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On The Threshold Behaviour of Heavy Top Production*

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Abstract

If top is heavy, as now seems likely, the $t\bar{t}$ threshold behaviour is given by perturbative QCD. The QCD threshold interaction can be formulated in terms of a potential, attractive or repulsive depending on whether the $t\bar{t}$ is in a colour singlet or octet state. This gives a suppression factor for octet production. Singlet production is enhanced, both above threshold and, by resonance formation, below it. While e^+e^- annihilation only proceeds in the singlet $t\bar{t}$ channel, hadron-hadron collisions contain a nontrivial mixture of the two. In this paper we review the relevant threshold factor formulae, and present phenomenological consequences for hadron colliders, current and future.

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1 Introduction

We can feel quite confident that the sixth flavour – top – exists. In the framework of the standard model, the observation of large B_d^0–$\overline{B}_d^0$ mixing by ARGUS [1] and CLEO [2] suggests that the top should be heavy [3]

$$m_t \simeq 100 - 150 \text{ GeV}$$

(more conservatively, taking into account existing uncertainties in the parameters of the standard model, $m_t \geq 60$ GeV). From the analysis of the recent experimental data [4] there exist the restrictions

$$60 \leq m_t \leq 180 - 200 \text{ GeV}$$

It is therefore interesting to consider the consequences of a heavy top scenario.

The properties of a heavy top will be in marked contrast to those of charm and bottom. Specifically, for large enough quark mass ($m_t \geq 100$ GeV), the influence of nonperturbative effects is small, and quark dynamics is governed only by electroweak and perturbative QCD effects. This has been discussed in detail in Refs. [5,6,7].

In the threshold region, the behaviour is calculable in terms of a QCD Coulomb-like interaction between the t and $\bar{t}$ at small enough distances, with an effective α_S that is reasonably small. This gives a situation closely similar to heavy lepton pair production, with α_{EM} replaced by α_S.

In hadron collisions, a $t\bar{t}$ pair may be produced either in a colour singlet state or in a colour octet one (while only the former is allowed in e^+e^- annihilation). The threshold interaction is then given by two-particle Coulomb-like potentials,

$$V^{(s)}(r) = -\frac{4}{3}\frac{\alpha_S(1/r)}{r} \tag{1}$$

for the colour singlet channel, and

$$V^{(8)}(r) = \frac{1}{6}\frac{\alpha_S(1/r)}{r} \tag{2}$$

for the colour octet one.

While the interaction is repulsive in the octet state, multiple soft gluon exchange between t and $\bar{t}$ can give bound states in the singlet channel, at a distance scale $r \simeq (\alpha_S m_t)^{-1}$. Thus the standard set of Coulomb-like bound states is formed below the continuum threshold (at least for the lower levels), see Refs. [8,9]. As we shall see, the interactions also imply significant modifications of the cross-sections above threshold.

As heavier and heavier top masses are considered, the top width increases rapidly [5],

$$\Gamma_t \approx (175\,\text{MeV})\left(\frac{m_t}{m_W}\right)^3 \tag{3}$$

for $m_t > m_W$. Therefore toponium states will be increasingly broad, and eventually (for $m_t \simeq 150$ GeV) merge into a smeared below-threshold 'continuum', when classical bound states do not have the time to form before weak decays.

To our knowledge, the effects of Coulomb rescattering of the nonrelativistic particles in QED were first discussed by Sommerfeld [10] and Sakharov [11]. Ref. [11] was devoted specifically to the effects of Coulomb attraction in lepton pair production. QCD Coulomb-like effects for the threshold behaviour in the colour singlet channel were rediscovered by Appelquist and Politzer [12].

A detailed study of threshold behaviour for the process $e^+e^- \to t\bar{t}$ was performed in Ref. [6], and of threshold behaviour for general processes in Ref. [7]. In the current paper, we will review the main results obtained in these publications, and perform a phenomenological study of consequences for heavy flavour production in hadron colliders. The results will be valid for the threshold region. This region could be of interest when special experimental cuts are imposed, but it also affects the total cross-section, since the effective parton luminosities sharply decrease with the pair invariant mass-squared, $\hat{s} = \tau s$, see Refs. [13,14].

2 Threshold Factors

The Coulomb and width effects modify drastically the threshold behaviour of the process $e^+e^- \to t\bar{t}$ [6]. (Also QED radiative corrections, connected with bremsstrahlung off the initial leptons, are of importance.) Thus, analogously to the QED case [11], for the singlet channel the Coulombic attraction leads to a sharp increase of the total cross-section. In the narrow width approximation, the standard threshold factor

$$\beta_t = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \tag{4}$$

(with $\hat{s} = s$ for e^+e^- annihilation) in the cross-section is replaced by

$$\beta_t |\Psi^{(s)}(0)|^2. \tag{5}$$

Here the squared wave function at the origin is given by

$$|\Psi^{(s)}(0)|^2 = \frac{X_{(s)}}{1 - \exp(-X_{(s)})}, \; X_{(s)} = \frac{4}{3}\frac{\pi\alpha_S}{\beta_t}. \tag{6}$$

In the limit $\beta_t \to 0$, the total threshold factor

$$\beta_t |\Psi^{(s)}(0)|^2 \to \frac{4}{3}\pi\alpha_S, \tag{7}$$

i.e. is non-vanishing.

