On the Interplay between Top Decay and Top Fragmentation*

Torbjörn Sjöstrand Theory Division, CERN CH-1211 Geneva 23, Switzerland

and

Peter Zerwas
Theorie Gruppe, DESY
Notkestrasse 85, D-2000 Hamburg 52, Germany

Abstract

For a range of top quark masses, the top decay time is comparable to its fragmentation time. Under these circumstances, the particle production between the original t and \bar{t} may have time to start, only to be cut short by the top decays. Additional production involves the b and \bar{b} quarks created in the decays. We show that the topology of the produced particles is changed as a function of the top lifetime. As a consequence, it could be possible to use experimental momentum measurements to probe the space-time structure of the top fragmentation and decay processes.

^{*}To appear in the proceedings of the EE500 Workshop on the physics of a 500 GeV e^+e^- linear collider, DESY, Hamburg, September 2 – 3, 1991

1 Introduction

Above the W threshold region, the top quark decay width increases cubically with the top mass

$$\Gamma_t \approx \frac{G_F \, m_t^3}{8 \sqrt{2} \, \pi} \sqrt{(1 - r_W - r_b)^2 - 4 r_W r_b} \, \left\{ (1 - r_b)^2 + (1 + r_b) r_W - 2 r_W^2 \right\},\tag{1}$$

where $r_W = m_W^2/m_t^2$ and $r_b = m_b^2/m_t^2$. This corresponds to $\Gamma_t \approx 0.1$ GeV for $m_t = 100$ GeV.

If we assume a non-perturbative linear confinement force like between light quarks, i.e. characterized by a string tension $\kappa \approx 1 \text{ GeV/fm}$, the average amount of energy lost by the top quark before it decays is given by

$$\Delta E = \kappa \langle t_t \rangle = \kappa \gamma \langle \tau_t \rangle = \frac{\kappa \gamma \hbar}{\Gamma_t}, \qquad (2)$$

where $\gamma = E/m$ is the ordinary Lorentz factor. At a CM energy of 500 GeV and a top mass of 100 GeV, this gives about 5 GeV of energy lost before the top decay takes place. This is an average, however, with large fluctuations.

Had the top not decayed, one could have written down an effective top fragmentation function, e.g. like that of Peterson $et\,al.$ [1], which describes how large a fraction z of the original top quark energy is retained by the top hadron. Based on naive extrapolations from measured charm and bottom fragmentation functions, one ends up with a similar number of a few GeV of energy that goes into hadrons other than the leading top one. In this sense, the top decay and top fragmentation processes are being played out on comparable timescales, and necessarily become intertwined.

One would therefore imagine a scenario as follows. A t and \bar{t} are produced at a single space-time point and move apart in opposite directions. As they move apart a 'string' is stretched between them, in which the energy lost to the non-perturbative confinement force is stored. After a while this string will begin to break by the production of $q\bar{q}$ pairs. The q from one such pair may combine with the \bar{q} from an adjacent one to form a colour singlet hadron. This fragmentation process begins in the middle of the string and spreads outwards, see e.g. [2].

However, before the process has run to its natural end, by the formation of a top meson at the t end of the string, the top quark decays. Let us, for simplicity, assume that the decay is a semileptonic one, $t \to b + \ell + \nu$, such that there is only one coloured decay product to keep track of. The b quark is kicked out at a direction different from the original t one, and starts to pull out a string of its own. This second string is attached to the remaining piece of the original $t-\bar{t}$ string that did not yet fragment, i.e. effectively to a \bar{q} moving in the t original direction. Fragmentation will now proceed in this new string piece, to produce a number of hadrons, including a B one. Similarly, the decay of the \bar{t} quark gives rise to a third string. We will therefore refer to this as a '3-string' topology.

If the top quark is long-lived, a top meson $t\overline{q}$ has time to form. When the t quark inside this meson decays, a string is again stretched between the b decay product and the \overline{q} spectator. Events of this type therefore qualitatively look much the same as the ones decribed above.

