CERN-TH.6369/92

Prompt Photons in Hadronic Events at LEP*

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Abstract

This preprint contains two separate, but interconnected, contributions on photon production by branchings $q \rightarrow q\gamma$ in the parton shower evolution of ordinary $e^+e^- \rightarrow \gamma^*/Z^0 \rightarrow q\overline{q}$ events. The first contribution describes the implementation of photon emission in the JETSET program, covering issues such as the basic shower evolution, matching to first-order matrix elements, and cut-off procedures. A few comments are also given on differences compared with other programs. The second contribution presents three interesting topics on the physics of photon emission off quarks: the emission cut-off scale, the competition between γ and g branchings, and the difference in angular-ordering properties between the two branching processes.

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*To appear in the proceedings of the Workshop on Photon Radiation from Quarks, Annecy, 2–3 December, 1991

CERN-TH.6369/92
January 1992

Photon Implementation in JETSET

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Abstract

Prompt photons can be emitted off quarks as part of the parton shower evolution algorithm in JETSET. The $q \to qg$ and $q \to q\gamma$ branching implementations are closely analogous, but there are a few differences between them. The basic scheme and the differences are discussed here.

1 Introduction

Photon emission as part of the parton shower was first implemented in JETSET version 7.2 of November 1989 [1], as a simple extension of the already existing QCD parton shower machinery [2].

Once a quark–antiquark pair has been formed from the hard process $e^+e^- \to \gamma^*/Z^0 \to q\overline{q}$, the quark and antiquark may radiate both gluons and photons. The two basic branchings $q \to qg$ and $q \to q\gamma$ appear as competing processes on an equal footing, and with a closely similar structure: the probability for a quark to branch at some given virtuality scale Q^2, with the daughter quark retaining a fraction z of the mother energy, is given by

$$d\mathcal{P} = \left\{ \frac{\alpha_S}{2\pi} C_F + \frac{\alpha_{em}}{2\pi} e_q^2 \right\} \frac{dQ^2}{Q^2} \frac{1+z^2}{1-z} dz. \qquad (1)$$

Here the first term corresponds to gluon emission and the second to photon one. Starting from the gluon emission description, it is thus only necessary to replace the strong coupling constant α_S by the electromagnetic one α_{em}, and the QCD colour Casimir factor $C_F = 4/3$ by the squared quark charge e_q^2.

If the process is studied in a little more detail, a number of differences appear, however, which break the perfect symmetry between photon and gluon emission:

- A gluon may branch further, via either of the two processes $g \to gg$ and $g \to q\overline{q}$. This means that a gluon emitted off the quark leg often has an effective non-zero mass. Also a photon may branch, into a quark or lepton pair, but this is not implemented in the program and therefore a photon is always taken to be massless. Even if photon branchings were to be included, they would be fairly infrequent because of the smallness of α_{em}, and therefore not affect the basic asymmetry between gluon and photon masses.
- Photons do not obey the requirement of angular ordering, whilst gluons do.
- The strong coupling constant is taken to run as a function of the Q^2 scale, while α_{em} is assumed fixed at its Thomson limit value, $\alpha_{em} \approx 1/137$.

- The lower cut-off Q_0 for shower evolution can be chosen separately for gluon and photon emission; for the default scale choice $\alpha_S(p_\perp^2)$, additionally, the shape of the gluon emission phase-space region is different from the photon one.

These points will be described in detail in the following sections. To set the stage, also the main points of the parton shower machinery as a whole will be reviewed.