It should be noted that, for $X_{(s)} \gg 1$, eq. (6) is twice as large as the well-known Schwinger [15] one-loop result (near threshold $1 + X_{(s)}/2$).

For the octet channel, the cross-section is decreased due to Coulombic repulsion, see eq. (2) and Ref. [7]. The naive threshold factor β_t is now replaced by

$$\beta_t |\Psi^{(8)}(0)|^2, \tag{8}$$

where

$$|\Psi^{(8)}(0)|^2 = \frac{X_{(8)}}{\exp(X_{(8)}) - 1}, \; X_{(8)} = \frac{1}{6}\frac{\pi\alpha_S}{\beta_t}. \tag{9}$$

A detailed analysis of the one-loop QCD corrections to the cross-section of heavy flavour pair production in hadron-hadron collisions has been performed by Nason et al. in Ref. [16];

see also Refs. [13]. In their formulae, the terms of $\mathcal{O}(\pi\alpha_S/\beta_t)$ coincide with the first terms in the decomposition of eqs. (6) and (9). The latter formulae may therefore be viewed as properly exponentiated versions of the first order result for the threshold behaviour.

Note that the threshold modification, eqs. (6) and (9), is non-negligible even rather far above the threshold. This in particular applies for the colour singlet channel, where the parameter $X_{(s)}$ is large. The equations will have to be taken with a grain of salt for large invariant $t\bar{t}$ masses, however. Not only is the derivation based on approximations not valid then, but additionally hard perturbative QCD effect (extra hard gluon emission etc.) have to be taken into account [16].

For a heavy top, the Coulomb effects in the singlet channel totally determine the formation of bound $t\bar{t}$ states, since perturbative QCD may be used. However, the contribution of these states to the total $t\bar{t}$ cross-section is rather small, of order α_S^3, and concentrated into a small region, $\Delta E \simeq \alpha_S^2 m_t$, of $t\bar{t}$ invariant masses.

In hadron-hadron collisions, the contributions of the different production mechanisms to the pair invariant mass distribution take the standard forms

$$\frac{d\sigma_{q\bar{q}}}{d\tau} = \sigma_{q\bar{q}\to t\bar{t}}(\hat{s}) \left(\frac{d\mathcal{L}_{q\bar{q}}}{d\tau}\right), \quad \frac{d\sigma_{gg}}{d\tau} = \sigma_{gg\to t\bar{t}}(\hat{s}) \left(\frac{d\mathcal{L}_{q\bar{q}}}{d\tau}\right), \tag{10}$$

where the bracketed expressions denote effective parton luminosities.

In the Born approximation, averaging over initial parton polarizations, the cross-sections for the subprocesses $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ are, near threshold, equal to (see e.g. [16,13,14])

$$\sigma^{(B)(th)}_{q\bar{q}\to t\bar{t}}(\hat{s}) = \frac{1}{9}\frac{\pi\alpha_S^2}{m_t^2}\beta_t, \quad \sigma^{(B)(th)}_{gg\to t\bar{t}}(\hat{s}) = \frac{7}{192}\frac{\pi\alpha_S^2}{m_t^2}\beta_t. \tag{11}$$

In $q\bar{q}$ collisions, where the process has to go via an intermediate s-channel gluon, $t\bar{t}$ pairs are produced exclusively in the colour octet state, while gg collisions give a mixture of octet and singlet contributions. The ratio of octet to singlet states is given by the ratio of colour factors

$$\frac{(d^{abc}/\sqrt{2})^2}{(\delta^{ab}/\sqrt{3})^2} = \frac{5}{2}. \tag{12}$$

Note that, in the $gg \to t\bar{t}$ process, the $t\bar{t}$ pair has spin-parity $J^P = 0^-$ ($S = 0$, $L = 0$), with incoming gluons having total spin $S_G = 1$ and relative orbital momentum $L_G = 1$.

Evidently, the Coulomb gluon exchanges lead to the enhancement of singlet state production and the suppression of octet state. Since $t\bar{t}$ pairs are produced at a characteristic distance $\simeq 1/m_t$, which is much less than the characteristic distance $r_{char} \simeq 1/p_{char} \simeq 1/(m_t\beta_t)$ ($\beta_t \sim \alpha_S$ for bound states), one can use the results of the analysis in Refs. [6,7]. For the octet part of the cross-section this gives

$$\sigma_{q\bar{q}\to t\bar{t}} = \sigma^{(B)}_{q\bar{q}\to t\bar{t}}|\Psi^{(8)}(0)|^2, \quad \sigma^{(8)}_{gg\to t\bar{t}} = \frac{5}{7}\sigma^{(B)}_{gg\to t\bar{t}}|\Psi^{(8)}(0)|^2. \tag{13}$$