In the other extreme, where the t and \bar{t} decay instantaneously, the b and b decay products emerge from a common origin, and are directly connected by a string. This string has no

memory of the original $t-\bar{t}$ direction, but is entirely defined by the b and \bar{b} momenta. This is thus a '1-string' topology. In the not-quite-instantaneous case, where the t and \bar{t} have some time to travel apart before they decay, but not long enough for the fragmentation to start, the string connecting the b and \bar{b} is going to consist of three different pieces, but these will be mixed up by the string motion [3], and the expected end result is close to that of the simple 1-string picture above.

One could also consider '2-string' configurations where, e.g., the t decays early on and the \bar{t} rather later, such that there is one initial string between the b and the \bar{t} , with a second formed when the \bar{t} decays. Such a situation is intermediate between the 1-string and 3-string scenarios, also in terms of expected event topologies. In the following we will therefore present comparisons between the two 'extremes', and leave the intermediate configuration aside.

The objective of this paper is to show that the 1-string and 3-string topologies give rise to different momentum distributions of produced particles. As a consequence, it could become possible to use experimental momentum measurements to probe the space-time structure of the top fragmentation and decay processes, maybe even to extract some crude measurement of the top lifetime.

2 The Model

To achieve a complete description of the full fragmentation and decay history is very difficult. In this paper we will therefore have to introduce a few simplifying assumptions. At the current stage, the whole exercise has to be taken more as a 'gedanken' study than a full-fledged description. Further work will be needed to go from a qualitative to a quantitative understanding.

One set of approximations deals with the experimental conditions assumed. All studies are for a 500 GeV e^+e^- collider, with effects of beamstrahlung and bremsstrahlung neglected, so that the CM frame of the event is well defined. Both t and \bar{t} are taken to decay semileptonically. These events are likely to be very characteristic, with little background from other processes; however, no background studies have been performed. It is further assumed that the original $t-\bar{t}$ event axis can be (reasonably well) reconstructed from the event topology, despite the presence of two neutrinos. Finally, the study is restricted to events where the b and \bar{b} decay products both come out at essentially the same azimuthal angle w.r.t. the $t-\bar{t}$ event axis, in the following defined as $\phi=0$. This latter condition could easily be relaxed, and in real life a comparison of events with varying $\Delta \phi = \phi_b - \phi_{\bar{b}}$ would be of interest on its own.

Several approximations are used in the theoretical description. The main one is the neglect of all perturbative gluon emission, from the original t and \bar{t} quarks, from the subsequent b and \bar{b} ones, and from interference terms — when the top is short-lived, the distinction of emission before and after the decay is not well-defined. Further approximations will be mentioned in their context. In section 4 we will attempt to estimate gluon emission effects; however, within a framework even more approximate than that in the current section.

Events are generated as follows. First the $t-\bar{t}$ system is allowed to fragment as if the top quarks were stable, i.e. a top hadron is produced at one end of the fragmenting string, an antitop at the other, and a variable number of ordinary hadrons in between. Some

of these could be baryons although, for simplicity, we will only consider mesons in the following. Fragmentation is performed by the standard JETSET 7.3 ('Lund') program [4], with $\epsilon_t = 10^{-4}$ in the Peterson *et al.* fragmentation function. A smaller ϵ_t value would be expected based purely on non-perturbative considerations, so some effects of gluon emissions are implicitly taken into account with this choice.

In the standard string fragmentation language there is a one-to-one correspondence between the momentum-energy and the space-time pictures of fragmentation [2], with constant of proportionality κ . This is clearly a classical picture, and can only be used sensibly for the energy and longitudinal momentum component. Transverse momenta are included in the fragmentation process, but the extent in transverse space coordinates is assumed vanishing. Each meson can therefore be associated with a unique space-time point, where the constituents q and \overline{q} meet for the first time. We will call this the production vertex of the meson.

Next the invariant t and \bar{t} decay times τ are selected, and translated into actual spacetime points where the decays take place. To first approximation $t = \gamma \tau$ with $\gamma = E_t/m_t$, and $x = \beta \gamma \tau$ with $\beta = p_t/E_t$, where E_t and p_t are the original top energy and momentum. In the program, account is also taken of the gradual reduction in top energy and momentum due to the confinement force, but this is a minor correction.

