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Coherent Parton Showers vs. Matrix Elements – Implications of PETRA/PEP Data

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Abstract:

Traditionally, matrix elements have been used to provide the probability for multijet configurations in e^+e^- annihilation events. Some serious discrepancies between PETRA/PEP data and this approach are brought to attention. Instead we develop a new coherent parton shower algorithm, well matched on to the three-jet matrix element, which provides a better description of existing data. A very low cutoff for the parton shower evolution, in the order of 1 GeV, seems indicated. The conclusion is that events with many partons, more than are accessible with present-day matrix elements, play an important rôle already at CM energies of 30 GeV. For extrapolations to higher energies, like TRISTAN/SLC/LEP, parton showers therefore offer more reliable predictions.

The study of multijet events, both in e^+e^- annihilation and in hadron physics, requires a knowledge of the cross-section for multiparton configurations. Two main approaches exist for obtaining this information, matrix elements and parton showers. Assuming that perturbation theory is valid, matrix elements are guaranteed to come arbitrarily close to the "exact" answer, if only calculations are carried out to sufficiently high orders. By contrast, parton showers are based on leading log approximations to the full answer, and are not expected to be as accurate.

In e^+e^-, matrix elements have been calculated to second order in α_s, i.e. including the Born term for $q\bar{q}gg$ and $q\bar{q}q'\bar{q}'$ production [1,2] and the one-loop corrections to $q\bar{q}g$ production [2,3]. Within the framework of the Lund Monte Carlo [4], these matrix elements have been available for a number of years, in conjunction with string (or, optionally, independent) fragmentation [5,6]. In general, a good description of the data is obtained. There are, however, three areas where serious disagreements have been observed.

(i) It has long been known [7] that the number of events with four clusters is significantly higher in the data than predicted. The same trend is also observed in the aplanarity distribution and other four-jet sensitive measures. Recently, a detailed study by JADE shows that the discrepancy in four-jet rate is roughly a factor of two [8], based on an α_s determined mostly by the three-jet rate. Large positive corrections to the latter rate were obtained when going from first to second order, so a significant change in the four-jet rate from second to third order does not seem unreasonable; a significant effect could come e.g. just from the choice of $Q^2 = s$ in $\alpha_s(Q^2)$ [7]. Considering the complexity of a third order calculation, an answer does not seem imminent.

(ii) Based on a comparison between events at 19 GeV and symmetric three-jet events at 29 GeV, Mark II concludes that gluon jets are softer than quark ones [9]. For Lund Monte Carlo events, no such corresponding difference is observed, although softer gluon jets are an inherent feature of string fragmentation. With one order of α_s "used up" in the 29 GeV data just by the three-jet requirement, the amount of additional scaling violations is simply underestimated (this applies to q and g jets alike, but is pushed into the g fragmentation function by the extraction procedure).

(iii) In a study of the energy-energy correlation asymmetry [10] JADE shows that a decent description of data at small angles can only be obtained with a very low matrix element cutoff, $y_{min} = m^2_{min}/s \approx 0.0125$ [11]. For this value, the three- and four-parton rate essentially saturates the total cross-section. Actually, similar conclusions are obtained using first order only, or using (ε,δ) rather than y_{min} cuts. The punch line is that the amount of remaining

two-parton configurations seems to be $\lesssim 5\%$. Under such circumstances, large higher order corrections are a natural expectation.

Turning to parton showers, these are based on an iterative use of the basic $q \to qg$, $g \to gg$ and $g \to q\bar{q}$ branchings, as given by the Altarelli-Parisi (AP) equations [12]

$$\frac{dP_{a \to bc}}{dt} = \int dz \, \frac{\alpha_s(Q^2)}{2\pi} \, P_{a \to bc}(z) \qquad (1)$$