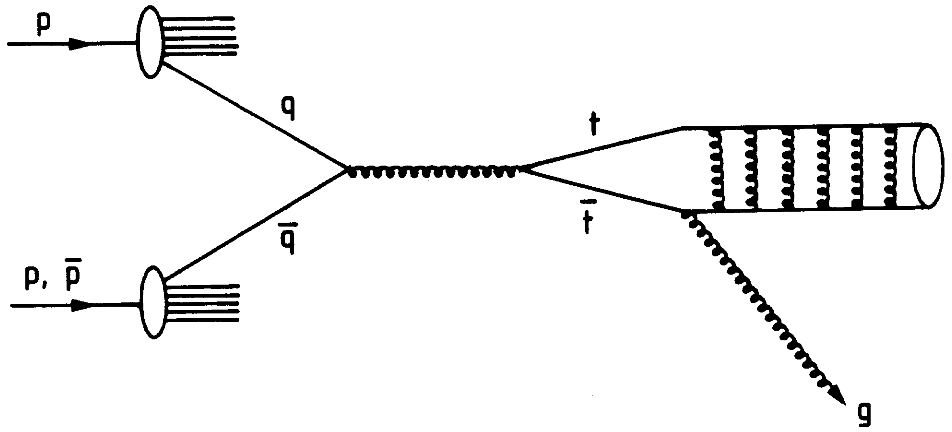

Figure 1: Formation of bound state by final state gluon emission.

The naive recipe for the singlet part is

$$\sigma^{(s)}_{gg\to t\bar{t}} = \frac{2}{7}\sigma^{(B)}_{gg\to t\bar{t}}|\Psi^{(s)}(0)|^2. \tag{14}$$

This formula does not explicitly take into account bound states, however. Doing this, one obtains [7]

$$\sigma^{(s)}_{gg\to t\bar{t}} = \frac{2}{7}\sigma^{(B)}_{gg\to t\bar{t}}\frac{4\pi}{m_t^2\beta_t}\Im G_{E+i\Gamma_t}(0,0). \tag{15}$$

Here E is the energy above or below threshold, $E = \sqrt{\hat{s}} - 2m_t$, Γ_t the top weak decay width, eq. (3), $G_{E+i\Gamma_t}(\vec{r},\vec{r}')$ the Green function of the $t\bar{t}$ system in the colour singlet state, and $\Im G$ its imaginary part [6,7]

$$\begin{aligned}\Im G_{E+i\Gamma_t}(0,0) &= \frac{m_t^2}{4\pi}\left[\frac{p_2}{m_t} + \frac{2p_0}{m_t}\arctan\frac{p_2}{p_1} + \right.\\ &+ \left.\sum_{n=1}^{\infty}\frac{2p_0^2}{m_t^2 n^4}\frac{\Gamma_t p_0 n + p_2(n^2\sqrt{E^2+\Gamma_t^2} + p_0^2/m_t)}{(E + p_0^2/(m_t n^2))^2 + \Gamma_t^2}\right],\end{aligned} \tag{16}$$

where

$$\begin{aligned}p_0 &= \frac{2}{3}m_t\alpha_S,\\ p_{1,2} &= \left[\frac{m_t}{2}\left(\sqrt{E^2+\Gamma_t^2} \mp E\right)\right]^{1/2}.\end{aligned} \tag{17}$$

The sum corresponds to the contribution from an infinite set of bound states, at energies $E_n = -p_0^2/(m_t n^2) = -4m_t\alpha_S^2/(9n^2)$, with total contributions $\propto 1/n^3$, and with a common width Γ_t given entirely by weak decays. In the subsequent section, we will compare the naive with the full expression.

Note that, especially at not so high hadron-hadron energies, where the $q\bar{q}$ mechanism of $t\bar{t}$ production dominates, the main source of bound state formation could be the 'bremsstrahlung process', $q\bar{q} \to (t\bar{t})g$, shown in Fig. 1. Here the emission of an additional gluon transforms the $t\bar{t}$ pair into a colour singlet state. Just the opposite would be expected for the colour singlet channel: emission of a gluon is suppressed, since it would give a repulsive colour octet $t\bar{t}$ state. Additional suppression would come from the dipole nature of such emission close to threshold, see Ref. [7]. Neither of these effects are included in the formalism above.