If the production vertex of a meson is not in the forward light-cone of either the t or \overline{t} decay points, it is assumed that the decays have no possibility to affect the meson formation. These 'early' mesons are retained. The number of 'early' particles is constrained by the joint effects of a hard top fragmentation function and a rapid top decay.

The remaining 'late' mesons are formed at a time where information on the t/\bar{t} decays would already have reached them, and therefore they would not have been formed as assumed till now. The late particles are therefore removed. This, in particular, includes the top and antitop mesons. Obviously, the clearcut borderline between 'early' and 'late' particles, as defined here, must be smeared by quantum mechanical uncertainties. However, we expect our choice to be a reasonable one in an average sense.

Now consider the removed particles on the top side. These form an unbroken chain in flavour in the string space-time picture. The net flavour content is therefore given by the t at one end and a \overline{q} at the other. The energy and momentum of this system is known. At the time it decays, the t quark is carrying a well-defined amount of energy and momentum, which is shared between its decay products. The remaining energy is partly sitting in the \overline{q} end and partly in the $t-\overline{q}$ string. For simplicity this string is assumed to be of negligible length, such that all energy can be assigned to the \overline{q} .

After the top decay, a string is therefore drawn out between the b and the \overline{q} , and this string fragments to produce a set of hadrons. Typically the \overline{q} is travelling in the t direction, while the b has been kicked out in some other direction.

Similarly, the 'late' particles on the \bar{t} side of the event are lumped together into one system, which after the \bar{t} decay gives rise to a fragmenting \bar{b} -q string. The final event therefore has hadrons produced either as 'early' particles in the original t- \bar{t} string, or else in the two strings formed after the t and \bar{t} decays.

Many of the primary particles produced in the string fragmentation are unstable and subsequently decay into the final observable ones. The B and \overline{B} mesons here play a prominent rôle, with many decay products.

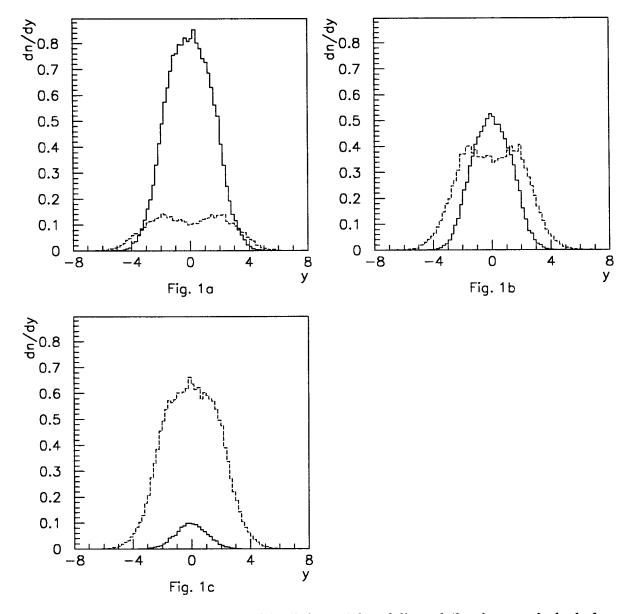


Figure 1: Rapidity distributions of 'early' particles, full, and 'late' ones, dashed, for three different top masses, 90 GeV in a, 100 GeV in b and 120 GeV in c.

By comparison, the 1-string configurations are simple to simulate. Since the t and \bar{t} here are assumed short-lived, one may make the approximation that the b and \bar{b} decay products are formed at a common origin, and thereafter pull out a string which fragments in the ordinary manner.

