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Multiple Hard Parton Interactions at HERA

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Abstract

At HERA, the large flux of almost real photons accompanying the electron beam leads to the copious photoproduction of jets. Regions of small momentum fractions x of the incoming particles are explored, where the density of partons is high. As a result, the probability for more than one hard partonic scattering occurring in a single γp collision could become significant. It is well known that this effect modifies the contribution of jets (minijets) to the total cross section. We discuss the latest HERA data on the total γp cross section in this context. The possible effects of multiple hard interactions on event shapes and jet cross sections at HERA have been studied using Monte Carlo programs. We review some of the available results, which in general indicate that the effects of multiple interactions should be significant and may already be manifest in the existing HERA data.

1. Introduction

For both protons and photons, QCD predicts a rapid increase in parton densities at low x, where x is the fraction of the beam particle's momentum which participates in the 'hard' scattering (interaction). In a naive treatment, this rise can lead to a corresponding (but ultimately unphysical) rise with increasing energy of perturbative QCD calculations of the jet contribution to the total cross section. However, the large number of small x partons contributing to jet production can mean that there is a significant probability for more than one hard scatter per γp interaction. Only the resolved part of the photon can undergo multiple interactions, i.e. the direct part is taken to be unaffected. The effects of multiple interactions can provide a mechanism for taming the rise in the QCD cross section [1] in accord with unitarity, as discussed in Sjöstrand's talk [2].

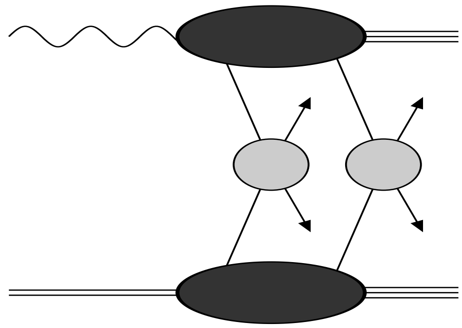

Figure 1: An example of a multiple scattering in a γp collision.

Additionally, there are clearly implications for the hadronic final state. In order to study these effects eikonal models have been implemented within the hard process generation of several Monte Carlo programs [3, 4, 5, 6]. Multiple parton scattering, as illustrated in Fig.1, is expected to affect jet rates. The average number of jets per event should be increased when partons from secondary hard scatters are of sufficiently high p_T to give jets in their own right. In addition, lower p_T secondary scatters produce extra transverse energy in the event which contributes to the pedestal energy underneath other jets in the event. Thus multiple scattering can influence jet cross sections even when no parton from the secondary scatters is itself of a high enough p_T to produce an observable jet. By boosting the transverse energy of jets in this way, multiple scattering leads to an increase in jet cross sections for jets above a certain E_T^{jet} cut, even though the total cross section is reduced.

The theory of multiple scattering is still not well understood; therefore phenomenology is based on models with several assumptions and unknown parameters. The effects of multiple scattering on the total cross section and on jet/event shapes probe these unknowns in somewhat different ways. Although they are related, it therefore makes sense to consider the two aspects separately. In sect.2 we outline the framework of models of multiple interactions, in sect.3 we discuss the HERA total cross section data for γp quantitatively from the minijet point of view. In sect.4 we present some of the expected effects of multiple interactions on jet cross sections, with reference to existing data, and in sect.5 we give some conclusions.

2. Modelling multiple interactions

When speaking of multiple scattering (alias multiple interactions), the basic building blocks are the standard $2 \to 2$ partonic interaction processes, such as $gg \to gg$, $qq \to qq$, $q\overline{q} \to gg$ and $qg \to qg$. An event may contain none, one, two or more such interactions; the novelty of the multiple interactions concept is that the 'two or more' occurrence may be the norm at high energies rather than a rare exception. The correlations between interactions occurring in the same event may be horribly complicated, but the hope is that it is feasible to approximate this by a simple factorized description in terms of several independent scatterings. This may be made plausible by viewing the incoming hadron and hadronlike photon as two pancakes, flattened by Lorentz contraction. When these pass through each other, different parts are causally separated, so the probability of an interaction between any pair of partons can be assumed independent of what occurs anywhere else. This gives a poissonian distribution in the number of interactions, for a fixed impact parameter between the two colliding hadrons/photons. In central collisions the average number of interactions should be larger than in peripheral ones; it is therefore necessary to introduce a model of how partons are distributed in the pancake. The 'eikonalization' procedure, so central to current descriptions, is a combination of these two concepts: a poissonian distribution at each fixed impact parameter plus an impact-parameter-dependent overlap function. This form may be integrated over the impact parameter to give the total cross section, the probability distribution of interactions, and even the number of additional interactions underlying a hard interaction.