for the probability that a branching $a \to bc$ will take place during a small change dt. A wide selection of algorithms have been developed, see e.g. [13,14], which mainly differ in the interpretation of the variables t, Q^2 and z. Many of the variations are formally of a subleading character, and therefore are not constrained by theoretical leading log analyses. The splitting variable z, with fraction z taken by b and 1-z by c, may be applied to E, or $E+p_z$, or $E+|p|$, or some other energy-momentum combination. The traditional Q^2 scale in α_s has been $Q^2 = m_a^2$, but studies of loop corrections [15] indicate that $Q^2 = z(1-z)m_a^2 \approx p_T^2$ is better. The conventional choice for the evolution parameter t is $t = \ln(m_a^2/\Lambda^2)$, but in the coherent algorithm of Marchesini-Webber (MW) [14] $t = \ln(E_a^2\xi_a/\Lambda^2)$, with $\xi_a \approx 1-\cos\theta_a$ and θ_a the opening angle when $a \to bc$. One also needs to specify the maximum value t_{max} from which the subsequent evolution of b and c should be started.

Very valuable input for model builders is provided by the theoretical studies of coherence effects [16]. In particular, it has been shown that an ordering in emission angles ($\theta_b < \theta_a$, $\theta_c < \theta_a$ if $a \to bc$) takes into account soft gluon interference terms. Analytic expressions are also obtained for the growth of final parton multiplicity with the original parton virtuality [17], and for the distribution of partons in angle and momentum [18]. This phenomenology is correctly included in the MW algorithm. The coherence studies contain information about subleading corrections, severely constraining the range of allowed models. It is therefore not possible to take just any conventional parton shower program and, by addition of the angular ordering constraint, obtain a correct coherent one. A case in point is the parton multiplicity growth, which in many conventional models is slower than given by coherent formulae (at least for subasymptotic energies, up to and including a 2 TeV linear collider or jets at the SSC), and which would be further reduced by a veto on branchings with nonordered angles.

Granted the superiority of coherent showers over conventional ones, and given the existence of the MW program, why not rest content with that? Although well suited for theoretical studies, there are a number of practical inconveniences with MW: it is not possible to fix the invariant mass of a system beforehand, the evolution is performed in a specific frame with a 90° angle (or smaller) between the two outgoing partons, parton masses are not known during the evolution and kinematics can only be reconstructed by a complicated three-pass procedure. We have therefore found it to be of interest to try to reach the same general phenomenology by travelling a different route. Similar studies have been reported by Odorico [19] and by Gottschalk [20], but from different points of view.

In order to avoid the limitations of the MW algorithm, the conventional interpretation $t = \ln(m^2/\Lambda^2)$ is adopted, i.e. parton virtualities are well defined during the evolution. The z variable is defined as energy fraction in the CM frame of the event. Formally, with $e^+e^- \to \gamma(0) \to q(1) + \bar{q}(2)$ and $q(1) \to q(3) + g(4)$, z is a Lorentz invariant

$$z_1 = \frac{p_0 p_3}{p_0 p_1} = \frac{E_3}{E_1} \qquad (2)$$

but the virtual photon four-momentum p_0 acts as a "gauge fixing" vector. In the evolution of parton 1, the daughters 3 and 4 are assumed massless. The allowed range $z_{1-} < z_1 < z_{1+}$ then is

$$z_{1\pm} = \frac{1}{2}\{1 \pm \beta_1\, \theta(m_1 - m_{min})\} = \frac{1}{2}\{1 \pm \frac{|\bar{p}_1|}{E_1}\, \theta(m_1 - m_{min})\} \qquad (3)$$

where m_{min} is some parton shower cutoff scale, to be discussed further, and $\theta(x) = 0$ for $x < 0$, $= 1$ for $x > 0$. Since the β_1 value depends on the mass of parton 2, the evolutions of the original partons 1 and 2 have to be carried out in parallel, starting from a maximum virtuality s. Once a branching is accepted, the mass of the decaying parton is fixed, together with its branching mode (for a quark always $q \to qg$, for a gluon either $g \to gg$ or $g \to q\bar{q}$) and the z value in the branching. Massless four-vectors p_3^0 and p_4^0 may be constructed from the knowledge of $\bar{p}_1$, m_1, z_1 and an isotropically selected azimuthal angle ϕ_1. The evolution of the daughters 3 and 4 may now be commenced, subject to the constraints $m_3 < E_3^0 = z_1 E_1$, $m_4 < E_4^0 = (1-z_1)E_1$ and $m_3 + m_4 < m_1$; $z_{3\pm}$ and $z_{4\pm}$ are defined in analogy with eq. (3). Once m_3 and m_4 have been found (together with z_3, z_4 and further branching modes), corrected on-mass-shell four-vectors p_3 and p_4 are given by

$$p_{3,4} = p_{3,4}^0 \pm (k_4 p_4^0 - k_3 p_3^0)$$
$$k_{3,4} = \frac{m_1^2 - \{(m_1^2 - m_3^2 - m_4^2)^2 - 4m_3^2 m_4^2\}^{1/2} \pm (m_4^2 - m_3^2)}{2m_1^2} \qquad (4)$$

