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Transverse Momentum as a Measure of Colour Topologies

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Abstract

Several distinct colour flow topologies are possible in multiparton configurations. A method is proposed to find the correct topology, based on a minimization of the total transverse momentum of produced particles. This method is studied for three-jet $Z^0 \to q\overline{q}g$ and four-jet $W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4$ events. It is shown how the basic picture is smeared, especially by parton-shower activity. The method therefore may not be sufficient on its own, but could still be a useful complement to others, and e.g. help provide some handle on colour rearrangement effects.

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When high-energy processes produce multiparton states, it is generally believed that the confinement property of QCD leads to the formation of colour flux tubes or vortex lines spanned between the partons. These tubes/vortices are here called strings, in anticipation of our use of hadronization based on the string model [1]. A quark or antiquark is attached to one end of a string. A gluon is attached to two string pieces, one for its colour and one for its anticolour index, and thus corresponds to a kink on the string.

The simplest kind of events, $e^+e^- \to \gamma^*/Z^0 \to q\overline{q}$, gives a single string stretched between the q and the $\overline{q}$. The direction of the colour flow can, in principle, be distinguished by flavour correlations [2], but will not be studied here. In next order, $q\overline{q}g$ events correspond to a string stretched from the q via the g to the $\overline{q}$. The colour topology is unique, but experimentally it is not normally known which of the three jets is the gluon one, so this gives a threefold experimental ambiguity.

From four partons onwards, true ambiguities of the topology exist, even when the identity of the partons is known. In $q\overline{q}gg$ events, the string can be drawn from the q to either of the two gluons, on to the other gluon and then to the $\overline{q}$. This gives two possible topologies. A third topology, not expected to leading order in N_C, is when one string runs directly between the q and $\overline{q}$ and another string in a closed loop between the two gluons. There is an experimental ambiguity, in picking the two quarks among the four partons, which gives a further factor of six, i.e. a total of eighteen possible topologies (reduced to fifteen if single and double strings are not distinguished).

Another four-jet final state is obtained in the process $e^+e^- \to W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4$. Since the W's are colour singlets, in principle each of $q_1\overline{q}_2$ and $q_3\overline{q}_4$ form a separate colour singlet string. However, by soft gluon exchange or some other mechanism, alternatively $q_1\overline{q}_4$ and $q_3\overline{q}_2$ may form two singlets. Since it would not normally be known which of the four jets are quarks and which antiquarks, there is a total of three experimental pairings of four jets, where the third corresponds to the unphysical flavour sets q_1q_3 and $\overline{q}_2\overline{q}_4$.

Among the topologies above, only the three-jet $q\overline{q}g$ events have been studied in detail. It has been shown that the string approach here correctly predicts the topology of particle flow, with a dip in the angular region between the q and $\overline{q}$ jets [3, 4]. This comes about because the qg and $\overline{q}g$ string pieces produce (soft) particles in the respective angular ranges, while there is no string directly between the q and $\overline{q}$. The same effect is also obtained as a consequence of colour coherence in perturbative soft-gluon emission [5] — normally these two approaches give the same qualitative picture. Although by now LEP 1 has produced large samples of four-jet events, the energy flow between these jets have not been studied in detail, presumably because of the large number of possible topologies.

LEP 2 will provide four-jets from W pairs, and here the issue of colour topology may become of great importance [6, 7]. If, by colour rearrangement, an original $q_1\overline{q}_2$ plus $q_3\overline{q}_4$ colour singlet configuration is turned into a $q_1\overline{q}_4$ plus $q_3\overline{q}_2$ one, particle production will be somewhat different. Methods to determine the W mass from LEP 2 four-jet events will then give different results. There is more than one model of the colour rearrangement process; therefore the uncertainty on the W mass could be as high as 100 MeV [8], i.e. larger than the expected statistical error of the order of 40 MeV. The final number will then be dominated by the mixed hadronic–leptonic channel $W^+W^- \to \ell\nu_\ell q\overline{q}$, which has about the same statistics. The hadronic events could be recuperated if colour rearrangement effects could be diagnosed from the data itself. Additionally, the ability to distinguish between various reconnection scenarios could provide information on the nature of the QCD vacuum, and therefore be of fundamental interest. Unfortunately, realistic reconnection scenarios give only very minute effects on the data (remember that the effect on the W mass is at most of the order of one per mille), and so attempts to find useful signals have been rather unsuccessful [7]. There is only one claim for having found a signal [9] where, in a few events, a central rapidity gap separates the particle production from two low-mass colour singlet reconnected systems. It has turned out, however, that neglected angular correlations in the W pair decays tend to reduce this signal to the border of observability [10, 11]. Therefore one would like to devise alternative methods to diagnose the appearance of colour rearrangement.