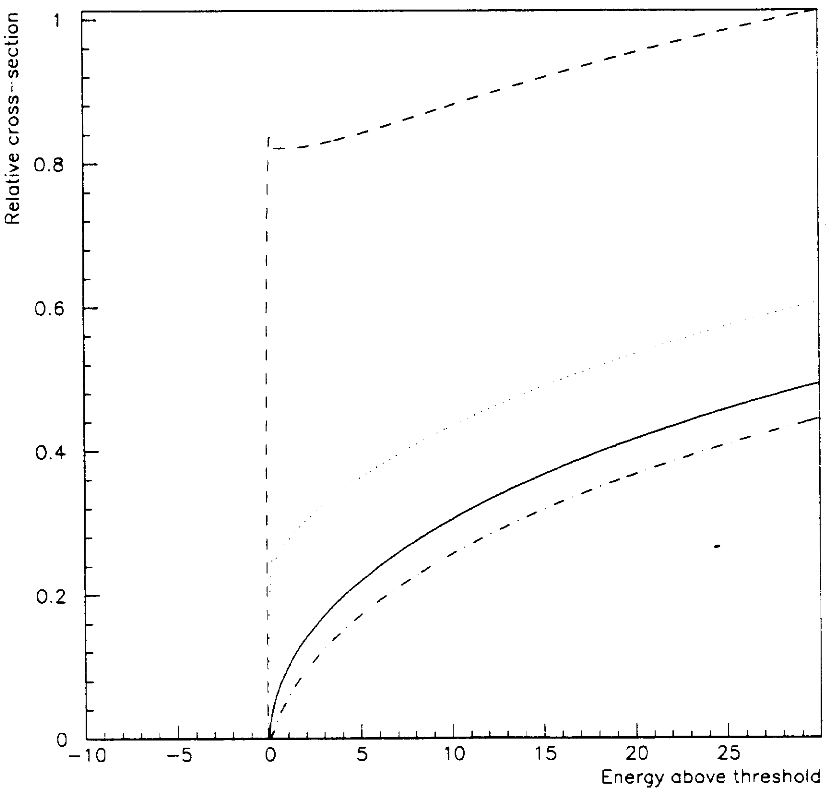

Figure 2: Threshold behaviour for $m_t = 100$ GeV and fixed $\alpha_S = 0.196$. Full line is the standard threshold factor β_t, dashed the enhanced singlet channel factor $\beta_t|\Psi^{(s)}(0)|^2$, and dash-dotted the suppressed octet channel factor $\beta_t|\Psi^{(8)}(0)|^2$. Dotted gives the combination relevant for the gg channel, 2/7 singlet and 5/7 octet. E is energy above nominal top threshold at $2m_t$.

3 Results

In the following, we will study the implications of the threshold modifications introduced above. For the calculations, the EHLQ set 1 structure functions [14], with $\Lambda = 200$ MeV, have been used throughout. Although the absolute results would be somewhat different with another choice, none of the qualitative features would be affected. For Monte Carlo integration, the PYTHIA program [17] has been used, with the full $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ Born cross-sections (rather than the approximate ones in eq. (11)).

Normally, a running α_S is used, with argument corresponding to the simplest dependence on the characteristic virtuality in the process, $Q^2 = \bar{p}_Q^2 \simeq m_t\sqrt{E^2 + \Gamma_t^2}$, i.e.

$$\alpha_S = \frac{12\pi}{23\ln(m_t\sqrt{E^2 + \Gamma_t^2}/\Lambda^2)}, \tag{18}$$

with $\Lambda = 200$ MeV. When using fixed α_S values, these were chosen to agree with the running α_S value at the first maximum of the cross-section (the lowest bound state).

Fig. 2 shows the importance of the enhancement and suppression factors in eqs. (6) and (9). Note that, due to $X_{(s)}$ being much larger than $X_{(8)}$, the singlet enhancement factor deviates more from unity than the octet suppression factor does, and that therefore there is a net increase in the gg channel.

Figures 3 - 5 illustrate the threshold behaviour in the colour singlet channel for various

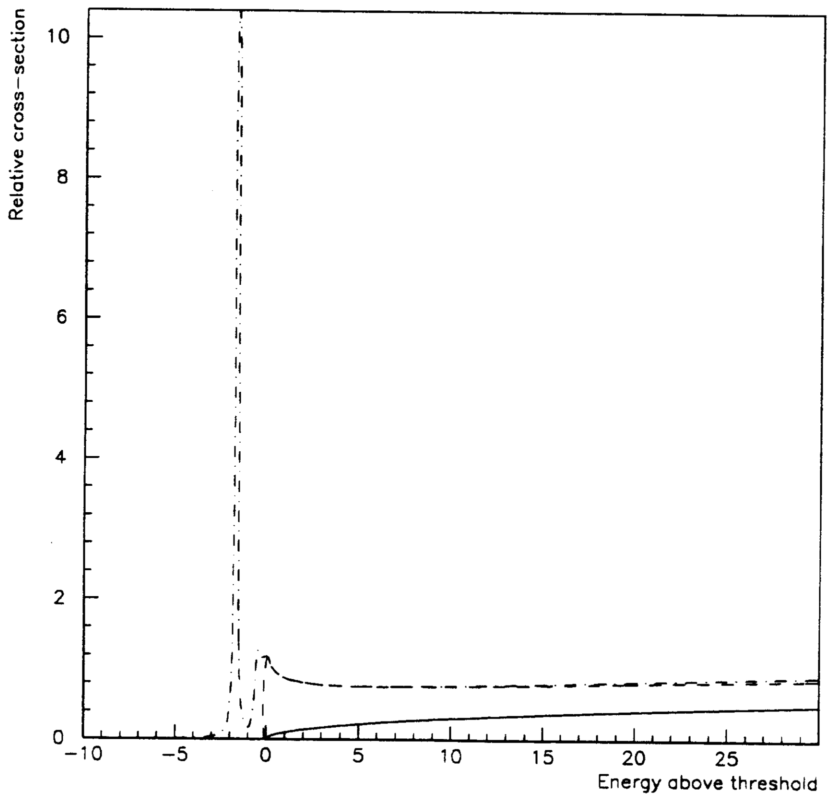

Figure 3: Threshold behaviour for colour singlet channel, with $m_t = 100$ GeV and running α_S. Full is standard threshold factor β_t, dashed the naive enhancement recipe $\beta_t|\Psi^{(s)}(0)|^2$, and dash-dotted the expression $4\pi\Im G_{E+i\Gamma_t}(0,0)/m_t^2$.