Several objections can be raised. For instance, scatterings are assumed to be disjoint, i.e. a parton does not undergo more than one scattering. This can be motivated by simple counting arguments: if each hadron contains N partons, the rate of two disjoint $2 \to 2$ scatterings is proportional to N^4 but that of two scatterings with a shared parton, i.e. a $3 \to 3$ process, is only proportional to N^3. A counterexample would be the occurrence of 'hot spots' caused by the cascading of a single parton. This is especially relevant since each $2 \to 2$ subprocess is embedded in a larger system of initial- and final-state radiation. Colour correlations between interactions are not well understood, and yet this is of large importance for the structure of the final event. In summary, current eikonalization approaches can only be viewed as sensible first approximations, and therefore the existence of several different models is an advantage.

Maybe the most ambitious approach is the DTU (dual topological unitarization) one. Here a hard pomeron term is given by scatterings with $p_T > p_{T min}$, a soft pomeron term includes nonperturbative scatterings at small p_T, and triple- and loop-pomeron graphs are added to incorporate also diffractive topologies in the description. The

p_{Tmin} scale can be changed freely over some range, since the nonperturbative pomeron term can be modelled to compensate for the change in the hard interaction rate. This approach is represented by the PHOJET program [6].

In less ambitious approaches, the issue of diffractive events is kept separate, and unitarization within the soft sector is simplified to allow (at most) one soft interaction. Here p_{Tmin} is a real parameter of the theory, to be determined by a comparison with data. Of course the use of a sharp cutoff in p_T is only an approximation to what has to be a smooth turn on in reality. Examples include the models found in PYTHIA [5] and HERWIG [3, 4]. These two are similar in philosophy but differ in the details. For simplicity we therefore concentrate on the latter in the following.

Let us start by considering a γp interaction at some fixed impact parameter, b, and centre-of-mass energy, s. In particular, we suppose the γ to be hadronlike (i.e. resolved [7]). We assume that the probability for the photon to interact in such a state is P_{res} and take the ρ-dominance form, i.e. $P_{res} = 4\pi\alpha_{\rm em}/f_\rho^2 \approx 1/300$. The mean number of jet pairs produced in this resolved-γ–p interaction is then

$$\langle n(b,s)\rangle = \mathcal{L}_{\rm partons} \otimes \hat{\sigma}_H \tag{1}$$

where $\mathcal{L}_{\rm partons}$ is the parton luminosity and $\hat{\sigma}_H$ is the cross section for a pair of partons to produce a pair of jets (i.e. partons with $p_T > p_{Tmin}$).

The convolution is because the parton cross section depends upon the parton energies. More specifically,

$$d\mathcal{L}_{\rm partons} = A(b) n_\gamma(x_\gamma) n_p(x_p) dx_\gamma dx_p \tag{2}$$

where $A(b)$ is a function which specifies the distribution of partons in impact parameter. It must satisfy

$$\int \pi db^2 A(b) = 1$$

in order that the parton luminosity integrated over all space is simply the product of the parton number densities. Factorizing the b dependence like this is an assumption. In particular we do not contemplate QCD effects which would spoil this, e.g. perhaps leading to 'hot spots' of partons. Also $n_i(x_i)$ is the number density of partons in hadron i which carry a fraction x_i of the hadron energy. For ease of notation we do not distinguish between parton types and have ignored any scale dependence of the number densities. For the proton, the number density is none other than the proton parton density, i.e. $n_p(x_p) \equiv f_p(x_p)$. However, since we are dealing with resolved photons, the number density n_γ is related to the photon parton density by a factor of P_{res}, i.e. $n_\gamma(x_\gamma) = f_\gamma(x_\gamma)/P_{res}$. Thus, after performing the convolution, we can write:

$$\langle n(b,s)\rangle = \frac{A(b)}{P_{res}}\sigma_H^{inc}(s), \tag{3}$$

where $\sigma_H^{inc}(s)$ is the inclusive cross section for γp to jets. Restoring the parton indices, it is given by

$$\sigma_H^{inc}(s) = \int_{p_{Tmin}^2}^{s/4} dp_T^2 \int_{4p_T^2/s}^{1} dx_\gamma \int_{4p_T^2/x_\gamma s}^{1} dx_p \sum_{ij} f_{i/\gamma}(x_\gamma, p_T^2) f_{j/p}(x_p, p_T^2)\, \frac{d\hat{\sigma}_{ij}(x_\gamma x_p s, p_T)}{dp_T^2}. \tag{4}$$