We now want to propose and study a method to select the correct string topology. The starting point is the observation that, were it not for various smearing effects, hadrons would be perfectly lined up with the string pieces spanned between the partons. In momentum space the hadrons would appear along hyperbolae with the respective endpoint parton directions as asymptotes. In a frame where a string piece has no transverse motion, i.e. where the two endpoint partons are moving apart back-to-back, the hadrons produced by this piece would have vanishing transverse momentum. A hadron could then successively be boosted to the rest frame of any parton pair until the pair is found for which the particle $p_\perp$ is vanishing. When smearing is introduced, this $p_\perp$ will no longer be vanishing, but it still would be reasonable that a "best bet" is to assign each hadron to the string piece with respect to which it has smallest $p_\perp$. The most likely string configuration would be the one where the sum of all hadron $p_\perp$'s is minimal. While the fluctuations for each single hadron is too large for usefulness, it could be hoped that the net effect of all the hadrons would be to single out the correct topology.

To be more precise, here is the scheme proposed, to be carried out for each event:

1. Use some jet clustering algorithm to identify the directions of the number of jets that should be used for the current application.
2. Enumerate the possible colour flow topologies allowed in the process.
3. Form one $\sum p_\perp$ measure for each topology, where the sum runs over all final-state particles in the event. For each particle, its $p_\perp$ is defined as the minimum of the $p_\perp$'s obtained by boosting the particle to the rest frame of each of the string pieces making up the current topology.
4. Identify the correct topology as the one with smallest $\sum p_\perp$.

Several effects could smear the simple picture and lead to incorrect conclusions. The main ones are:

- Errors in the reconstruction of jet directions.
- Additional $p_\perp$ caused by perturbative QCD branchings, predominantly gluon emission.
- The $p_\perp$ generated by the fragmentation process.
- Secondary decays of unstable hadrons.

A convenient test bed for the relative importance of these effects is provided by $q\bar{q}g$ events. In symmetric three-jet events at Z^0 energies, without any parton-shower activity, the gluon is correctly identified in 87% of the cases. This should be compared with the 33% expected from a random picking among the three alternatives. When complete Z^0 events events are generated (with PYTHIA 5.7 and JETSET 7.4 [12]), three jets are found (in 10–20% of the events) and the method above is used, the success rate is around 55–60%. The "correct" answer is here found by tracing the original quark and antiquark through the shower history and associating them with the jets they are closest to in angle. The conclusion of this kind of studies is that the method does work, though maybe not as well as one might have hoped, and that the main cause of errors is the perturbative gluon emission.

The studies can be extended to four-jet $q\bar{q}gg$ events, although the large number of colour flow topologies here makes the method rather inefficient for selecting the correct topology. A more modest objective is to establish differences between string and independent fragmentation [13] models. In the latter approach, particle production is aligned along the direction connecting a parton with the origin in the c.m. frame of the event, again with some smearing effects. This approach is already disfavoured by the three-jet studies [4], but is a convenient reference. We have not completed a full study, but can illustrate with results for a simple four-jet cross topology with q and $\bar{q}$ back-to-back. Fig. 1a gives the difference in $\sum p_\perp$ between the worst topology (where none of the string pieces run the way assumed) and the right one. Results are for charged particles, and the $\sum p_\perp$ has been normalized to the number N of charged particles per event. The independent fragmentation distribution is symmetric around the origin, as it should, while the string approach shows a clear offset. Without knowledge of the correct answer, one is reduced to studying a measure such as the difference between the maximal and the minimal $\sum p_\perp$ among all the twelve possible topologies, Fig. 1b. By construction this is a positive number, also for independent fragmentation, but an additional shift is visible for the string approach. Identification of one or both quarks, e.g. by b quark tagging or energy ordering (quarks are likely to have higher energy than gluons), would cut down on the number of topologies to be compared and therefore enhance the signal. Some experimental studies along these lines could therefore be interesting.