3 Results

In Fig. 1 is shown the rapidity distribution of particles produced in the original $t-\overline{t}$ string, subdivided into 'early' and 'late' ones. Not shown are the T and \overline{T} mesons, which are always 'late', and which have a very spiked rapidity distribution. As the top mass is increased, and

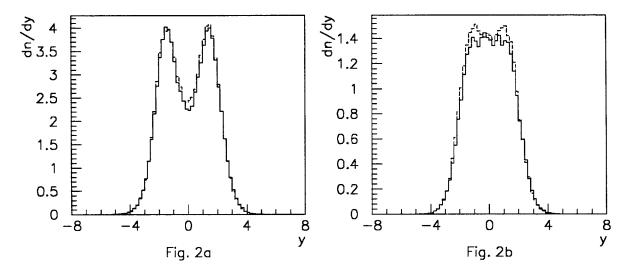


Figure 2: Rapidity distribution of final charged particles, in **a** for all particles and in **b** with particles from the B and \overline{B} decays removed. Full lines are for the 3-string topology and dashed for the 1-string one. Top mass is 100 GeV.

the top lifetime therefore decreased, the number of 'early' particles is decreased. As can be seen, the switchover is quite rapid, and takes place at around $m_t \approx 100$ GeV.

The fraction of 3-string configurations, i.e. where at least one 'early' meson is formed before either t or \bar{t} decays, is 94% for $m_t = 90$ GeV, 71% for 100 GeV and 18% for 120 GeV. All results refer to a CM energy of 500 GeV; because of the decay time dilatation factor (and also because of the fragmentation function behaviour) the fraction of 3-string configurations would be lower at lower energies and higher at higher ones.

In the following comparisons we will concentrate on the 100 GeV mass case, but actually the choice of top mass is not that crucial, so long as results are presented separately for 3-string and 1-string events, normalized per event of the given types.

The distributions in Fig. 2 show little difference between the 3-string and the 1-string topologies. To observe a difference, it is necessary to consider more specific distributions. One such is the particle flow in the event plane. This plane is defined by the original t and \bar{t} quarks and the produced b and \bar{b} ones; as explained above we specialize to $\Delta \phi = 0$. Therefore $\theta_t = 0$, $\theta_{\bar{t}} = \pi$, and $0 \le \theta_b, \theta_{\bar{b}} \le \pi$. The resulting final particle distributions are

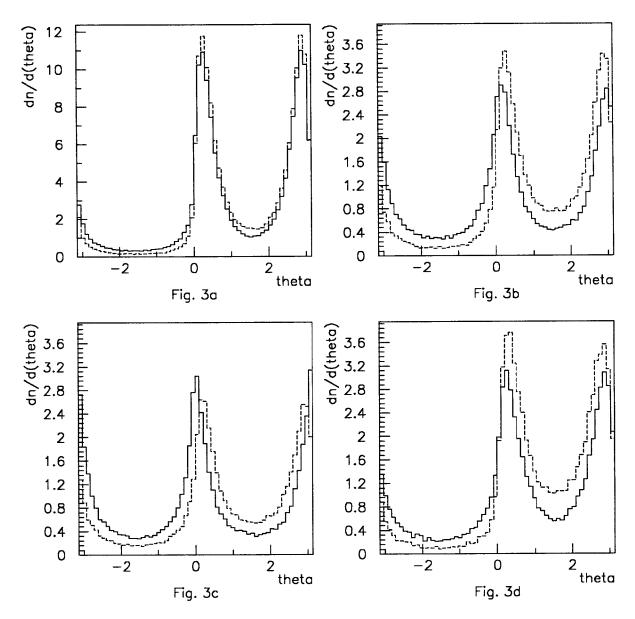


Figure 3: Particle distribution $dn/d\theta$ in the event plane. In **a** all final charged particles are considered, with 100 GeV top mass. In **b**, **c**, and **d**, particles from the B and \overline{B} decays have been removed, and the top mass is 100 GeV, 90 GeV and 120 GeV, respectively. Full lines are for the 3-string topology and dashed for the 1-string one.

shown in Fig. 3a and, with B and \overline{B} decay products removed, in Fig. 3b.

A clear difference now emerges, in that the 3-string topology gives a comparable number of particles in the regions around $\theta = +\pi/2$ and $\theta = -\pi/2$, while 1-string events have a larger production rate in the former region and a smaller one in the latter region. This may be readily understood as follows.