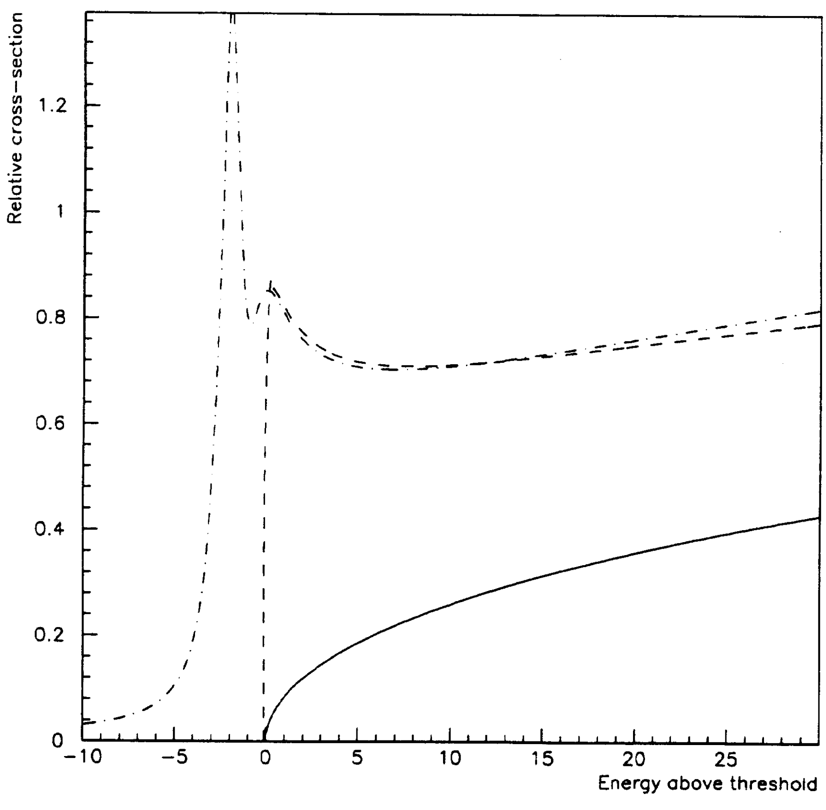

Figure 4: Threshold behaviour for colour singlet channel, with $m_t = 140$ GeV and running α_S. Notation as in Fig. 3.

values of the top mass, $m_t = 100$, 140 and 200 GeV. Even for 100 GeV, only the lowest bound state shows up as an explicit peak in the cross-section, while all the rest merge into a broad bump. At 200 GeV the $t\bar{t}$ levels completely overlap, and the resonance structure practically washed out and buried in the continuum [5,6,7].

Above threshold, the simple enhancement factor formula, eq. (6), agrees well with the full answer involving $\Im G_{E+i\Gamma_t}(0,0)$, but the simple answer does not include the effect of bound state formation below threshold. On the other hand, $\Im G_{E+i\Gamma_t}(0,0)$ is derived specifically with respect to threshold behaviour, and the formula breaks down far away from the threshold region. Specifically, the first term in eq. (16) is proportional to $\sqrt{E}$, and will give an unphysical asymptotic behaviour. One should also note that, for Γ_t large, the simple Breit-Wigner type shape assumed gives little damping for producing a $t\bar{t}$ pair far below threshold. When folded with structure functions, which are peaked at small values, this would give a large rate for very low-mass $t\bar{t}$ pair production. A more detailed description would here be necessary, so the formula should not be trusted more than maybe 20 - 30 GeV below threshold.

The Figures 2–5 have illustrated the threshold modifications arising from Coulomb effects. In Figs. 6–9 consequences are shown for current and future hadron colliders, in terms of the cross-section as function of the $t\bar{t}$ invariant mass. Monte Carlo results are for 10000 events at each energy, and have been obtained by a sampling of the correct Born term expression, multiplied by the relevant modification factors.

At 630 GeV, where the $q\bar{q}$ mechanism dominates, the total cross-section is reduced by about 10%. As the energy is increased, the gg channel becomes the more important, and so the total cross-section is increased by Coulomb effects. At *LHC* and *SSC* the increase is roughly 10%. The crossover is at around *TeVI* energies, see Fig. 10. The shape of the invariant mass distribution is also changed somewhat by the introduction of threshold factors, not unexpectedly with the effect most marked close to threshold.

Note that the *analytical* expressions for the cross-sections of the processes $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$ in the threshold region change drastically. The lack of a large *numerical* difference in the total hadron-hadron production cross-section is connected, on the one hand, with the relatively small colour factor 2/7 for the singlet channel, and, on the other hand, with the smallness of the damping parameter $X_{(8)}$ for the dominant octet channel. Further, in hadron collisions, the integration over a large range of $t\bar{t}$ masses and summation over contributing structure functions reduce the observable Coulomb effects.

4 Summary

Let us emphasize that our purpose here was not the detailed description of all QCD corrections, but the demonstration of the Coulombic $\pi\alpha_S/\beta$ effects, in differential and total cross-sections. The reader can find a comprehensive analysis of the one-loop QCD corrections to the heavy quark pair production in [16], and in these proceedings in [13]. The two approaches are complementary. One aspect not included in the one-loop formulae, but included here, is bound state production. Likely the one-loop approach will still be reasonably good in a dual sense, i.e. in giving the total cross-section near to threshold, even if not the detailed shape.

