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Multiple Parton-Parton Interactions in an Impact Parameter Picture

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Abstract:

The possibility of having several independent parton-parton interactions in a hadron-hadron collision is studied. A simple framework is developed for the effects of varying impact parameters. The consequences of having several different matter distributions inside the colliding hadrons are explored. Properties studied include multiplicity distributions, minijet rates, average transverse momentum dependence on multiplicity and E_T density outside jet cores.

The event structure in hadronic collisions is very complex, and the task of understanding this structure is correspondingly difficult. It is often convenient to imagine the physics subdivided into a number of components, like hard central interactions, initial and final state radiation, structure functions, beam jet structure and fragmentation. In this note we will discuss so-called beam jet or minimum bias physics, but in a way that links it with the hard interaction physics. Specifically, we will argue that hadronic events contain a varying number of semihard interactions, with average interaction rate given by perturbative QCD and the variation between different events by Poissonian statistics for each impact parameter separately. Some first evidence for multiple interactions has already been obtained by the AFS Collaboration [1]. The picture also has support in Collider data, as will be shown in this paper. Additionally, consequences for higher energies will be covered.

The differential cross-section for a hard parton-parton interaction is given by perturbative QCD, as a convolution of the hard scattering matrix elements and the structure functions of the incoming hadrons. The integrated cross-section of all interactions with transverse momentum $p_T > p_{Tmin}$, $\sigma_{hard}(p_{Tmin})$, is divergent for $p_{Tmin} \to 0$. At present Collider energies, $\sigma_{hard}(p_{Tmin})$ becomes comparable with the total cross-section for $p_{Tmin} \approx 1.5 - 2.0$ GeV. This need not lead to contradictions: $\sigma_{hard}(p_{Tmin})$ does not give the hadron-hadron cross-section but the parton-parton one. Each of the two incoming hadrons may be viewed as a beam of partons, with the possibility of several parton-parton interactions when the hadrons pass through each other [2], so that $\sigma_{hard} > \sigma_{tot}$ is perfectly allowed.

In [3] we argue that Collider data indicate a significant probability for multiple interactions at 540 GeV. Specifically, if there is at most one hard interaction per event, the predicted multiplicity distribution is much narrower than the experimental one [4], and the experimentally observed forward-backward multiplicity correlations [5] are almost absent. These observations are based on a detailed simulation of complete hadronic events, using the Lund Monte Carlo for Hadronic Processes, PYTHIA [6], as follows. Hard interactions above some p_{Tmin} cutoff are chosen according to the correct QCD cross-section. Associated initial and final state radiation are included for these events. Additional low-p_T events are generated to fill out the total (inelastic, non-diffractive) cross-section. Jet universality is assumed, i.e. the underlying fragmentation mechanism in hadron physics is taken to be no different from that in e^+e^- annihilation. In the latter process, the Lund string fragmentation model [7] provides an accurate description of most phenomenology known to date. The way strings are stretched in hadron physics is more complicated than in e^+e^- annihilation, however. Our standard assumption is that in low-p_T events there are two strings being stretched, e.g. for $p\bar{p}$ collisions one between a quark in the p and an antiquark in the $\bar{p}$ and one between the remaining diquark and antidiquark. In high-p_T events the strings are stretched out to the two scattered partons [6], in such a way that the simple two-string picture is recovered when the p_T of the hard interaction is allowed to vanish (at least for the dominant one-gluon-exchange graphs).

If, for any given hadron-hadron collision, several different parton interactions above p_{Tmin} are assumed to take place (essentially) independently of each other, one obtains a Poissonian multiplicity distribution in the number of interactions, with mean given by $\sigma_{hard}(p_{Tmin})/\sigma_{tot}$, where σ_{tot} is the total inelastic, nondiffractive cross-section. With a varying number of interactions in an event, the multiplicity fluctuations are increased, and strong forward-backward multiplicity correlations are introduced. Results are sensitive to the choice of p_{Tmin} value, with a reasonable description of the multiplicity distribution obtained for $p_{Tmin} \approx 1.6$ GeV. In the region of low multiplicity, it is here important to include the effects of surviving double diffractive events. Forward-backward multiplicity correlations, the rate of "hot spots" and other phenomena are also well described with the same p_{Tmin} choice [3].