In the 3-string scenario, the original $t-\bar{t}$ string piece has no preferred azimuthal direction, but produces particles isotropically, which translates into comparable particle numbers at $\theta \approx \pm \pi/2$. The b and \bar{b} jets, which both are found in the $\theta > 0$ hemisphere, give clear peaks

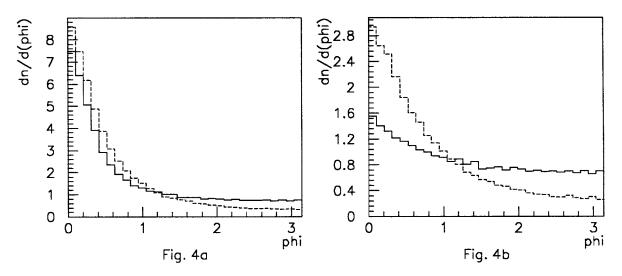


Figure 4: Distribution $dn/d\phi$ of final charged particles in the rapidity range |y| < 1. In a all particles are considered, while particles from B and \overline{B} decays have been removed in b. Full lines are for the 3-string topology and dashed for the 1-string one. The top mass is 100 GeV.

in the particle flow, but only rather few b jet particles leak to the $\theta \approx +\pi/2$ region.

In the 1-string topology, all particle production takes place in a string that is boosted in a direction intermediate to the b and \bar{b} ones. Particle production is isotropic in azimuth in the rest frame of the string, but the overall boost shifts particles from the $\theta \approx -\pi/2$ side to the $\theta \approx +\pi/2$ one. This is similar to the well-known 'string effect' in ordinary three-jet events [5, 6].

In the 3-string configuration there is no string directly between the b and the \overline{b} . Rather the b is connected to a \overline{q} remnant moving in the original t direction, i.e. away from the \overline{b} direction. Not only is this difference reflected in the regions between the b and \overline{b} jets, but also in the profile of these jets themselves, with the peaks of particle production shifted closer to each other in the 1-string and away from each other in the 3-string topology, all with respect to the original parton directions. Also this phenomenon has a correspondence in ordinary three-jet events [7].

The top mass dependence of these results is shown in Figs. 3c and 3d. Qualitatively, the picture remains the same. At larger top mass, however, the b jets have larger energies and, since the top quarks move slower, the b jets also appear at larger angles away from the top direction of motion. Both of these effects act to enhance particle production around $\theta = +\pi/2$. In the 1-string scenario, the boost of the string is enhanced when the b and \bar{b} come closer, and therefore the depletion in the $\theta \approx -\pi/2$ region is more marked. Also for the 3-string topologies this latter region is less populated at higher top masses, simply because the average number of 'early' particles is less. (By definition, each 3-string event is required to contain at least one 'early' particle, but for a low top mass it often contains several.)

An alternative way to show these results is to consider $dn/d\phi$, the azimuthal distribution of particles around the $t-\bar{t}$ axis, with $\phi=0$ defined by the b and \bar{b} direction. The presence of the b jets tends to dominate the picture at large rapidities; therefore only the central region |y|<1 is considered. This distribution is shown in Fig. 4a. As before, the picture becomes

clearer if B and \overline{B} decay products are removed, Fig. 4b. In particular, consider the region $\phi > \pi/2$, where the 3-string topology gives an almost flat distribution, thus reflecting the isotropic ϕ distribution of particles produced in the original $t-\overline{t}$ string. For 1-string events, on the other hand, the boost effect leads to a continuously decreasing distribution, with marked variation also for $\phi > \pi/2$.

4 Gluon Emission

All results so far have neglected the effects of gluon radiation. In this section we will try to include such effects approximately. It has not been included in the main part of the study, since it is associated with additional uncertainties, and has only been studied for a simplified space-time picture of the fragmentation, as described below.

The formulae for gluon emission have been taken from [8] (with the insertion of a factor 1/2 in the first term of eq. (3); we assume this is just a typo), which have been obtained in the soft gluon limit. The total emission cross-section can be divided into two parts, one which only depends on the t, \bar{t} and g momenta, and one which additionally depends on the b and \bar{b} momenta. The time ordering of emission is not obvious but, to make some headway, we assume that the first part can be used to describe the very early emission, which takes place before the hadronization has started at all. The second part we will, somewhat loosely, refer to as emission off the b and \bar{b} quarks.