In order to investigate further the structure of events containing multiple interactions we need to know the probability distribution for having m (and only m) scatters in a given resolved-γ–p event, P_m. In order to do this we assume that the separate scatters are uncorrelated, i.e. they obey poissonian statistics. Thus

$$P_m = \frac{(\langle n(b,s)\rangle)^m}{m!}\exp(-\langle n(b,s)\rangle). \tag{5}$$

This formula is central to the Monte Carlo implementation in HERWIG.

We can now ask for the total cross section for $\gamma p \to$ partons with $p_T > p_{Tmin}$.

$$\begin{aligned} \sigma_H(s) &= \pi P_{res} \int db^2 \sum_{m=1}^{\infty} P_m \\ &= \pi P_{res} \int db^2 [1 - \exp(-\langle n(b,s)\rangle)]. \end{aligned} \tag{6}$$

Since the total inclusive cross section (σ_H^{inc}) counts all jet pairs (even ones which occur in the same event) we expect it to be larger than σ_H by a factor equal to the mean number of multiple interactions per event (i.e. averaged over impact parameter). This is easy to see. Let $\langle n(s)\rangle$ be the average number of jet pairs produced in resolved-γ–p events which contain at least one pair of jets, then

$$\begin{aligned} \langle n(s)\rangle &= \frac{\int db^2 \sum_{m=1}^{\infty} m P_m}{\int db^2 \sum_{m=1}^{\infty} P_m} \\ &= \frac{\int db^2 \langle n(b,s)\rangle}{\int db^2 [1-\exp(-\langle n(b,s)\rangle)]} \\ &= \frac{\sigma_H^{incl}(s)}{\sigma_H(s)}. \end{aligned} \tag{7}$$

Note that σ_H must always be less than the total γp cross section, whereas σ_H^{incl} need not be.

In order to study the details of the hadronic final state in the presence of multiple interactions it is most convenient to use a Monte Carlo simulation. The HERWIG Monte Carlo generates the required number of hard scatters and the associated initial and final state parton showering. The outgoing partons and remnant jets are then fragmented to the hadronic final state. Two modifications are made to the simple eikonal model in the implementation. Firstly, energy conservation is imposed, i.e. after the backward evolution of all the hard scatters in an event, the energy remaining in the hadronic remnants must be greater than zero. Secondly, if during the backward evolution of the first scatter the splitting $q\bar{q} \leftarrow \gamma$ is arrived at before the evolution cut off scale, the event is classified as an 'anomalous' event and no multiple interactions are allowed.

3. Total γp cross section

The 'minijet' contribution, $\sigma_H(s)$, is clearly only one contribution to the total cross section. To compare with experiment we must add the soft cross section σ_T^{soft}, which is assumed to contain all of the physics below p_{Tmin}. Furthermore there is a (small) cross section, σ_T^{dir}, for the production of single jet pairs (with $p_T > p'_{Tmin}$) by unresolved (i.e. direct) photons. Often it is assumed that $p'_{Tmin} = p_{Tmin}$, but this differs between models. Of course simply adding together cross sections is only a first guess.

As an example of multiple interaction approaches to the total γp cross section, we take the eikonalized minijet calculations for $\sigma_H(s)$ of ref.[8], which used the parton distribution functions of refs.[9, 10] for the proton and photon respectively, and add to them various choices of parametrisations of σ_T^{soft}. We adjust p_{Tmin} and the parameters of σ_T^{soft} accordingly, and compare to the recent data on $\sigma_T^{\gamma p}$ [11, 12]. This is similar to what was done in ref.[13], except there the old HERA data were used [14, 15]. For a fuller discussion of this approach and its theoretical uncertainties see refs.[2, 8, 13].