The presence of some regularization of the divergent parton-parton cross-section can be motivated by the fact that the incoming hadrons are colour singlets: a gluon of small p_T, and hence large transverse wavelength, will not resolve the individual colour charges inside the hadrons and therefore effectively decouple. For the subsequent study it makes sense to use a more continuous regularization than the sharp cutoff at p_{Tmin}, and to extend the generation of semihard interactions to $p_T = 0$. The matrix elements, which normally diverge like dp_T^2/p_T^4, are therefore multiplied by a factor $p_T^4/(p_{T0}^2+p_T^2)^2$. Further, α_s is evaluated at a scale $p_{T0}^2+p_T^2$ rather than at p_T^2. With $p_{T0} \approx 1.8$ GeV we reproduce the same phenomenology at 540 GeV as with a sharp cutoff at $p_{Tmin} \approx 1.6$ GeV.

Up to this point it has been assumed that the initial state of all hadron collisions is the same, whereas in fact each collision is also characterized by a varying impact parameter *b* (in this paper *b* is to be thought of as a distance of closest approach, not as the Fourier transform of the momentum transfer). A small b value corresponds to a large overlap between the two colliding hadrons, and hence an enhanced probability for multiple interactions. A large b, on the other hand, corresponds to a grazing collision, with a large probability that no parton interactions at all take place. This effect will tend to broaden the minimum bias multiplicity distribution at higher energies. At present energies the change in the multiplicity distribution is less dramatic, since the mean number of interactions is small anyhow. It may explain the "pedestal effect", however: events containing hard interactions are biased towards small impact parameters, and hence have a larger than average multiple interaction probability.

In order to quantify this, one may assume a spherically symmetric distribution of matter inside a hadron, $P(\bar{x})\ d^3x$. For simplicity, the same spatial distribution is taken to apply for partons of all species and momenta. Four different parametrizations have been compared, to check how sensitive results are to this choice. The distributions are a solid sphere $P_1(\bar{x}) \propto \theta(a-|\bar{x}|)$, a Gaussian $P_2(\bar{x}) \propto \exp(-x^2/a^2)$, an exponential $P_3(\bar{x}) \propto \exp(-|\bar{x}|/a)$ and a double Gaussian $P_4(\bar{x}) \propto a_1^{-3} \exp(-x^2/a_1^2) + a_2^{-3} \exp(-x^2/a_2^2)$. The last possibility may be thought of as representing a hadron where half the hadronic matter appears in local concentrations, be that a hard hadronic core surrounded by a pion cloud (as in the chiral bag model [8]) or separate cores around the three valence quarks. The ratio $a_1/a_2=5$ has been used throughout. For a collision with impact parameter b, the integrated overlap between the colliding hadrons is then given by

$$\mathcal{O}(b) = \iint d^3x\ dt\ P_{boosted}(x-\frac{b}{2},\ y,\ z-vt)\ P_{boosted}(x+\frac{b}{2},\ y,\ z+vt) \qquad (1)$$

where v is the velocity in the CM frame and $P_{boosted}$ the suitably Lorentz contracted $P(\bar{x})$. By a scale change in z, $P_{boosted}$ can be replaced by P. After a further scale change in t one obtains

$$\begin{aligned}\mathcal{O}(b) &\propto \iint d^3x\ dt\ P(x-\frac{b}{2},\ y,\ z-\frac{t}{2})\ P(x+\frac{b}{2},\ y,\ z+\frac{t}{2}) \\ &= \int dt \int d^3x\ P(x,\ y,\ z)\ P(x,\ y,\ z-(b^2+t^2)^{1/2})\end{aligned} \qquad (2)$$

The average number of interactions is now assumed to be proportional to this overlap

$$\langle n_{int}(b)\rangle = k\ \mathcal{O}(b) \qquad (3)$$

where the constant of proportionality k is related to the integrated parton-parton cross-section, and hence increases with the CM energy. For a given

impact parameter, the number of interactions is assumed to be distributed according to a Poissonian. If the matter distribution has a tail to infinity (which is true for the examples above, except for the solid sphere), events may be obtained with arbitrarily large b values. In order to obtain finite cross-sections, one has to assume that each event contains at least one semihard interaction. The probability that two hadrons, passing each other with an impact parameter b, will actually interact is then given by

$$P_{int}(b) = 1 - \exp(-k\mathcal{O}(b)) \qquad (4)$$

For a given p_{T0} scale, the ratio of the (regularized) integrated parton-parton cross-section and the total (inelastic, nondiffractive) cross-section can be used to determine k.