Thus, in the 3-string scenario, the t, \bar{t} and a number of gluons emerge from a common origin, and this system fragments. The 'late' mesons are identified and removed. Thereafter the top decays are performed, and give rise to b quarks. Based on the t, \bar{t} , b and \bar{b} momenta, the remaining gluon emission terms are used to generate a further number of gluons. These gluons are taken to form part of either the $b-\bar{q}$ or the $\bar{b}-q$ strings, depending on whether their longitudinal momentum component (along the original $t-\bar{t}$ direction) is positive or negative. Finally, the two strings above are fragmented.

Since the formulae of [8] are for the soft gluon limit, where emission is (almost) independent, they can be used to describe the emission of several gluons. We allow for the emission of up to four gluons off the original t and \bar{t} , by making four tries. In each try, a gluon energy is selected between 1 and 20 GeV according to dE_g/E_g , and distributed evenly in angles. The emission probability is evaluated, assuming a fix (first order) $\alpha_S = 0.25$, and compared with a maximum weight, for Monte Carlo acceptance or rejection of the gluon. Since accepted gluons add to the energy and momentum of the event, a boost is made to the new CM frame, and in this frame all three-momenta are rescaled by a common factor to restore the correct energy.

The gluons emitted off the tops are fairly isotropically distributed, with only a moderate enhancement in the top directions. The remaining terms, which depend on the b momenta, are strongly peaked in the b and \bar{b} directions. A bg and $\bar{b}g$ invariant mass of at least 6 GeV is imposed to cut out the collinear regions. Since gluons are again picked uniformly in angle, a total of 20 tries is made, to allow acceptance probabilities below unity. Gluon energies are allowed in the range between 1 and 10 GeV. The latter number is lower than the 20 GeV used previously, since we are here concerned with gluons that are associated mainly with the top decay process, where the allowed phase space is smaller than in the primary $t\bar{t}$ production.

Energy and momentum conservation are assured as above, but now the procedure is applied in the rest frame of the respective decaying top quark.

The average number of gluons per event is 1.3 for emission directly off the t and \bar{t} , and 1.0 for the b and \bar{b} emission part. The latter number is very sensitive to the choice of cut-off scale around the b directions. The numbers also depend on the α_S value; 0.25 is about what is required in first order to describe gluon emission at PETRA energies, and should not be confused with second-order α_S determinations for the harder gluon jets at LEP energies.

The upper gluon energy cut-off can be motivated both by the validity of the formulae used, and by reasonable experimental cut-offs: too hard gluons would appear as additional jets. The resulting multi-jet events would be studied separately, and not be allowed to contaminate the event samples of interest here. In fact, even the scenario here is likely to be more pessimistic, in terms of gluon jet contamination, than a real experimental study.

In the 1-string scenario, all gluon emission is handled before fragmentation is allowed to take place. For consistency the same gluon emission scheme as above is used, including the division of emission into two parts, which is not strictly necessary in this case.

The string model is perfectly capable of dealing with the fragmentation of strings stretched from a q end, via a number of intermediate gluons, to a \overline{q} end [3]. However, the trivial relation between the space-time picture and the energy-momentum one is not easy to extend from a simple $q\overline{q}$ string to a more complicated one. We have therefore made use of a much more crude picture, which should be reasonable in an average sense, but is less well suited for the study of fine details.

First, independently of the motion of the string, one expects all $q\overline{q}$ pair production vertices to be centred around a hyperbola of constant invariant time, $\tau = \langle \tau \rangle$. We will make the approximation to assume that all the vertices are actually on this hyperbola. By adding corrections related to the mass of the produced meson, it is then possible to find the invariant production time τ_i of particle i. The assumed production space-time point x_i is then obtained from the known four-momentum p_i by $x_i = \tau_i p_i/m_i$.

Second, while the transverse momentum related to gluon emission does correspond to a true transverse space separation, transverse momentum fluctuations in the fragmentation process cannot be equated with a transverse space separation. Unfortunately, the two kinds of transverse momenta are mixed up. Therefore, again, an average picture is used. When defining the x_i points above, the transverse momentum component is set to zero if it is smaller than $\langle p_{\perp} \rangle$, the average fragmentation p_{\perp} , and reduced by this amount if it is above $\langle p_{\perp} \rangle$.