The recent results for the total γp cross section at centre of mass energies of $\simeq 200$ GeV are as follows. The H1 collaboration ($\langle W \rangle = \langle \sqrt{s} \rangle = 197$ GeV) find $\sigma_T^{\gamma p} = (165 \pm 2.3 \pm 10.9)\,\mu$b and ZEUS ($\langle W \rangle = 180$ GeV) obtain $\sigma_T^{\gamma p} = (143 \pm 17)\,\mu$b. In the H1 case, the first error is statistical and the second systematic. The ZEUS error has the systematic and statistical errors added in quadrature.

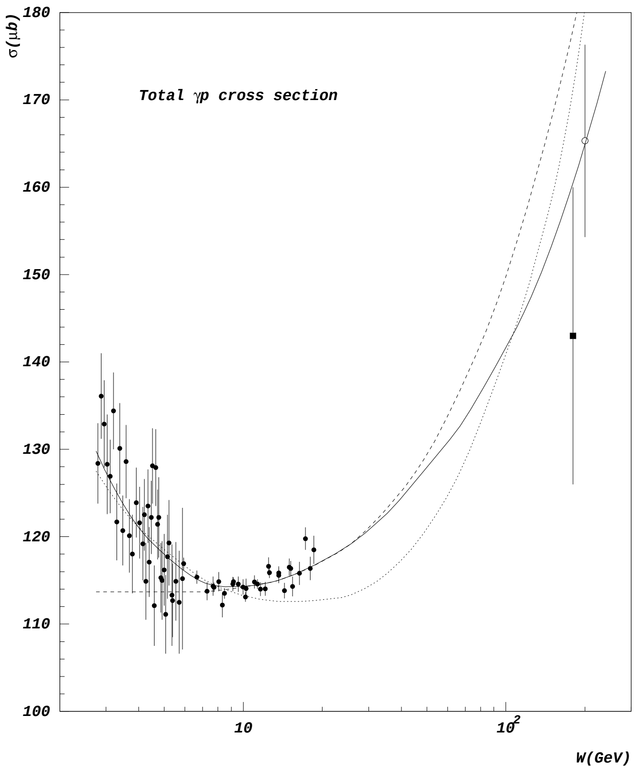

Figure 2: Energy dependence of the total γp cross section. See text for the description of the curves.

In the simplest variant, σ_T^{soft} is taken to have the form:

$$\sigma_T^{soft}(s) = A + B/\sqrt{s} \tag{8}$$

where A and B are constants. This ansatz implies that the entire rise in $\sigma_T^{\gamma p}$ at high energies is due to minijets. It has very little theoretical motivation and makes rather an ad hoc separation between hard and soft physics, endowing p_{Tmin} with great physical significance. In fig.2 we show as the dotted curve the best fit to the data that can be achieved with such an ansatz added to the results of ref.[8] for $\sigma_H(s)$, calculated with $p_{Tmin} = 2$ GeV . We take $A = 107.98\,\mu$b and $B = 54.34\,\mu$b GeV. This leads to an unconvincing description of the the data in the 10–18 GeV range [16], which show evidence of a rise with energy. If one attempts to attribute the low energy rise entirely to the minijet contribution then one would be forced to choose very low values of p_{Tmin}, in the 1–1.5 GeV range (see ref.[2]), which would lead to much too high a prediction at HERA, as was found in ref.[17]. This is illustrated by the broken curve in fig.2, where we take a constant $\sigma_T^{soft} = 114\mu$b and $p_{Tmin} = 1.5$ GeV in $\sigma_H(s)$. Again the need to fit the low energy rise leads to a dangerously high cross section at HERA energies (no attempt was made to fit the data below $\simeq 10$ GeV: any attempt to do so must worsen the fit to the higher energy data).

Also in fig.2 we show (the solid line) the results of ref.[8] added to the following form for σ_T^{soft}

$$\sigma_T^{soft}(s) = As^{\epsilon} + B/\sqrt{s}. \tag{9}$$

The choice $A = 78.4\,\mu$b, $\epsilon = 0.058$ and $B = 117.05\,\mu$b GeV, with $p_{Tmin} = 3$ GeV in $\sigma_H(s)$, ensures an excellent fit to the low energy data (i.e. $W \leq 18$ GeV) [16].