We now turn to the main application of this letter, namely four-jet W^+W^- events. Since the two W decay vertices are less than or of the order of 0.1 fm apart, while typical hadronic distances are of the order of 1 fm, the two W decays occur almost on top of each other. QCD interconnection effects could appear at all stages of the process, namely in the original perturbative parton cascades, in the subsequent soft hadronization stage, and in the final hadronic state. It can be shown that perturbative effects are strongly suppressed [7], but no similar arguments hold for the other two. Bose-Einstein effects in the hadronic state could be the largest individual source of W mass uncertainty [14], but it is the least well studied. The presence of such Bose-Einstein effects presumably could be established from the data itself, while reconnection in the hadronization stage is less easy to diagnose. We will study whether the $\sum p_\perp$ measure offers any help here.

The reconnection models used as references in this work are:

1. Reconnection after the perturbative shower stage but before the hadronization, with reconnection occuring at the 'origin' of the showering systems. This 'intermediate' model is the simplest of the more realistic ones.
2. Reconnection when strings overlap based on cylindrical geometry. A 'bag model' based on a type I superconductor analogy. The reconnection probability is proportional to the overlap integral between the field strengths, with each field having a Gaussian fall-off in the transverse direction, the radius being about 0.5 fm. The model contains a free strength parameter that can be modified to give any reconnection probability.
3. Reconnection when strings cross. In this model the strings mimic the behaviour of the vortex lines in a type II superconductor, where all topological information is given by a one-dimensional region at the core of the string.
4. Reconnections occur in such a way that the string 'length' is minimized. As a measure of this length the so-called λ-measure is used, which essentially represents

the rapidity range for particle production counted along the string. This can also be seen as a measure of the potential energy of the string.

Models 1 through 3 is described in [7] and the last in [9]. Further models have been proposed [15].

For the study, events should have a clear four-jet structure. To achieve this we demand that each jet must have some minimum energy and that the angle between any jet pair must not be too small [7]. When applied to the expected statistics of LEP 2, the number of events left after the cuts will be about 2500 per experiment; therefore statistics will be a problem when different models are compared with each other.

Three different algorithms are used to identify which jet pairs belong together:

1. The $q_1\overline{q}_2q_3\overline{q}_4$ configuration before parton showers can be matched one-to-one with the reconstructed jets after hadronization. This is done by minimizing the products of the four (jet+q) invariant masses. The original quark information is not available in an experimental situation, so this measure can only be used as a theory reference.
2. The invariant mass of jet pairs from the same W should be close to the known W-mass of about 80 GeV. Among the three possible jet pairings, therefore the one is selected which has minimal $|m_{ij}-80|+|m_{kl}-80|$, where i, j, k, l are the four jets. We have picked this method rather than a few similar ones since it has (marginally) the best correlation with the reference method above. This method is mainly probing the electroweak aspect of W pair production, namely the W mass spectrum, while it should be less dependent on the QCD stages of showering and hadronization.
3. The $\sum p_\perp$ method introduced in this letter provides an alternative measure, that rather should be sensitive to the QCD stages and less so to the electroweak one. Without colour reconnection it should (hopefully) agree with the previous two, while it could give interesting differences if reconnection occurs.