2 Evolution Variable and Sudakov Factor

In the JETSET shower algorithm, the evolution variable Q^2 is associated with the mass squared of the branching parton, $Q^2 = m^2$. For convenience, we also make use of its logarithm $t = \ln(Q^2/\Lambda^2) = \ln(m^2/\Lambda^2)$. For a given t value we define the integral of the branching probability over all allowed z values,

$$\mathcal{I}_{a\to bc}(t) = \int_{z_{min}(t)}^{z_{max}(t)} dz\, \frac{\alpha_S}{2\pi}\, P_{a\to bc}(z)\,, \tag{2}$$

where $P_{a\to bc}(z)$ are the splitting kernels for $q \to qg$, $g \to gg$, and $g \to q\overline{q}$, respectively. The corresponding integral for photon emission is

$$\mathcal{I}_{q\to q\gamma}(t) = \int_{z_{min}(t)}^{z_{max}(t)} dz\, \frac{\alpha_{em}}{2\pi}\, e_q^2\, \frac{1+z^2}{1-z}\,. \tag{3}$$

Here z descibes the energy sharing between the two daughters, with the first taking a fraction z of the mother energy and the second a fraction $1 - z$.

Starting from a maximum virtuality t_{max}, the probability for a branching to take place at a given scale t is now given by

$$\frac{d\mathcal{P}}{dt} = \sum_{b,c} \mathcal{I}_{a\to bc}(t) \times \exp\left\{ - \int_t^{t_{max}} dt' \sum_{b,c} \mathcal{I}_{a\to bc}(t') \right\}\,. \tag{4}$$

The first factor is the naive branching probability, the second the suppression due to the conservation of probability: if a parton has already branched at a $t' > t$, it can no longer branch at t. This is nothing but the exponential factor which is familiar from radioactive decay; in parton-shower language it is referred to as the Sudakov form factor. (Stricly speaking, the Sudakov is defined from the lower cut-off scale t_0 to t; the above exponential is thus the ratio of two Sudakovs, one to t_{max} and one to t.)

Once a quark has branched, the daughter quark can branch in its turn, with maximum virtuality related to the t selected in the preceding branching. In the limit of soft photon emission off a lepton, the total probability to produce a photon of given energy, either at the first branching, the second one, the third one, etc., adds to give back the naive probability, without any Sudakov. In this sense, the inclusive photon emission rate in the shower is defined already by the $e^+e^- \to \ell\overline{\ell}\gamma$ matrix element. However, for a quark there is competition with gluon emission. The largeness of α_S means that gluons remove a significant fraction of the original quark energy. For each step of gluon emission in the shower, the allowed phase space for photon emission is reduced. Therefore the net amount of photon emission in a given region of phase space is reduced compared with naive predictions. The suppression is especially significant for hard photons. The ordering of possible branchings then comes to play a rôle: a photon emission which is classified as 'early', in the sense of a t close to t_{max}, is not suppressed much in production rate by 'earlier' branchings, whilst a 'late' one is.

As we noted above, the JETSET shower is based on $Q^2 = m^2$, which should be compared with the $Q^2 = p_\perp^2 \approx z(1-z)m^2$ of Ariadne [3] and $Q^2 \approx E^2\theta^2/2 \approx m^2/(2z(1-z))$ of HERWIG [4]. All other things being equal (which, in real life, they are not) one would thus expect a larger rate of photons at large $p_\perp$ from Ariadne and a smaller from HERWIG, based on the Sudakov form factor plus momentum conservation. In HERWIG, on the other hand, preference is given to photons emitted at large angles, also with small energies, since these emissions are considered first in that program.

3 First Branchings and Matrix Element Matching

The parton-shower language does not guarantee agreement with matrix element results for hard gluon or photon emission. In JETSET a special matching procedure is used to ensure agreement at least with first-order matrix elements [2]. The matching is based on a mapping of the parton shower variables on to the three-jet phase space. To produce a three-jet event $e^+e^- \to q(1)\overline{q}(2)g(3)$, in the shower language one will pass through an intermediate state, where either the q or the $\overline{q}$ is off the mass shell. If the former is the case then

$$\begin{aligned} m^2 &= (p_1+p_3)^2 = E_{cm}^2(1-x_2), \\ z &= \frac{E_1}{E_1+E_3} = \frac{x_1}{x_1+x_3} = \frac{x_1}{2-x_2}, \end{aligned} \tag{5}$$

