas the trend with the boost IF and the SF schemes is the opposite. Thus the jet cross sections reconstructed will be different, as will the total E_T flow. The relevancy of the latter point for the "NA5 effect" has been discussed by Corcoran [18], who shows that the E_T flow cross sections may be shifted by factors of two or more by different choices of momentum conservation scheme.

The second kind of momentum conservation effects deals with the change among the high-p_T partons themselves, specifically when there are three (or more) of them. For properties in the transverse momentum plane, the phenomena discussed for $e^+e^- \rightarrow q\bar{q}g$ events are applicable: the rescaling IF scheme will tend to make the three-jet structure more pronounced and the boost IF scheme the events more two-jetlike. In the SF model, the string drawing is more complicated than in e^+e^-, but the two high-p_T partons which combine to give the smallest invariant mass are the ones most likely to come from the branching of a common parent parton, and thus be connected with a string piece. The string effect will then tend to result in reconstructed jet axes for these two partons shifted "inwards" towards the intermediate direction, so that the event again is more two-jetlike.

We will now study the size of these effects, simulating parton configurations from hadron collisions at different energies, and fragmenting each configuration according to all the various possibilities outlined above. Momentum conservation is carried out for the system as a whole, i.e. without distinguishing high-p_T and beam jets, but similar results are obtained also with a separate treatment of the two jet classes. Rather than using a matrix element description, initial [19] and final [20] state parton showers are used to generate events with a varying number of partons. This latter approach may lead to a larger uncertainty in the absolute rate of three-jet events, but provides a more realistic picture of the underlying physics, where an experimentally defined jet may often come from the fragmentation of several nearby partons. Most of the results have been obtained with the Duke-Owens set 2 structure functions with Λ = 0.4 GeV [21], but results are not sensitive to this choice.

In order to quantify three-jetness, we have used procedures in the spirit of the AFS, UA1 and UA2 ones. Since results at a given energy come out roughly the same for all three methods, which all are based on calorimetric definitions of jet energies and directions, the figures presented below are mostly based on our simplified version of the AFS algorithm. Here a detector is supposed to stretch between -1 and +1 in pseudorapidity, in a grid of 16x48 cells in pseudorapidity η times azimuth ϕ. A jet is defined by the requirement

that all cells within $\Delta R = ((\Delta\eta)^2+(\Delta\phi)^2)^{1/2} < 0.5$ of a cluster initiator (tried out in falling sequence of Σp_T) have a $\Sigma p_T > 0.1 \cdot E_{Tmin}$. All events with $(\Sigma p_T)_{jets} > E_{Tmin}$ are accepted and enter in the denominator of the three-jet fraction. If an accepted event contains at least three jets, the three largest transverse momentum vectors $\bar{p}_{jet} = p_{Tjet}(\cos\phi_{jet},\sin\phi_{jet},0)$ are defined; these vectors are boosted to their CM frame and energy fractions $x_i = 2p_{jeti}/\Sigma p_{jet}$ with $x_1 > x_2 > x_3$ are defined in this frame. The opening angle between the two most nearby jets is defined as $\omega = \mathrm{acos}(1-2(x_2+x_3-1)/(x_2x_3))$. A bona fide three-jet event is now required to have $x_3 > 0.4$ and $\omega > 60^o$. The UA2 algorithm is similar to the AFS one but, e.g., contains no cuts on the internal three-jet configuration. Such cuts are present in the UA1 algorithm, where the main difference is the requirement of a certain minimum two-jet or three-jet invariant mass M_{min}, rather than an E_{Tmin}; only the subsequent angular cuts introduce an effective E_{Tmin}. In the UA2 algorithm the three-jet and two-jet samples are disjoint and in the UA1 one almost so; we have therefore chosen to define the three-jet fraction as $N_3/(N_2+N_3)$ rather than as N_3/N_2 (with N_i the number of i-jet events).