The threshold modifications are especially important for e^+e^- annihilation. This process is entirely colour singlet, and a significant enhancement of the cross-section is therefore to be expected. This is important to take into account, e.g. for the determination of the top

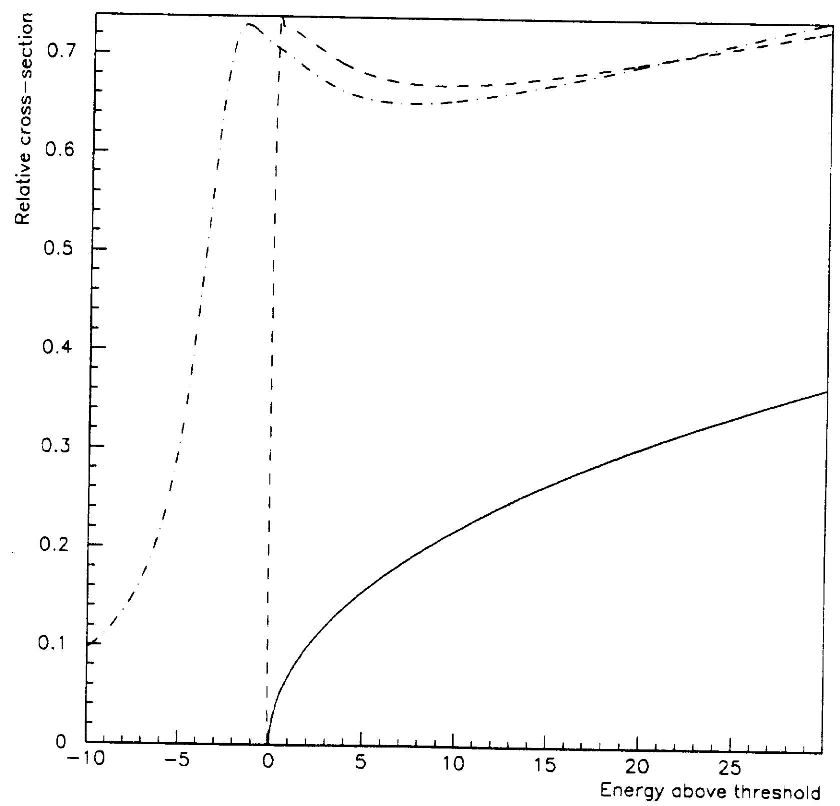

Figure 5: Threshold behaviour for colour singlet channel, with $m_t = 200$ GeV and running α_S. Notation as in Fig. 3.

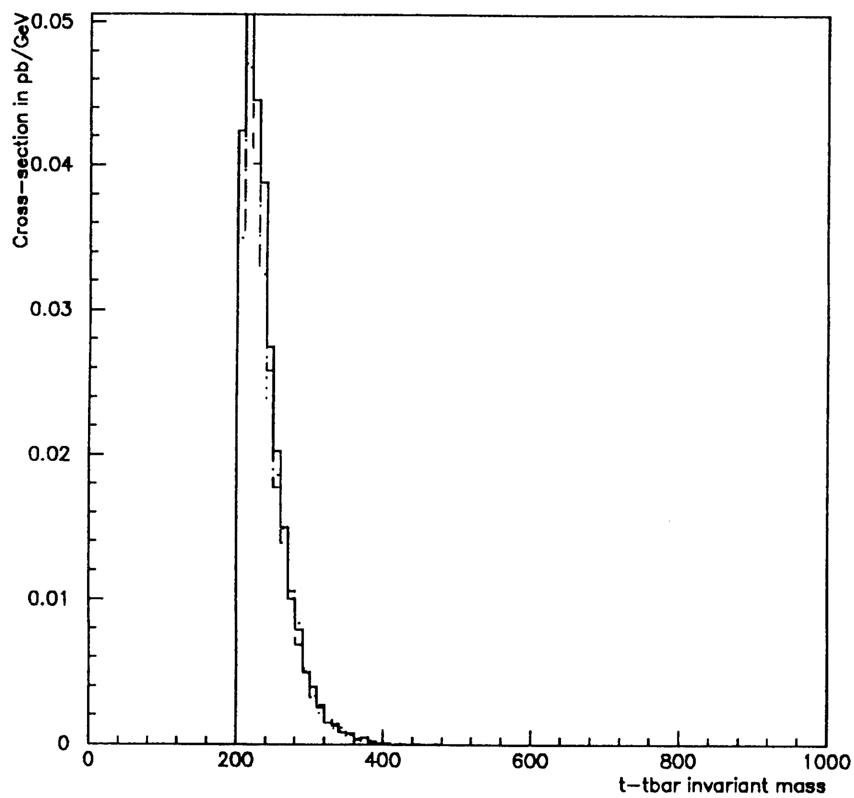

Figure 6: Invariant mass distribution of $t\bar{t}$ pairs for the $Sp\bar{p}S$ collider at 630 GeV, with $m_t =$ 100 GeV. Full line is without any Coulomb corrections, dashed and dotted with corrections, using a fixed or running α_S, respectively. Contribution below threshold is not included.