The agreement between these three methods is shown in Table 1, with and without reconnection, the former for model 1. As should be expected, algorithm 2 comes close to the "correct" answer of number 1, and is not significantly affected by colour rearrangement. The $\sum p_\perp$ method, algorithm 3, shows the expected dependence on colour rearrangement, with a smaller success rate when colour rearrangement is included. Note, however, that the success rate does not drop below the naive 33% number, indicating that the $\sum p_\perp$ method is also picking up other aspects of events, such as the jet topology. The results in the table are for a 170 GeV energy, but we do not expect a significant energy dependence.

Both methods 2 and 3 can be applied to data, so therefore the correlation is an observable. The rate of reconnection could be extracted, by interpolation between the two extremes of no and complete colour rearrangement. Statistically it should be feasible to establish a signal for reconnections, if they occur at a rate above the 10–20% level. The systematic errors on the correlation method may be large, however, especially for the model-dependent change when reconnection is included. It is therefore important to study whether a differential distribution would better highlight qualitative differences.

Algorithm 2 can be used to identify the best hypothesis for which jets should be paired to form the two W's, and also the worst hypothesis, where $|m_{ij}-80|+|m_{kl}-80|$ is maximal. The $\sum p_\perp$ can be calculated for each of these two extremes. When the strings are reconnected, the first sum should increase and the second one decrease relative to the no-reconnection case. The signal is therefore enhanced by making use of the difference, $\Delta = (\sum p_\perp)_{\mathrm{worst}} - (\sum p_\perp)_{\mathrm{best}}$, which should decrease in case of reconnection. The subtraction furthermore has the advantage of removing some spurious fluctuations from the comparison. The main example is high-momentum particles, where the assumed string hyperbolae attach well with the four jet directions and therefore all three string hypotheses give the same contribution to $\sum p_\perp$. (Our studies show that particles with momenta above 3 GeV add little to the discrimination between the string hypotheses.)

The Δ measure is plotted for models 1–4 in Fig. 2. Note that the results for models 1 and 4 correspond to 100% reconnection, while the reconnection rate in models 2 and 3 is about 30%. (The reconnection fraction could be varied in all models, but the effects of reconnection are not linear in this fraction for models 2 and 3, so a reasonable value is preferred here.) All reconnection models show the expected shift towards smaller Δ values, and the magnitude of the shift is comparable once differences in assumed reconnection fractions are removed. Remaining differences imply that one cannot model-independently extract a reconnection rate from the data. The signal for reconnection may be enhanced, compared with the results of Table 1, by cuts on Δ, e.g. by only considering the fraction of events with $\Delta < 0$. This is at the price of a reduced statistics, however, so the balance is not so clear. It may be better to use measures that gauge the full shape of the curve, given that the physics of the no-reconnection scenario is presumed well-known (by extrapolation from the Z^0 results).

Prospects look promising to diagnose colour rearrangement along these lines but, as before, the combination of low effects and small statistics could give marginal results. Furthermore, hopes should not be raised too high that this would immediately imply a scheme to correct a W mass measurements for the reconnection effects: of the models above, number 3 shifts the W mass downwards while the others shift it upwards [7, 8], and yet they all shift the Δ distribution in the same direction. Clearly the study of reconnection effects ultimately must be based on a host of different measures, the $\sum p_\perp$ one and others.

It may be of some interest to understand why effects are not larger. Several simulations have been performed with various simplified toy models to study this issue [16]. It turns out that there are two main mechanisms that smear distributions and make them less easily distinguished. One is parton showers, just as for the $Z^0 \to q\overline{q}g$ process studied above. The other is the geometry of the process, namely that the helicity structure of the $W^+W^- \to q_1\overline{q}_2q_3\overline{q}_4$ process is such that q_1 and q_3 tend to go in the same general direction, as do $\overline{q}_2$ and $\overline{q}_4$ [10]. The overall change of string topology by reconnection therefore is not as drastic as if the q_1 had tended to be close to $\overline{q}_4$ and vice versa. Had it been possible to remove these effects, i.e. study events without shower activity and with q_1 and $\overline{q}_4$ reasonably close in angle, the original and the colour-reconnected Δ distributions would almost completely separate. In practice, only a modest reduction of shower activity could be obtained by requiring that all four jets be reasonably narrow, while tagging of quark vs. antiquark (e.g. with charm) would leave very few events. Therefore no simple solutions have been found.