where $x_i = 2E_i/E_{cm}$. The $\overline{q}$ emission case is obtained with $1 \leftrightarrow 2$. The parton shower splitting expression in terms of m^2 and z, eq. (1) can therefore be translated into the following differential three-jet rate:

$$\begin{aligned} \frac{1}{\sigma}\frac{d\sigma_{PS}}{dx_1\,dx_2} &= \frac{\alpha_S}{2\pi}C_F\frac{1}{(1-x_1)(1-x_2)} \times \\ &\times \left\{\frac{1-x_1}{x_3}\left(1+\left(\frac{x_1}{2-x_2}\right)^2\right)+\frac{1-x_2}{x_3}\left(1+\left(\frac{x_2}{2-x_1}\right)^2\right)\right\}, \end{aligned} \tag{6}$$

where the first term inside the curly bracket comes from emission off the quark and the second term from emission off the antiquark. The corresponding expression in matrix-element language is

$$\frac{1}{\sigma}\frac{d\sigma_{ME}}{dx_1\,dx_2} = \frac{\alpha_S}{2\pi}C_F\frac{1}{(1-x_1)(1-x_2)}\left\{x_1^2+x_2^2\right\}. \tag{7}$$

With the kinematics choice of JETSET, the matrix-element expression is always smaller than the parton shower one. It is therefore possible to run the shower as usual, but to impose an extra weight factor $d\sigma_{ME}/d\sigma_{PS}$, which is just the ratio of the expressions in curly brackets. If a branching is rejected, the evolution is continued from the rejected Q^2 value onwards. The weighting procedure is applied to the first branching of both the q and the $\overline{q}$, in each case with the (nominal) assumption that the other does not branch, so that the relations of eq. (5) are applicable.

Compared with the standard matrix-element treatment, a few differences remain. The shower one automatically contains the Sudakov form factor and an α_S running as a function of the $p_\perp^2$ scale of the branching. The shower also allows all partons to evolve further, which means that the naive kinematics assumed for a comparison with matrix elements is modified by subsequent branchings.

Additionally, since both the initial q and $\overline{q}$ may be off the mass shell, a check is necessary to ensure that the sum of q and $\overline{q}$ masses is smaller than the total c.m. energy. This is slightly different from Ariadne, where only one 'first branching' exists. In some kinematical configurations, with very hard gluons or photons, one would therefore expect a slightly higher production rate in Ariadne, although both programs nominally match the same first-order matrix elements. For QCD emission this can be compensated by the choice of Λ scale, which indeed is higher in JETSET than in Ariadne.

When photon emission is included in the shower, exactly the same matrix-element and parton-shower expressions appear, except that the factor $\alpha_S C_F$ is replaced by $\alpha_{em}\, e_q^2$. The same machinery can therefore be used to match the photon-emission probability to the first-order matrix elements, i.e. the weight factor is again the ratio of the curly brackets in eq. (7) and in eq. (6).

In fact, the JETSET machinery is constructed to handle also cases where the quark and antiquark charges are not opposite and compensating, such as in $W^\pm$ decays. The parton-shower expression in eq. (6) is then modified by the appearance of different squared charges in front of the two terms inside the curly brackets, while radiation zeros appear in the matrix-element expression. However, the relation $d\sigma_{ME}/d\sigma_{PS} < 1$ still holds, so that the matrix-element weighting procedure works as before.

4 Subsequent Branchings and Angular Ordering

As formulated in section 2, the parton shower does not contain angular ordering, i.e. it is not guaranteed that the opening angle of a parton branching is constrained from above by that of the preceding branching. In a pure QCD shower in JETSET, angular ordering is imposed by the veto algorithm, i.e. if a branching is constructed that does not obey angular ordering, it is rejected and the evolution in Q^2 is continued from the rejected Q^2 value onwards.

The first branchings of the q and $\overline{q}$ are not affected by the angular-ordering requirement — since the opening angle between the q and $\overline{q}$ is 180°, any angle would anyway be smaller than this — but here instead the matrix-element matching procedure is used. Subsequently, each opening angle is compared with that of the preceding branching in the shower.