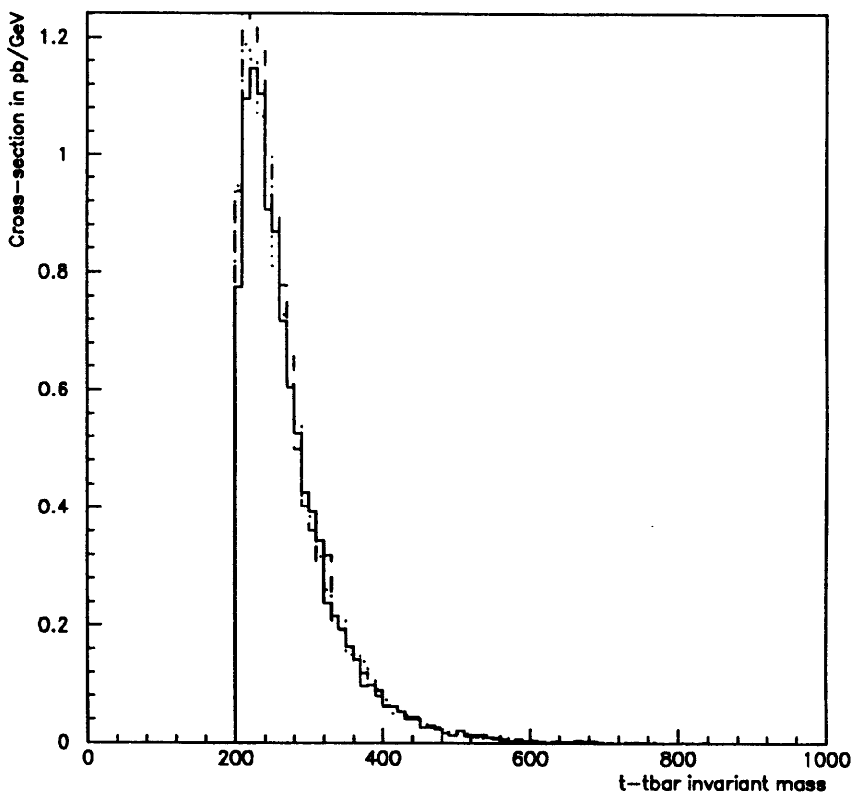

Figure 7: Invariant mass distribution of $t\bar{t}$ pairs for the $TeVI$ collider at 1.8 TeV, with m_t = 100 GeV. Notation as in Fig. 6.

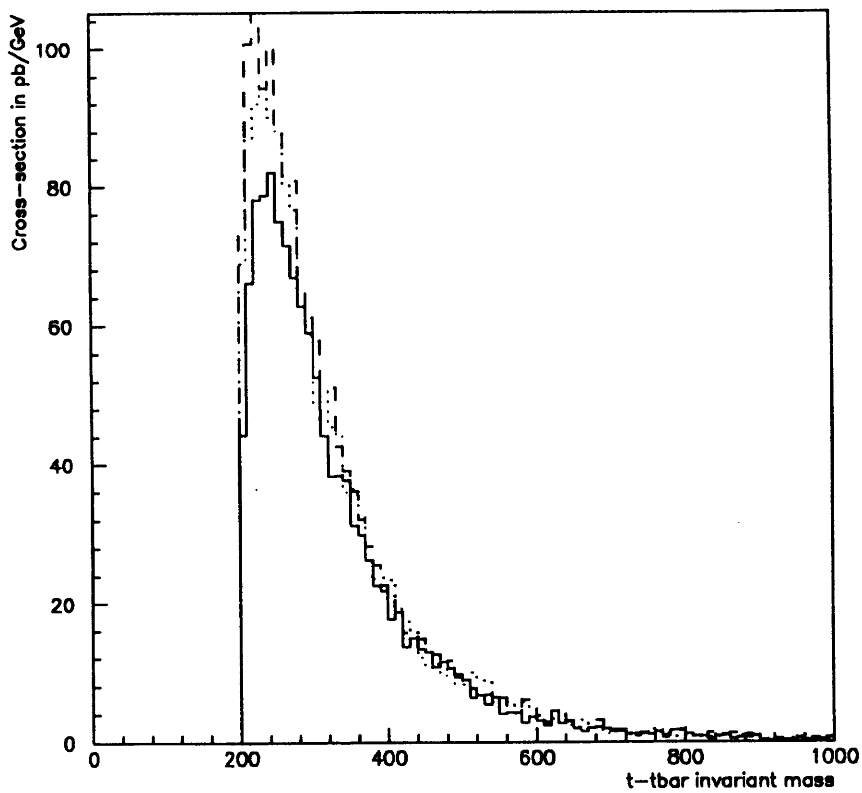

Figure 8: Invariant mass distribution of $t\bar{t}$ pairs for the LHC collider at 15 TeV, with m_t = 100 GeV. Notation as in Fig. 6.