This latter approach clearly provides the best description of the data, but is slightly unorthodox in attributing only part (around half) of the rise in cross section to minijets. We could argue that this is a rational view of the situation, less extreme than attributing all of the rise to σ_T^{soft}, as in the soft Pomeron approach [18]. We emphasize that the minijet question is not a stark choice between the two extremes of either minijets providing the entire rise with energy of the total cross section or being absent: indeed, jets are undeniably produced in both $\overline{p}p$ [19] and $\gamma\gamma$ [20], as well as γp, reactions. The important issue is whether they make a significant contribution to the total cross section at existing energies. In this picture, with $p_{Tmin} = 3\,$GeV, they do; $\sigma_H(s)$ being around 20 μb at $\sqrt{s} = 200$ GeV, although this is very sensitive to the choice of p_{Tmin}. The increase of the soft cross section could still have its origin in multiple interactions in the region $p_T < p_{Tmin}$, but then without a simple perturbative description.

4. Hadronic final state

In this section we illustrate the possible size and nature of the effect of multiple interactions on some aspects of jet cross sections and the hadronic final state at HERA. Here the experiments typically measure jets with transverse energies as low as 6 GeV. To simulate these events with Monte Carlo models, they use values for the minimum transverse momentum of a hard scatter of around $2 < p_{Tmin} < 3$ GeV. In addition, cuts on the γp CM energy are made, forcing it to lie typically in a range of around 120 GeV $\leq \sqrt{s} \leq$ 265 GeV. For these choices the mean number of hard scatters per event was found to be at least 1.04 (higher for the lower p_{Tmin} values), taking the MRS D_- [21] proton and GRV [22] photon parton densities.

For the events we generate, since the cross section falls rapidly with increasing p_T, we

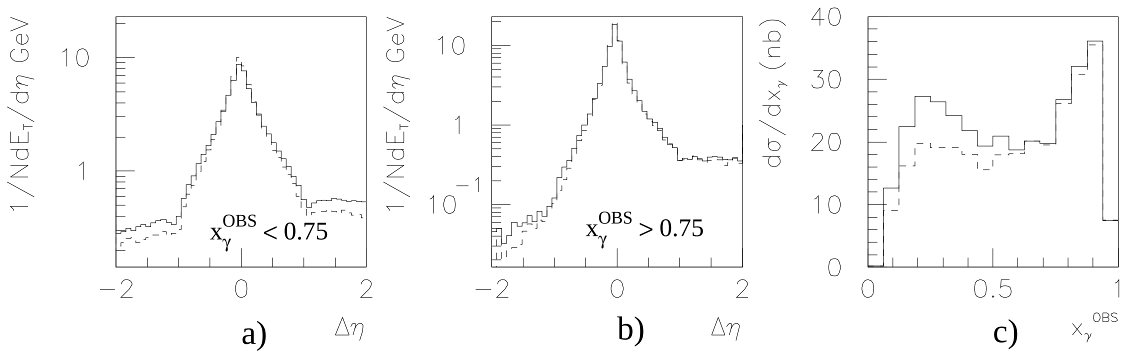

Figure 3: The E_T jet profile in η, for a) $x_\gamma^{obs} < 0.75$ and b) $x_\gamma^{obs} \geq 0.75$. c) x_γ^{obs} distribution for dijets, at HERA energies, with direct contribution. Including multiple scattering (solid line) and with multiple scattering turned off (broken line).

expect that most of the multiple interactions will have $p_T \sim p_{Tmin}$ and so their main effect will be to increase the mean number of observed jets by boosting the underlying E_T in the event. Events which contain multiple interactions with high enough p_T to be observed as jets in their own right are relatively rare, but their observation (they appear as pairs of back-to-back jets) would provide striking evidence in support of the existence of multiple interactions.

Jet finding was performed using a cone algorithm with a cone radius $R = 1$ [23]. Jets have $E_T \geq 6$ GeV and pseudorapidity $-2 \leq \eta^{jet} \leq 2$. The additional E_T can be seen directly in the jet profile, Fig.3(a), where the E_T in the jets is plotted against η relative to the jet axis. The pedestal energy is increased with the inclusion of multiple interactions. A variable found to be useful at HERA [24] for distinguishing between direct and resolved photon events is the 'observable x_γ'. It is defined by

$$x_\gamma^{obs} = \frac{\sum_{jets} E_T^{jet} e^{-\eta^{jet}}}{2E_\gamma} \tag{10}$$