In other words, the meaning of the z_1 variable is somewhat reinterpreted post facto (while it rather is m_1 that is adjusted in the MW algorithm, to solve the same kind of problem).

The principles for the branching $1 \to 3 + 4$ can be iterated until all remaining partons are below the cutoff mass m_{min}. This algorithm produces sensible conventional parton showers. Angular ordering may be introduced as follows. The opening angles θ_a for $a \to bc$ is approximately

$$\theta_a \approx \frac{p_{Tb}}{E_b} + \frac{p_{Tc}}{E_c} \approx \{z_a(1-z_a)\}^{1/2} m_a \{\frac{1}{z_a E_a} + \frac{1}{(1-z_a)E_a}\} = \frac{1}{\{z_a(1-z_a)\}^{1/2}} \frac{m_a}{E_a} \qquad (5)$$

so the requirement $\theta_3 < \theta_1$ is reduced to

$$\frac{z_3(1-z_3)}{m_3^2} > \frac{1 - z_1}{z_1 m_1^2} \qquad (6)$$

where $E_3 = z_1 E_1$ has been used to eliminate the energy factors. If a branching of parton 3 does not fulfill this ordering condition, the branching is rejected and the evolution continued.

Angular ordering provides no constraints on θ_1 and θ_2, since the opening angle between the initial q and $\bar{q}$ is $\theta_0 = 180^{\circ}$ (not so for the MW algorithm, where $\theta_0 = 90^{\circ}$). Instead, we use this freedom to match on to the three-jet matrix element, as follows. A three-jet event $q(x_1)\bar{q}(x_2)g(x_3)$, with $x_i = 2E_i/s^{1/2}$ in the CM frame, can be obtained either by $\gamma(0) \to q(1^*) + \bar{q}(2)$ followed by $q(1^*) \to q(1) + g(3)$ or by $\gamma(0) \to q(1) + \bar{q}(2^*)$ followed by $\bar{q}(2^*) \to \bar{q}(2) + g(3)$. In the first case one obtains

$$m^2 = m_{1^*}^2 = (1 - x_2)s \quad \Rightarrow \quad dt = \frac{dm^2}{m^2} = \frac{dx_2}{1 - x_2}$$
$$z = \frac{p_0 p_1}{p_0 p_{1^*}} = \frac{x_1}{x_1 + x_3} = \frac{x_1}{2 - x_2} \quad \Rightarrow \quad dz = \frac{dx_1}{2 - x_2} \qquad (7)$$

In the definitions of x_1 and x_2, masses other than m_{1^*} are neglected. Combined with the second possibility, with labels 1 and 2 exchanged, eq. (1) gives the same singularity structure as the first order three-jet matrix element

$$\frac{1}{\sigma}\frac{d\sigma}{dx_1 dx_2} = \frac{2}{3}\frac{\alpha_s}{\pi}\frac{A(x_1,x_2)}{(1-x_1)(1-x_2)} \tag{8}$$

$$A_{shower}(x_1,x_2) = 1 + \frac{1-x_1}{(1-x_1)+(1-x_2)}\left(\frac{x_1}{2-x_2}\right)^2 + \frac{1-x_2}{(1-x_1)+(1-x_2)}\left(\frac{x_2}{2-x_1}\right)^2 \tag{9}$$