Finally, since 'late' and 'early' particles could now appear intermixed, no particle is allowed to be called 'late' if a particle with lower rank (closer in flavour to the t or \bar{t} , respectively) has been classified as 'early'.

The effects of gluon emission may be studied in Figs. 5 and 6. Let us first note that Figs. 3b and 5a should have agreed, since both are without gluon emission. The difference reflects the modified definition of 'early' and 'late' particles. One may observe a non-negligible change inside jet cores, but the regions between the jets show the same behaviour in both figures, and it is on these regions that we will concentrate.

The net effect of gluon emission, seen in Fig. 5d, is to reduce the difference between the 3-string and 1-string scenarios. This comes about by a net increase of particle production at all angles, including in the regions between jets. Since the increase is about the same

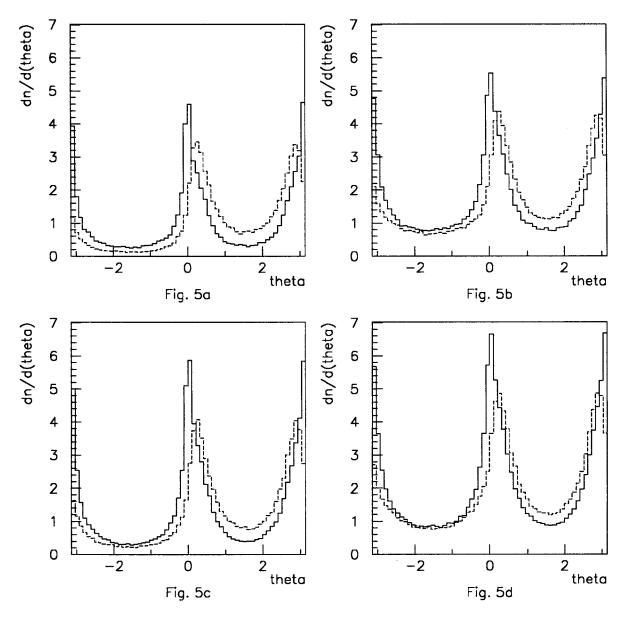


Figure 5: Particle distribution $dn/d\theta$ in the event plane. Only charged particles not from B or \overline{B} decays are considered, and the top mass is 100 GeV. In \mathbf{a} no gluon emission is included, in \mathbf{b} there is only emission from the t and \overline{t} , in \mathbf{c} only from the b and \overline{b} , and in \mathbf{d} both emission types are included. Full lines are for the 3-string topology and dashed for the 1-string one.

in both scenarios, a difference remains in absolute numbers, but becomes small in relative terms. Figs. 5b and 5c show that the bulk of the effect comes from the emission off the top quarks, as might have been expected: the emission off the b and \bar{b} quarks is concentrated close to the b and \bar{b} directions, and therefore does not significantly affect particle production between the jets.

The character of a net shift upwards is even better seen in a comparison between Figs. 6a and 6b, which shows the azimuthal particle distribution with and without gluon emission.