In Table 1 results are presented for the three-jet fraction obtained in the IF schemes, normalized to the fraction in the SF model. The deviation from unity gauges the extent to which different α_s values would be obtained, applying different fragmentation models to the same data. The reduction from a typical IF/SF ratio of 1.5 at ISR to 1.1 at the Sp$\bar{p}$S is mainly due to the change in E_{Tmin} values, roughly consistent with a $1/E_{Tmin}$ dependence of fragmentation corrections. The change in total CM energy is of secondary importance. The pattern among the IF alternatives resembles the one observed in e^+e^- annihilation, but with some differences as to details. This is to be expected: in hadron physics the three-jet fraction is affected by fragmentation corrections to the three-jet and two-jet rates separately, whereas the total rate in e^+e^- is independent of fragmentation effects. Thus the small dependence on m_{cut}, the cutoff mass of parton shower evolution, is deceptive; individually the two-jet and the three-jet rates are affected strongly by the choice of m_{cut} value (and hence the effective shape of fragmentation spectra). At 63 (630) GeV, with the AFS method and E_{Tmin} = 22 (80) GeV, a change of m_{cut} from 6 (15) to 2 GeV reduces the event rate by a factor 2.02 (1.54) in SF and by between 3.04 (1.96) and 3.64 (2.38) for the IF alternatives.

The AFS, UA1 and UA2 α_s determinations, all based on IF scenarios, do not show the expected Q^2 dependence of α_s. Studies by the AFS group [1] indicate that the large difference between the UA1 and UA2 values may be due to the different three-jet criteria used, while the AFS and UA1 values should be more comparable. The results in Table 1 can be used to estimate that, had SF rather than IF been used in the experimental analysis, AFS would have obtained $\alpha_s \approx 0.28$, UA1 $\alpha_s \approx 0.18$ and UA2 $\alpha_s \approx 0.26$. For five flavours, the first order α_s expression is $\alpha_s(Q^2) = 12\pi/(23 \ln(Q^2/\Lambda^2))$. If one takes $Q^2 \approx (E_{Tmin}/4)^2$, both the AFS and UA1 SF-corrected α_s values are reproduced with $\Lambda \approx 0.3$ GeV. The trend of the data would therefore be consistent with expectations from perturbative QCD. It would also be consistent with α_s values in e^+e^-, where $\alpha_s \approx 0.25 - 0.30$ are obtained with SF at around 30 GeV [17]. Second order corrections bring this down to $\alpha_s \approx 0.18$; a similar development in hadron physics would not be unreasonable.

In summary, we have shown that fragmentation effects can not be neglected in the determination of α_s from three-jet fractions (or, for that matter, total jet rates), particularly not for small E_{Tmin} values. Of the models available in hadron physics today, SF is the theoretically most sound, and also seems to lead to a consistent pattern of α_s values. An improved understanding of the interplay between parton production and fragmentation would probably require a second round of analysis by the experimental groups, in which the choice of a complete model for hadronic events is given more emphasis.

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Table 1

Fraction of three-jet events for different IF models, normalized to the fraction obtained in the SF model. For the AFS- and UA2-inspired algorithms the third column gives E_{Tmin}, for the UA1-inspired one M_{min}. Statistical errors are not quoted, but are generally expected to be less than ±0.05.

E_{cm}	alg	E_{Tmin} (M_{min})	m_{cut}	g=q pc=n	g=q pc=r	g=q pc=b	g=q$\bar{q}$ pc=n	g=q$\bar{q}$ pc=r	g=q$\bar{q}$ pc=b
63	AFS	22	6	1.62	2.08	1.76	1.34	1.73	1.18
63	AFS	22	2	1.24	1.67	1.73	1.15	1.69	1.33
63	UA1	25	4	1.63	2.08	1.51	1.18	2.47	1.39
63	UA2	22	4	1.12	1.51	1.20	0.89	1.48	0.90
540	UA1	150	6	1.08	1.15	1.11	1.09	1.21	1.12
630	UA2	70	6	1.17	1.26	1.17	1.05	1.18	1.06
630	AFS	22	6	1.44	1.59	1.37	1.34	1.74	1.36
630	AFS	40	6	1.18	1.25	1.14	1.13	1.23	1.08
630	AFS	80	15	1.08	1.14	1.12	1.06	1.12	1.07
630	AFS	80	6	1.13	1.12	1.11	1.05	1.12	1.08
630	AFS	80	2	1.13	1.07	1.07	1.04	1.17	1.07
630	AFS	160	6	1.01	1.05	1.03	0.98	0.96	1.02