$$A_{matrix}(x_1,x_2) = x_1^2 + x_2^2 \tag{10}$$

In the limit $x_1 \to 1$ ($x_2 \to 1$) $A_{shower} = A_{matrix} = 1 + x_2^2$ ($= 1 + x_1^2$), with $A_{matrix}(x_1,x_2) \leq A_{shower}(x_1,x_2)$ everywhere. It is therefore possible to reproduce the matrix element by generating the branchings of partons 1 and 2 according to the shower algorithm, but only accept these branchings with probability $A_{matrix}(x_1,x_2)/A_{shower}(x_1,x_2)$. One should note that some implicit differences remain, to the advantage of parton showers. The matrix element Q^2 scale is $Q^2 = s$, the parton shower one $Q^2 = z(1-z)m^2$. Further, in a parton shower the probability of a first branching at a given m is reduced by the probability that no branching has taken place at a larger virtuality, i.e. by the Sudakov form factor.

For a conventional parton shower, a cutoff can be introduced in the form of a mass m_{min}, below which partons are assumed not to radiate. Including quark masses, the following expressions have been used

$$m_{min,g} = m_{min}$$
$$m_{min,q} = \frac{m_{min}}{2} + \left\{\frac{m_{min}^2}{4} + m_q^2\right\}^{1/2} \tag{11}$$

with $m_u = m_d = 0.325$ GeV, $m_s = 0.5$ GeV, $m_c = 1.6$ GeV and $m_b = 5.0$ GeV. In coherent showers, the $\alpha_s(z(1-z)m^2)$ factor is not defined for $z(1-z)m^2 \leq \Lambda^2$. Therefore a further constraint $z(1-z)m^2 > m_{min}^2/4$ is required (the factor 1/4 is included because $z(1-z) \leq 1/4$).

This completes the description of our coherent parton shower algorithm. Further details will appear separately [21], including a discussion on the choice of z interpretation, which is constrained by the need to reproduce the coherent predicition for final parton multiplicity growth with initial virtuality. A host of other properties will also be studied, with particular attention to high energy extrapolations. The actual program code will be made publicly available in JETSET version 6.3.

A few parameters have to be fixed by comparison with data, in particular one obtains $\Lambda \approx 0.40$ GeV. This value is not directly comparable to the ordinary $\overline{MS}$ Λ. Our standard shower cutoff is $m_{min} = 1$ GeV, see (iii) below. While fragmentation issues are formally disconnected from the matrix element or parton shower treatment, there exists a "grey zone" of medium soft gluons that could either be simulated explicitly or be included in the effective fragmentation parameters. A particular advantage with the string fragmentation scheme is the infrared stability: adding a gluon to an event, the net effect on final hadrons vanishes continuously when the gluon energy or emission angle vanish [6]. This means that parameter values vary rather slowly with the cutoff used. Among fragmentation parameters [4,5], those related to the relative flavour production are not affected by the issues discussed here. The transverse momentum of primary hadrons is assumed distributed according to Gaussian distributions in p_x and p_y separately, with $\langle p_T^2 \rangle = 2\sigma^2$. The symmetric fragmentation function, expressed in the fraction z of remaining energy-momentum that a particle takes, is given by

$$f(z) = \frac{1}{z}(1-z)^a \exp\left(-\frac{b\,m_T^2}{z}\right) \tag{12}$$

with m_T the transverse mass of the hadron. For second order matrix elements and $y = 0.02$, i.e. $m_{min} \approx 4.5$ GeV, one obtains $\sigma \approx 0.40$ GeV, $a \approx 1$ and $b \approx 0.7$ GeV^{-2}. For parton showers with $m_{min} = 1$ GeV, these values are changed to $\sigma \approx 0.35$ GeV, $a \approx 0.5$ and $b \approx 0.9$ GeV^{-2}, i.e. $\langle p_T \rangle$ has to be decreased and f(z) made harder when more gluons are simulated explicitly.

A wide selection of event measures have been studied, like multiplicity, thrust, oblateness, sphericity and aplanarity distributions and x and p_T spectra [22]. The parton shower approach fares as well as or better than the matrix element one for all of them. As to the three specific points raised in the beginning of this letter, the following results are obtained.

(i) The percentage of four-cluster events (with $y_{cut} = 0.04$ in the clustering algorithm) is 3.75 ± 0.16 in the JADE data, 2.85 ± 0.12 with matrix elements and 3.47 ± 0.22 with parton showers [8,23]. The same numbers for the three-cluster fraction is 40.2 ± 0.4, 44.0 ± 0.3 and 42.1 ± 0.6, respectively. Results for the aplanarity distribution are presented in Fig. 1.