Using the equations above, the probability distribution in b of events may be obtained, and for each b the average number of interactions to be expected. For practical applications, it is more useful to define a factor $f(b) = \mathcal{O}(b)/\langle\mathcal{O}\rangle$, and study the probability distribution dP/df. A large f value corresponds to a central collision, with a high probability of several interactions, while a small f corresponds to a peripheral collision with the minimal number of one interaction. The larger a tail the hadronic matter distribution has, the wider is the dP/df distribution.

Before comparing with data, a comment about the relation to other work on multiple interactions and varying impact parameters is appropriate. The concept of multiple interactions has been explored particularly within the framework of dual topological unitarization [9], where interactions are described by nonperturbative rules, and are customarily considered to involve negligible transverse momenta. The importance of hard interactions has been studied [2], but then usually only for the specific case of two interactions. By contrast, we allow an arbitrary number of hard or semihard interactions, in accordance with the perturbative QCD cross-section (suitably truncated at small p_T). Also the importance of the impact parameter for multiplicity distributions has been studied by a number of authors (see e.g. the reference list in [10]). In particular, the so-called eikonal $\Omega(b)$ may be related to our overlap $\mathcal{O}(b)$ by the substitution $k\mathcal{O}(b) \rightarrow \Omega(b)$ in eq. (4). The physical interpretation is quite different, however: the eikonal of the optical model is based on the scattering of (the wave package of) one incoming hadron in the potential of the other, while our $\mathcal{O}(b)$ is obtained as a convolution of the assumed spatial distribution of (more or less pointlike) partons inside the colliding hadrons. Both points of view give a proton that becomes "blacker" for any given fixed *b* when the energy is increased (and hence the k value), so that the total cross-section increases, cf. [11]. Finally, one should note that we here study not only the parton-parton interactions themselves, but also the related colour flow and subsequent fragmentation. This offers the ability to study and predict properties not only for the inclusive event sample, but also for events obtained e.g. with high-p_T jet or minijet triggers.

In order to compare with the UA1 minijet phenomenology at 900 GeV [12,13], a cluster algorithm is used to find jets with a $E_{Tjet} > 5$ GeV in the region $|\eta| \leq 2.5$, $30^o \leq |\phi| \leq 150^o$. Detector smearing effects are included cell-by-cell, by assuming the detected neutral energy to be spread around the true one according to a Gaussian with width $0.6 \cdot E_T^{1/2}$, truncated so that the energy is never negative or larger than four times the true one. The model gives a minijet event rate of 13 %, to be compared with the experimental one 17.2 %. This is without the inclusion of any K-factor in the QCD cross-section. Agreement can be obtained by using a constant K≈1.5, but also by applying the results of recent loop calculations by Ellis and Sexton [14], namely that first order corrections to the jet cross-section can be minimized if the Q^2 scale in α_s is chosen to be roughly $0.075 \cdot p_T^2$ rather than p_T^2. In the following, the latter recipe is adopted, i.e. an $\alpha_s(0.075(p_T^2+p_{T0}^2))$ is used. The scale $p_{T0} \approx 2.1$ GeV is here somewhat larger than before because of the increased cross-section.