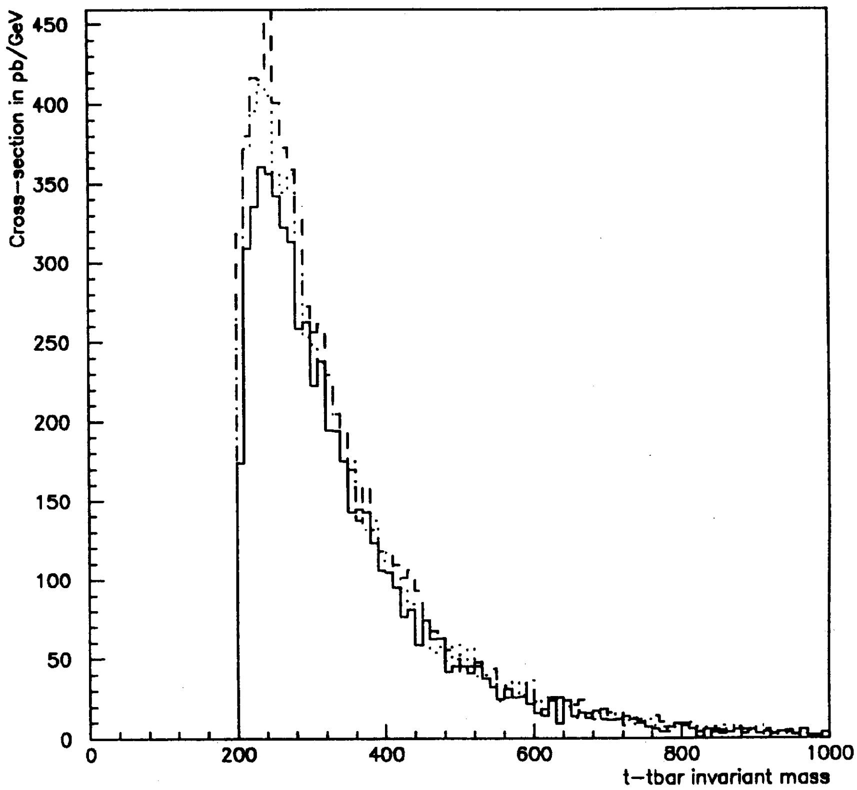

Figure 9: Invariant mass distribution of $t\bar{t}$ pairs for the SSC collider at 40 TeV, with $m_t =$ 100 GeV. Notation as in Fig. 6.

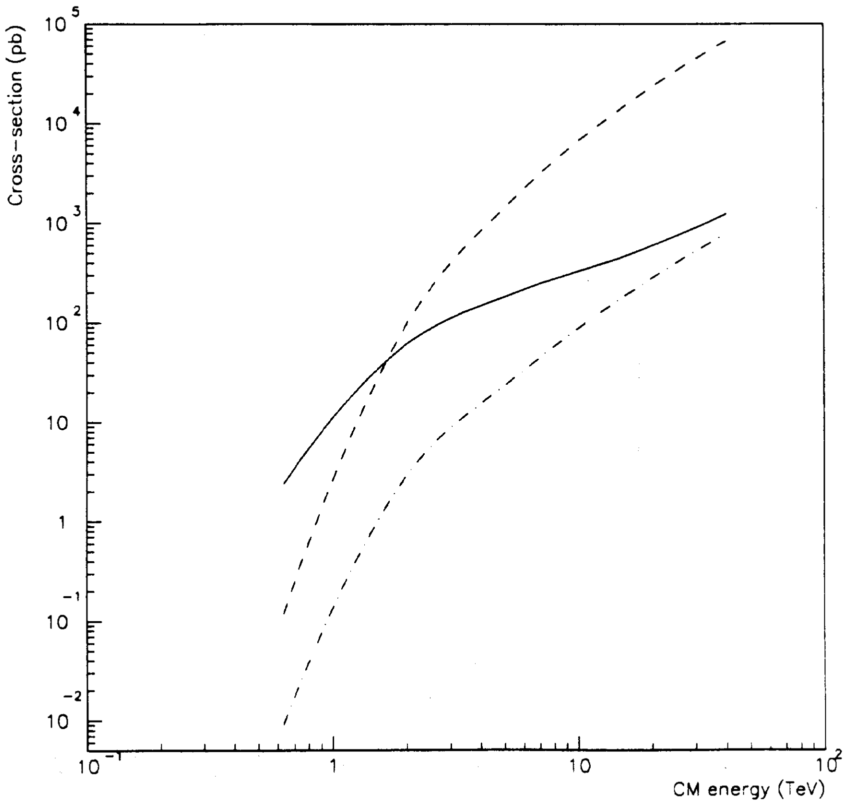

Figure 10: The energy dependence of the total cross-section for $t\bar{t}$ production, subdivided into contribution from $q\bar{q} \to t\bar{t}$ (full), $gg \to t\bar{t}$ continuum (dashed), and $gg \to t\bar{t}$ bound state (dash-dotted). Results are for $m_t = 100$ GeV and running α_S.

quark mass, once a signal is seen. We also mention that, for $m_t \leq 100$ GeV, one may hope to extract information on $\alpha_S(k_n)$, by a comparison of cross-sections in the lowest resonance states (here $k_n = (2/3)\alpha_S(k_n)m_t$ is the momentum of the relative t and $\bar{t}$ motion in the n^{th} Coulombic state).

For hadron colliders, effects are not as spectacular. In particular, the total top cross-section is only changed by at most 10% over a wide range of CM energies. As irony has it, the effect is actually one of decreasing the cross-section at lower energies; i.e. it does not make top searches any easier at current machines. Once top is found, again the correction factors studied here are of relevance for mass determinations. In particular, at all CM energies, there is a net enhancement very close to the threshold, due to the removal of the β_t phase space factor for 2/7 of the $gg \to t\bar{t}$ cross-section.

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