In summary, we have introduced a $\sum p_\perp$ measure as a diagnostic of the colour topology of hadronic events. The fuzzy nature of hadronic final states somewhat limits the usefulness of the method. In particular, the more drastic effects associated with perturbative gluon emission tend to obscure the subtler effects of different colour topologies. Therefore the $\sum p_\perp$ measure is no panacea, but could still be a useful addition to the (not so large) tool box of methods to characterize the nonperturbative stage of hadronic events. Applications include three- and four-jet events at Z^0 energies and, in particular, W pair decay to four jets at LEP 2. In the latter process, it could be possible to detect the effects of colour reconnection with this approach. Further details on the studies reported here may be found in [16].

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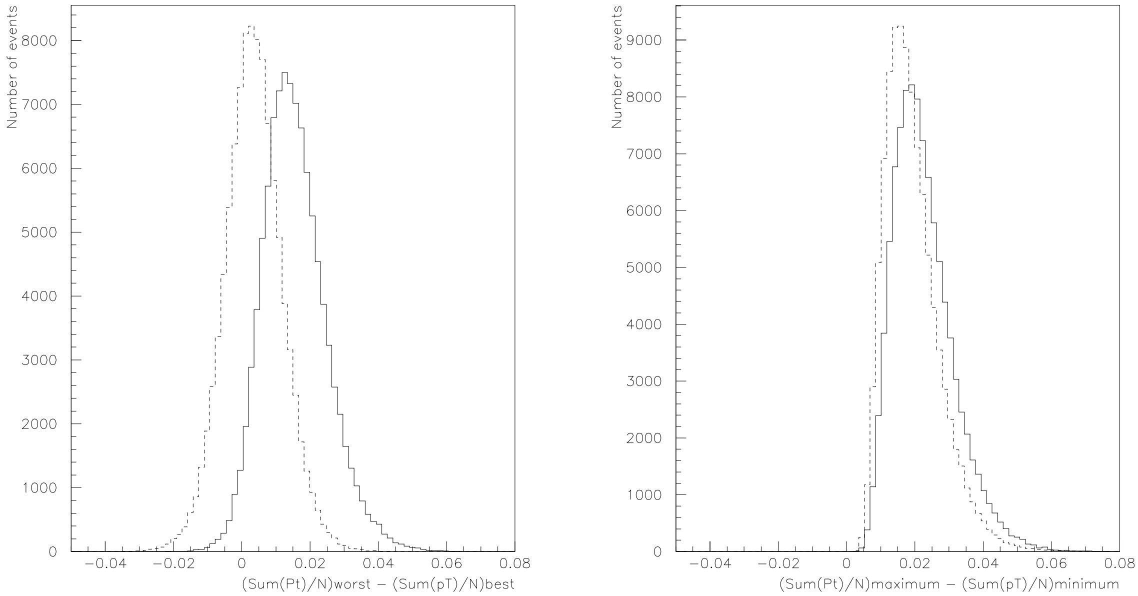
no reconnection			with reconnection (model 1)		
algorithm	1	2	algorithm	1	2
2	85%	–	2	85%	–
3	68%	65%	3	46%	46%