With these choices, good agreement can be obtained with the minijet phenomenology. One example is the average transverse momentum $\langle p_T \rangle$ of charged particles as a function of n_{ch}, the charged multiplicity in $|y| \leq 2.5$, separately for the jet and nojet samples as shown in Fig. 1. Fragmentation parameters are here fixed by e^+e^- data, and cannot be tuned. Two other aspects require mentioning, however. Firstly, if an event with few charged particles still has a jet with $E_{Tjet} > 5$ GeV, it would be natural to assume a larger $\langle p_T \rangle$ the smaller n_{ch} is, at least for models with a positive correlation between the production of charged and of neutral particles. Indeed, it is only the simulation of calorimetric fluctuations, which sometimes lead to "false" minijet triggers, that allow a fairly flat $\langle p_T \rangle$ for small n_{ch} in the jet sample. A detailed simulation of the UA1 detector is therefore probably needed to fully understand this region. Secondly, the rise of $\langle p_T \rangle$ in the region $n_{ch} > 50$ is sensitive to the nature of the multiple interactions. If all interactions after the hardest one are assumed to be of the gg→gg type, with the two scattered gluons in a colour singlet, the amount of multiplicity

obtained per interaction is essentially maximized, and the $\langle p_T \rangle$ in the jet sample tends to decrease with increasing multiplicity for $n_{ch} > 40$. The situation is improved if a large fraction of the scattered partons are quarks, or if the colours of the outgoing gluons are correlated so as to reduce the total string length, since in either case the transverse momentum of a hard interaction is shared between fewer particles. While the former option gives a fair agreement with the UA5 multiplicity distributions at 540 and 900 Gev [4,15], the latter two give too low a dispersion, however. The best solution proves to be an even mixture of these three possibilities. This still leads to a good correspondence with the UA1 $\langle p_T \rangle$ data, see Fig. 1, without lowering the dispersion significantly.

The question is how sensitive results are to variation in impact parameter and to the different parametrizations. In the UA5 data the most sensitive measure is the dispersion. At 540 GeV, the multiplicity distribution is not broadened significantly by the variation in impact parameters; at most it gives a factor two increase of rate in the high-multiplicity tail. Extrapolations to higher energies are significantly affected, however. For a fixed impact parameter, the width of the scaled multiplicity distribution will be maximal in the 1 TeV region, and then start slowly shrinking. Already at 900 GeV there is a significant deviation from the data. A varying impact parameter gives a larger dispersion. The choice of parametrization is also important, to the extent that the double Gaussian gives the best agreement of the four different alternatives. The fixed impact parameter picture gives $\langle n_{ch} \rangle = 34.6$ and dispersion $D = 15.6$, while a double Gaussian gives $\langle n_{ch} \rangle = 35.3$ and $D = 18.9$ and the UA5 data are $\langle n_{ch} \rangle = 34.6 \pm 1.2$ and $D = 20.2 \pm 0.6$ [15]. Even more glaring are the differences when extrapolating to the SSC, the planned 40 TeV pp collider, Fig. 2. The results for the solid sphere and the exponential matter distributions are not shown in Fig. 2 for the sake of clarity; these do not differ that much from the Gaussian alternative.

Much of the minijet phenomenology can be reproduced without any reference to varying impact parameters. The pedestal effect is an exception. Fig. 3 shows the amount of transverse energy in the region $1 \leq |\eta-\eta_{jet}| \leq 2$ and $|\phi-\phi_{jet}| \leq 90^o$ as a function of the E_{Tjet} trigger used. The pedestal does increase with increasing E_{Tjet} also in a model with a fixed impact parameter, but significantly less than in the data [13]. In a varying impact parameter picture results once again depend upon the parametrization used. Agreement is obtained provided the double Gaussian parametrization is used whereas, for example, the Gaussian does not increase the pedestal sufficiently. The fact that the UA1 data seem to favour the double Gaussian distribution is in keeping with the high double parton scattering cross-section measured by AFS [1].

In summary, the fact that the hadrons are composite, extended object gives a distinct character to hadronic interaction, quite different from e.g. e^+e^- annihilation events. Compositeness implies the possibility of having several hard (or at least semihard) parton-parton interactions in a given event. An extended wave function gives a variable overlap of the colliding hadrons, with subsequent fluctuations in interaction probability. Here we have argued that an understanding of the multiplicity distribution requires multiple parton-parton interactions and that the pedestal effect requires the introduction of a variable impact parameter. Also other phenomena are well described in this framework.