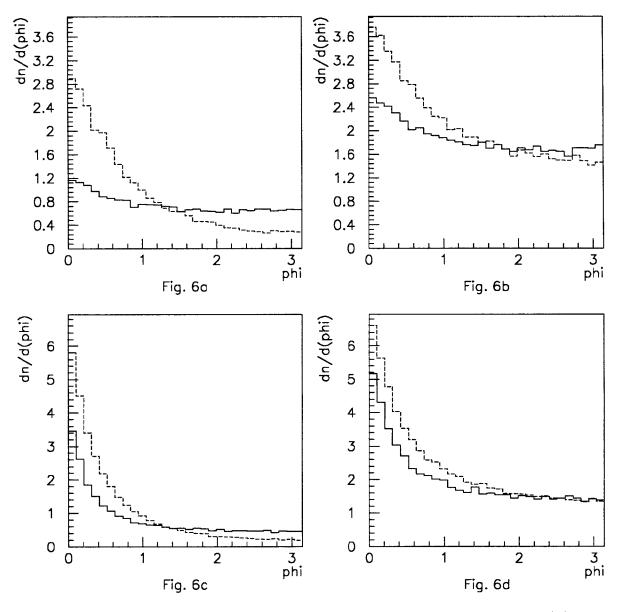


Figure 6: Distribution $dn/d\phi$ of final charged particles in the rapidity range |y| < 1, with particles from B and \overline{B} decays removed. Figures $\bf a$ and $\bf b$ are for a top mass of 100 GeV, $\bf c$ and $\bf d$ for a 120 GeV top mass. The figures $\bf a$ and $\bf c$ are without any gluon emission, $\bf b$ and $\bf d$ with full gluon emission included. Full lines are for the 3-string topology and dashed for the 1-string one.

In particular, the absolute variation with azimuthal angle could still be used to distinguish the two scenarios.

Since the number of 'early' particles goes up when gluon emission is included, the relative composition of 3-string and 1-string events is changed a bit in favour of the 3-string ones. One could therefore imagine going to higher top masses. As is seen from Figs. 6c and 6d, however, higher top masses do mean less favourable conditions if one wants to observe differences. This is related to the increased number of b jets at central rapidities, caused by

the lowered top velocity and the increased phase space available for the b quarks.

In summary, gluon emission effects will complicate comparisons, but not make them impossible. Further cuts, not included here, could be used to remove those events that are particularly distorted by gluon emission effects.

5 Summary

In this report we have made a first attempt to study the topology of particle production in the region of top masses where the top decay and top fragmentation times are comparable. A number of simplifications were introduced. Further work could therefore sharpen the predictions.

Some conclusions can already be drawn, however. A drastic change in event histories takes place between a top mass of 90 GeV and one of 120 GeV. At the lower mass, the familiar picture (e.g. from b fragmentation) still holds, where the $t\bar{t}$ system fragments into ordinary hadrons before the top decay takes place, while at 120 GeV the situation is opposite. The two possibilities turn out to give very similar results for a number of distributions, like the particle rapidity spectrum, which means that they would not be easily distinguishable. However, we have shown that more detailed studies of 'string effects' could distinguish between the 3-string topologies of late top decays and the 1-string ones of early decays.

Relative differences are significantly reduced when gluon emission effects are included. However, most of the changes are related to hard gluons emitted off the t and \bar{t} . A large fraction of the offending events could therefore be removed by straightforward event shape cuts.

If the relative admixture of the different topologies could be measured experimentally, it would offer the possibility to perform a direct measurement of the top lifetime. Given the many uncertainties that would remain also after a more detailed study, it is hard to see how the lifetime could be measured to better than a factor of two, but even this could be of some interest. The method would only work at an intermediate mass range, since the 1-string topology is dominant from 120 GeV onwards.

Whatever the top mass, this kind of studies would help us test our understanding of the fragmentation process. After all, the 1-string configuration that dominates at large top masses represents a completely new situation compared to what we observe at current energies, and would thus offer an interesting field of study.

References

- [1] C. Peterson, D. Schlatter, I. Schmitt and P. Zerwas, Phys. Rev. D27 (1983) 105
- [2] B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, Phys. Rep. 97 (1983) 31
- [3] T. Sjöstrand, Nucl. Phys. **B248** (1984) 469
- [4] T. Sjöstrand, Computer Physics Commun. 39 (1986) 347;
 - T. Sjöstrand and M. Bengtsson, Computer Physics Commun. 43 (1987) 367

- [5] B. Andersson, G. Gustafson and T. Sjöstrand, Phys. Lett. 94B (1980) 211
- [6] First observation in:
 JADE Collaboration, W. Bartel et al., Phys. Lett. 101B (1981) 129
 For a recent review, see e.g.:
 T. Hebbeker, Aachen preprint PITHA 91/17, to appear in the Proceedings of the International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, Geneva, July 1991
- [7] JADE Collaboration, W. Bartel et al., Phys. Lett. 157B (1985) 340
- [8] G. Jikia, Phys. Lett. **B257** (1991) 196