(ii) With parton showers, gluon jets are seen to be softer than quark ones. In our studies, the effect does not come out as big as in the Mark II data, but this may be due partly to problems in exactly reproducing the experimental algorithm.

(iii) The energy-energy correlation asymmetry is presented in Fig. 3 for parton shower cutoffs m_{min} = 1.5 GeV, 3 GeV and 6 GeV, respectively (with σ and b tuned to give the same mean multiplicity and transverse momentum in all four cases). Only the lower values provide a fair description in the region of small angles. For the regions of low sphericity or high thrust, improvements in the agreement with data can also be observed when m_{min} is brought down, even between 1.5 and 1 GeV. The main conclusion is the same as in the matrix element case: very few two-parton events are left at around 30 GeV (for the cutoffs above, the numbers are 3.2%, 15% and 45%, respectively). A matrix element $m_{min} \approx 4$ GeV can give roughly the same two-parton fraction as a parton shower $m_{min} \approx 1.5$ GeV because no Sudakov form factor is present in the former approach, so that the two-parton fraction dies away faster with decreasing cutoff, and becomes negative if the cut is chosen too small. The average number of partons per event is still larger with parton showers, 5.6 compared to 3.1. Also with the MW algorithm a small m_{min} has been advocated, $m_{min,g} = 2m_g \approx 1.4$ GeV. This value is mainly coming from $m_{min,g}$ being the main parameter of the subsequent cluster fragmentation, however, whereas our m_{min} is more decoupled from the fragmentation parameters of the string model.

In summary, we believe the time has come to put more emphasis on parton shower programs in the study of e^+e^- phenomenology. There may be instances when the matrix element approach is the only valid one, e.g. for $\Lambda_{\overline{MS}}$ determinations, but the amount of "medium soft" gluon emission is probably underestimated in this approach, a shortcoming that will become even more serious with the imminent advent of TRISTAN, SLC and LEP. The parton shower algorithm presented here may provide a convenient alternative to the MW one: the mass of the system is fixed beforehand, parton masses and momenta are found during the evolution, and variables are defined in a Lorentz invariant fashion. A "gauge vector" p_0 is still required, however. The wide-angle hard gluon emission, where parton showers are least reliable, is constrained to agree with the three-jet matrix element. A good agreement with existing e^+e^- data is found, with $\Lambda \approx 0.40$ GeV and a surprisingly low $m_{min} \approx 1$ GeV.

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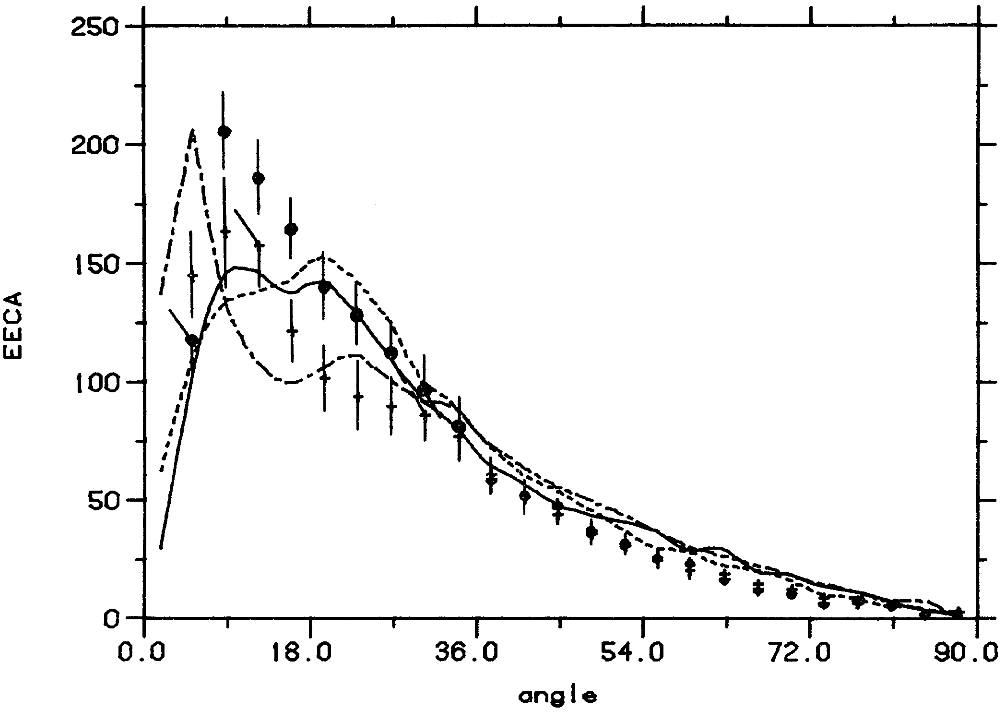
Figure Captions