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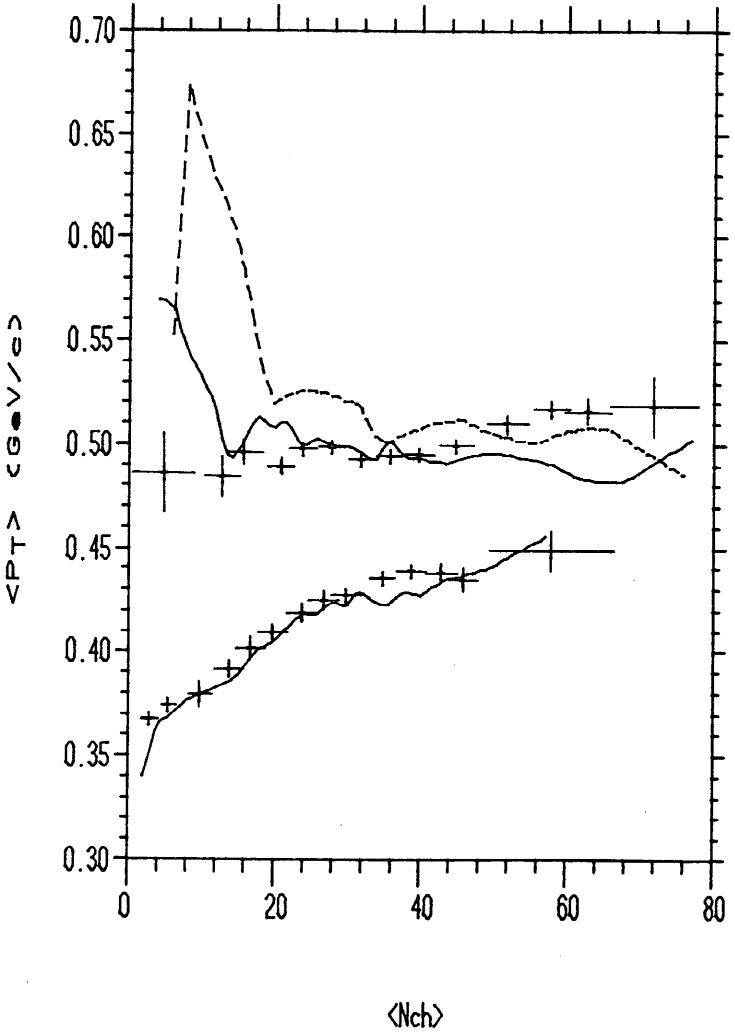
Figure Captions

Fig. 1. Average transverse momentum of charged particles as a function of the charged particle multiplicity. Data from UA1 at 900 GeV [12], for the jet (upper data points) and nojet samples separately. Full lines show model results with the double Gaussian distribution and detector smearing included; dashed line is for the jet sample when smearing is not included.

Fig. 2. Predicted scaled multiplicity distributions at 40 TeV. Dashed with fixed impact parameter, dotted with Gaussian and full with double Gaussian matter distribution.