Figure Caption

Fig. 1. A slightly exaggerated picture of momentum conservation effects in three-jet events. In a) the momenta of initial partons are full arrows and of jets after fragmentation dashed, with dotted indicating final momentum imbalance. In b) - d) the momenta before conservation are dashed, after full. Rescaling method in b), boost method in c), Lund strings (along which particles are sitting) in d).

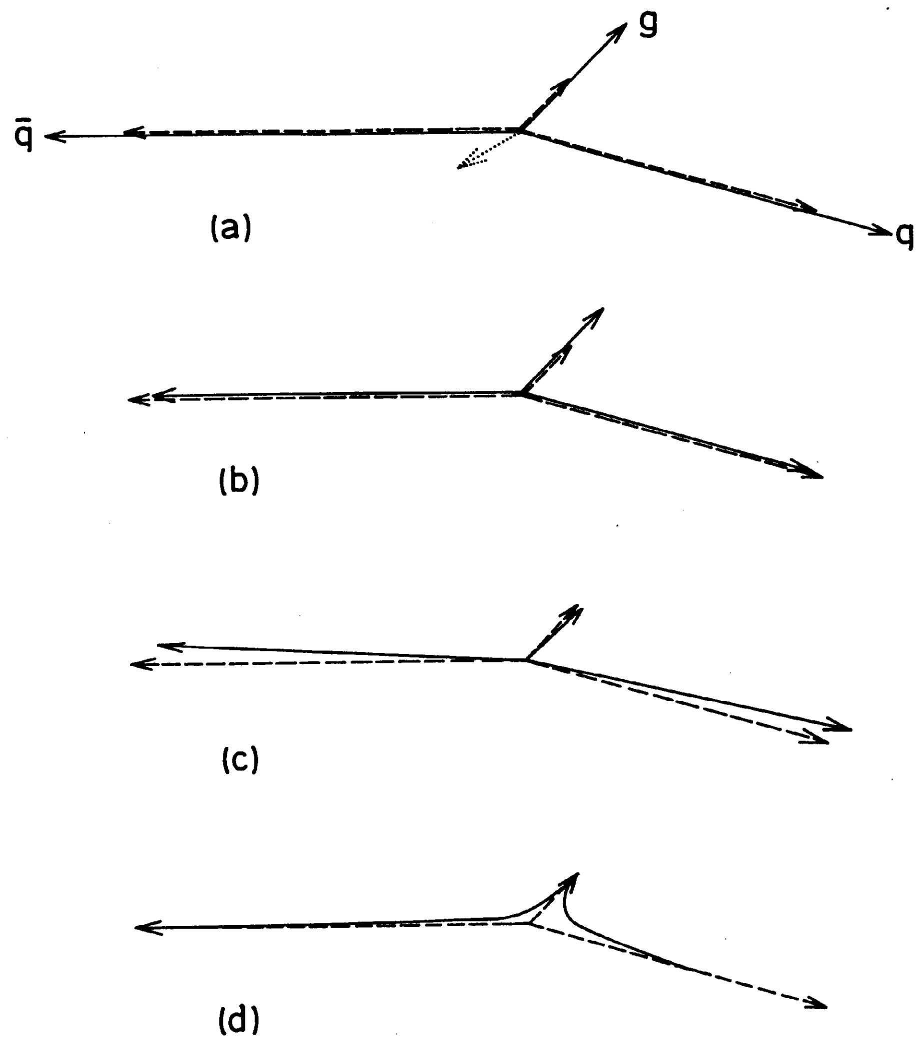

Fig. 1