and the sum is over the two highest E_T jets in the event. It is the fraction of the photon's energy manifest in the two jets of highest E_T, and in leading order is exactly the fraction of the photon energy which enters the hard scatter, i.e. direct events are peaked at $x_\gamma^{obs} = 1$ whilst resolved events have $x_\gamma^{obs} < 1$. The x_γ cut isolates the resolved interactions, and as expected multiple interactions have no effect in the direct case, Fig.3(b). The extra E_T enhances the inclusive jet rate around $\eta^{jet} = 1$, and an increased sensitivity to multiple scattering can be seen in the η^{jet} and E_T^{jet}

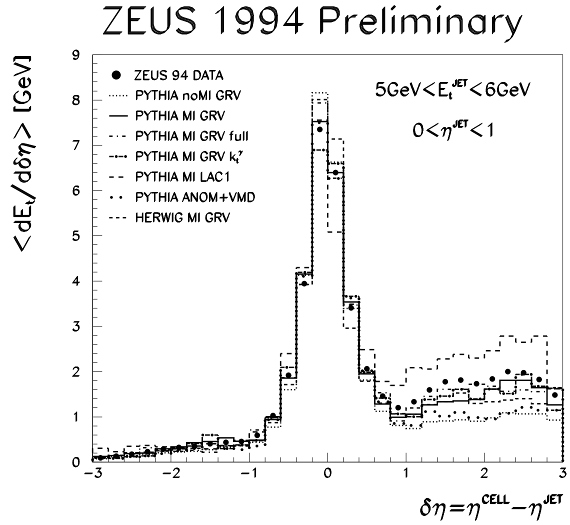

Figure 4: Jet profile. The solid circles are uncorrected ZEUS data. Uncorrected transverse energy flow seen in the calorimeter around the jet axis $\langle dE_T/d\delta\eta\rangle$ where $\delta\eta = \eta^{cell} - \eta^{jet}$, for cells within one radian in ϕ of the jet axis, for jets in the range 5 GeV$< E_T^{jet} <$ 6 GeV and with $0 < \eta^{jet} < 1$.

distributions in dijet events, which are shown in ref.[4]. An enhancement at high η^{jet} and low E_T^{jet} is seen. All plots in Fig.3 have the direct contribution generated using HERWIG included, hence the rise at $x_\gamma^{obs} \sim 0.8$ in Fig.3(c).

It can be seen from Fig.3(c) that the effect of multiple scattering is greater in the low x_γ^{obs} region. Multiple interactions are more likely to occur here as it is in this region that the higher parton densities occur; also the energy conservation constraint is less restrictive.

In Fig.4 various Monte Carlo models are compared to preliminary ZEUS data [25]. The jet profile is shown, and the data are not corrected for detector effects. The simulated events have been passed through a full simulation of the experiment. Jets have 5 GeV $< E_T^{jet} <$ 6 GeV and pseudorapidities in the range $0 < \eta^{jet} < 1$. These cuts remove the effect of the forward edge of the calorimeter acceptance and allow us to study the effect of the different models on the jet profile independently of the E_T^{jet} and η^{jet} distribution of the models. The comparison with the various models confirms the general conclusion made by H1 [26] that introducing multiple interactions can improve the agreement between Monte Carlo models and the data. However, there is a large amount of freedom in the models and, in particular, the effect of introducing multiple interactions depends strongly on the parton density of the photon (compare the histograms for the multiple interaction models using LAC1 and GRV). For further comparisons with HERA data, the reader is referred to the talk of Steve Maxfield [27] and references therein.

5. Conclusions

Multiple scattering must occur at some CM energy, in order that unitarity is not violated: the relevant point here is whether it is present at HERA. Clearly answering this question using total cross section data is going to be very difficult, given the uncertainties in the soft physics. This point was made very clear by our discussion of sect.3, and in ref.[2].

However, jet cross sections seem to be a more hopeful place to look. In sect.4, the effect on the hadronic final state of multiple parton scattering in γp interactions has been simulated by interfacing an eikonal model of multiple parton interactions with HERWIG. Models indicate that the effect of multiple scattering is significant at HERA energies. For reasonable experimental cuts, the inclusion of multiple scattering leads to significant changes in inclusive and dijet cross sections which should be understood before attempting to unfold to parton distribution functions.

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