Fig. 3. Amount of transverse energy in $1 \leqslant |\eta-\eta_{jet}| \leqslant 2$, $|\phi-\phi_{jet}| \leqslant 90^{o}$ as a function of E_{Tjet} trigger. Data points UA1 at 630 GeV [12], dashed line with fixed impact parameter, dotted with Gaussian and full with double Gaussian hadronic matter distribution.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0.65
0.60
0.55
0.50
0.45
0.40
0.35
0.30
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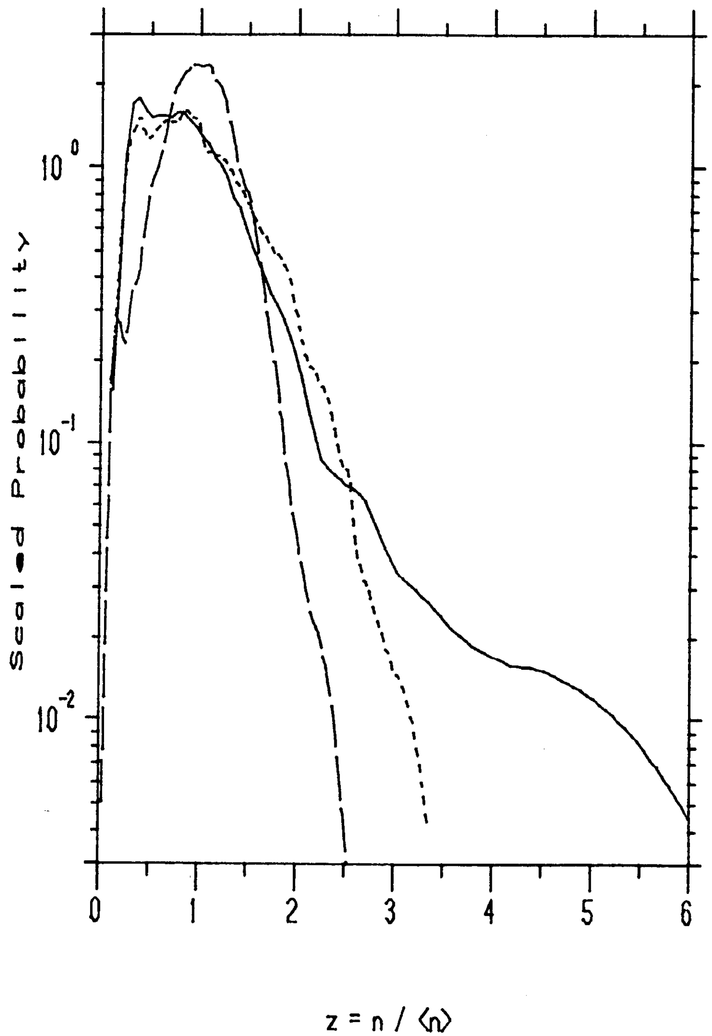
0
20
40
60
80
<Nch>
Fig 1


10^0
10^-1
10^-2
Scaled Probability
0
1
2
3
4
5
6
z = n / <n>
Fig 2

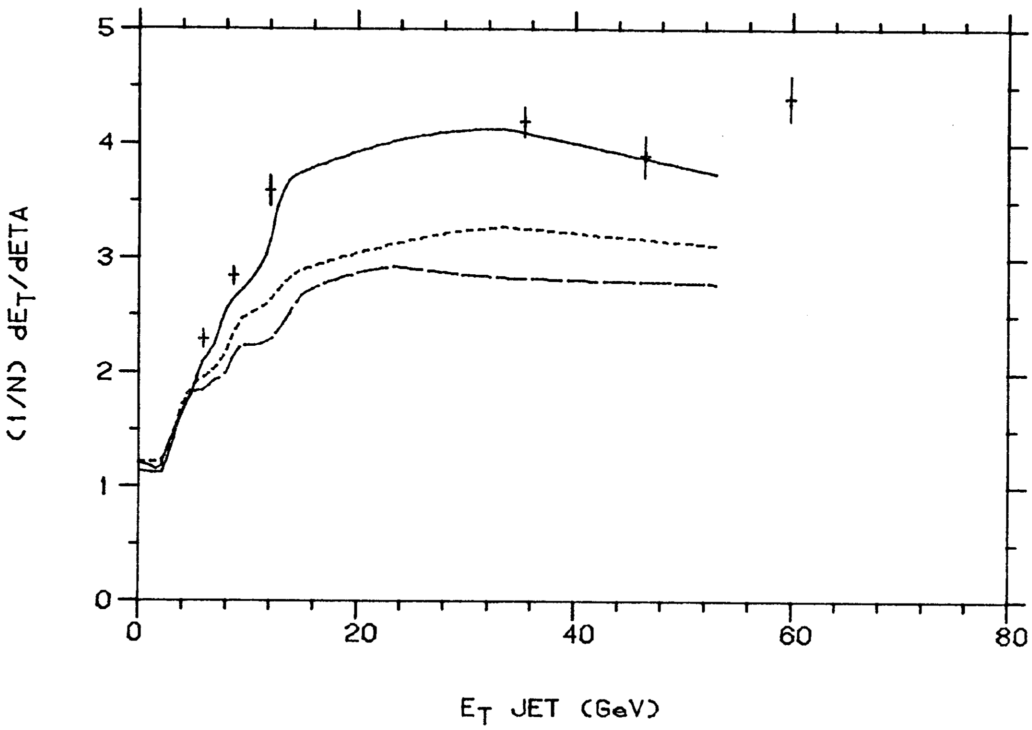


Fig 3