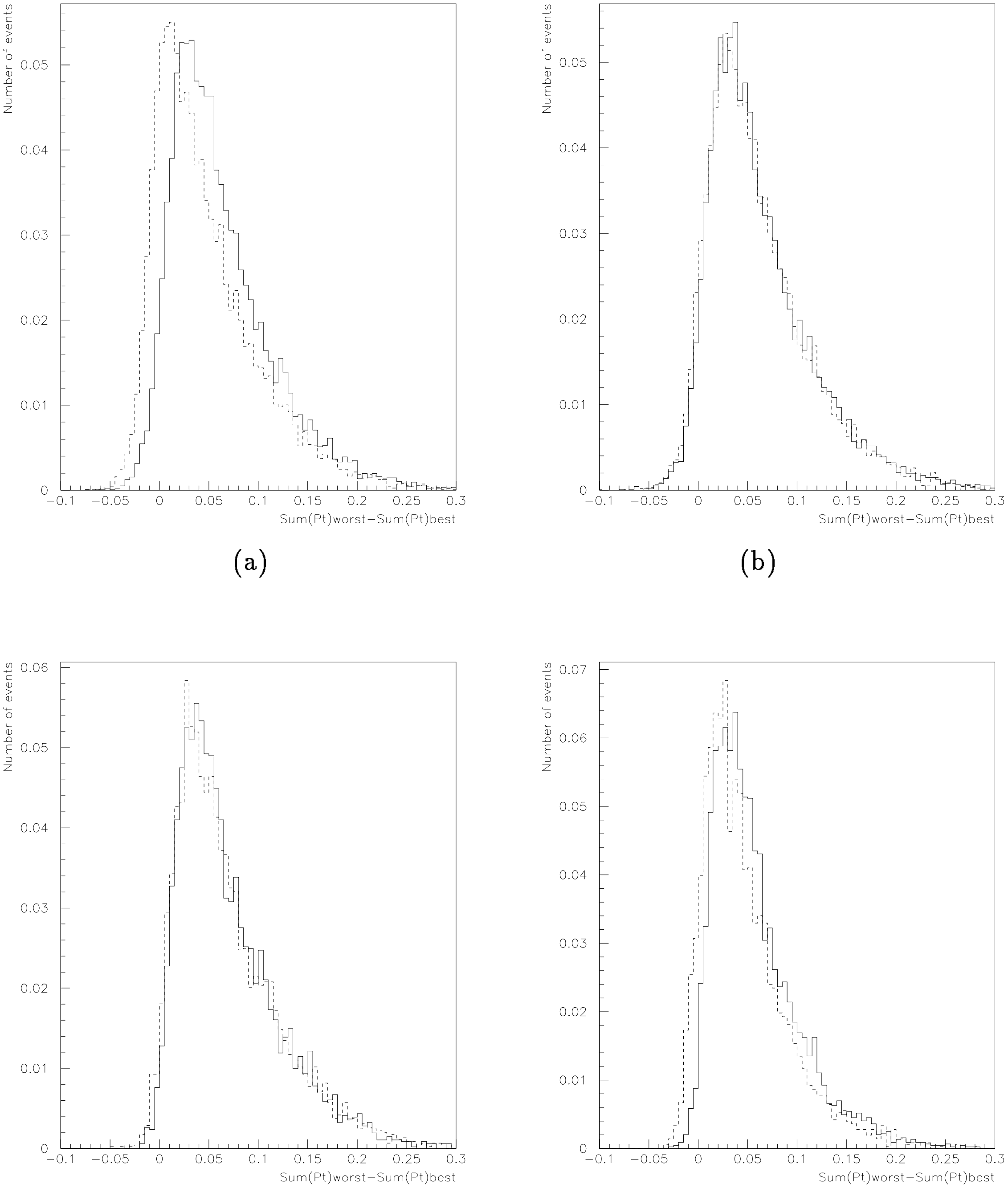
Table 1: Fraction of agreement in jet pair identification between the three algorithms.



(a)

(b)

Figure 1: The distribution of $\sum p_\perp/N$ for **a**: worst−right, and **b**: maximal−minimal, for Lund string (full) and independent (dashed) fragmentation respectively.



(a) (b) (c) (d)

Figure 2: Distribution of $\Delta = (\sum p_\perp)_{\mathbf{worst}} - (\sum p_\perp)_{\mathbf{best}}$. Dashed lines are reconnected events according to models **a**: 1 (intermediate), **b**: 2 (bag model), **c**: 3 (type II superconductor), and **d**: 4 (λ-measure), while full lines always are without reconnection.