Fig. 1. Aplanarity distribution at 29 GeV, Mark II data points [22] compared with parton showers, full line, and matrix elements, dashed.

Fig. 2. The energy-energy correlation asymmetry, JADE [11] (points) and MAC [24] (crosses) data points compared with parton showers, full line with m_{min} = 1.5 GeV, dashed with m_{min} = 3 GeV and dash-dotted with m_{min} = 6 GeV. JADE data and model figures are for 34 GeV, while MAC results are for 29 GeV. The model results are not changed much between the two energies, but in the large-angle region the 29 GeV model curves are slightly below the MAC data (in other words, the Λ value used is in good agreement with suitably averaged experimental data).



250
200
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50
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EECA
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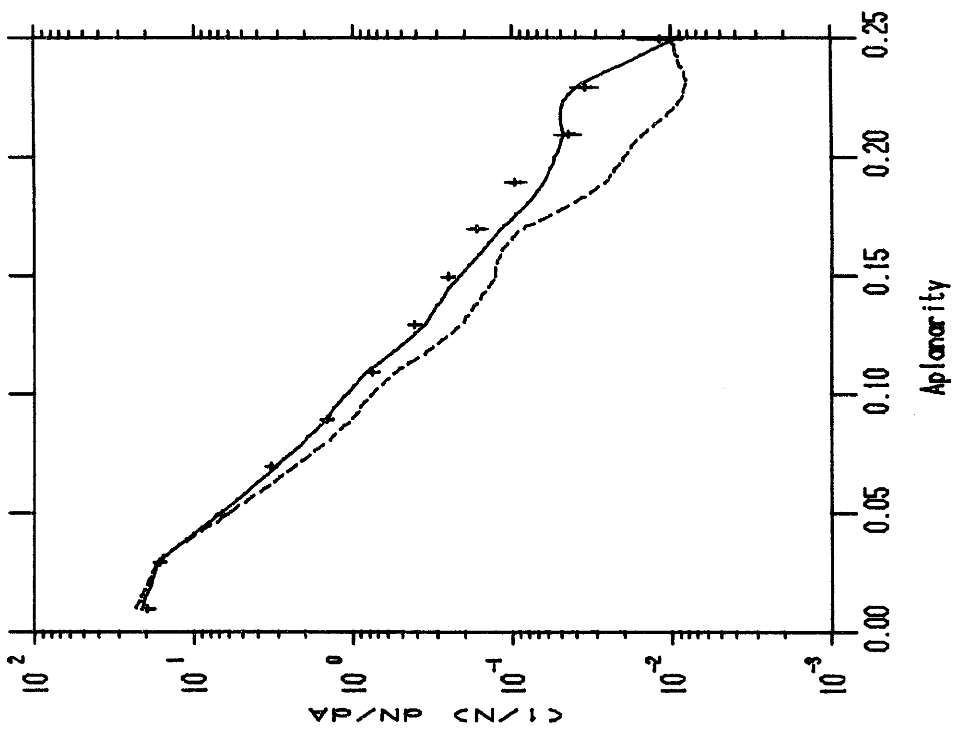
36.0
54.0
72.0
90.0
angle

Fig 2



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10^{1}
10^{0}
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10^{-2}
10^{-3}
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0.00
0.05
0.10
0.15
0.20
0.25
Aplanarity

Fig 1