For a branching $a \to bc$ the kinematical approximation

$$\theta \approx \frac{p_{\perp b}}{E_b} + \frac{p_{\perp c}}{E_c} \approx \sqrt{z(1-z)}m_a \left(\frac{1}{zE_a} + \frac{1}{(1-z)E_a} \right) = \frac{1}{\sqrt{z(1-z)}} \frac{m_a}{E_a} \tag{8}$$

is used to derive the opening angle.

Since photons do not obey angular ordering [5], the check on angular ordering is not performed when a photon is emitted. This enhances the fraction of $q \to q\gamma$ branchings compared with the naive implications of eq. (1), and especially the rate of fairly low-energy photons at wide angle to the event axis. When a gluon is emitted in the branching after a photon, its emission angle is restricted by that of the preceding QCD branching in the shower, i.e. the photon emission angle does not enter.

5 Coupling Constants and Cut-offs

The electromagnetic coupling constant for the emission of photons on the mass shell is $\alpha_{em} \approx 1/137$. For the strong coupling constant several alternatives are available, the default being the first-order expression $\alpha_S(p_\perp^2)$, where $p_\perp$ is defined by the approximate expression $p_\perp^2 \approx z(1-z)m^2$.

The emission probabilities contain soft and collinear divergences (the latter in principle regulated by quark masses for emission off quarks). Therefore a cut-off scale Q_0 is introduced in the QCD shower, which is used to derive effective masses

$$\begin{aligned} m_{eff,g} &= \frac{1}{2}Q_0, \\ m_{eff,q} &= \sqrt{m_q^2 + \frac{1}{4}Q_0^2}. \end{aligned} \tag{9}$$

A gluon can therefore not branch unless its mass is at least twice $m_{eff,g}$, i.e. Q_0, while a quark cannot branch unless the mass is above $m_{eff,q} + m_{eff,g}$. This provides a lower cut-off for the shower evolution in Q^2, and also constrains the allowed z range of branchings, cf. eq. (2), since each parton must have an energy at least equivalent to its rest mass.

However, once it has been decided that a parton cannot branch any further, that parton is put on the mass shell, i.e. 'final-state' gluons are massless. This affects the kinematics of the final branchings of the shower, such that energies below the cut-off scale are possible but damped. In this respect JETSET is different from HERWIG, where non-vanishing gluon masses are retained.

If also photon emission is included, a separate Q_0 scale is introduced for the QED part of the shower, exactly reproducing the QCD one above. By default the two Q_0 scales are chosen equal, and have the value 1 GeV. If anything, one would be inclined to allow a lower cut-off for photon emission than for gluon one. In that case the allowed z range of eq. (3) will be larger than that of eq. (2), and at the end of the shower evolution only photon emission will be allowed.

With the default choice of $p_\perp^2$ as scale in α_S, a further cut-off is introduced on the allowed phase space of gluon emission, not present in the options with fixed α_S or with $\alpha_S(m^2)$, nor in the QED shower. A minimum requirement, to ensure a well-defined α_S, is that $p_\perp/\Lambda > 1.1$, but additionally JETSET requires that $p_\perp > Q_0/2$. This latter requirement is not a necessity, but it makes sense when $p_\perp$ is taken to be the preferred scale of the branching process, rather than e.g. m. It reduces the allowed z range, compared with the purely kinematical constraints. Since the $p_\perp$ cut is not present for photon emission, the relative ratio of photon to gluon emission off a quark is enhanced at small virtualities compared with naive expectations; in actual fact this enhancement is largely compensated by the running of α_S, which acts in the opposite direction. The main consequence, however, is that the gluon energy spectrum is peaked at around Q_0 and rapidly vanishes for energies below that, whilst the photon spectum extends all the way to zero energy.

6 Summary

The photon emission algorithm in JETSET has been implemented as a simple extension of the standard parton shower machinery. Except for some points about what to do close to the lower cut-off scales (details which therefore only affect the soft part of the photon energy spectrum), the extensions are all very simple and minimal. The discrepancies observed between the JETSET predictions and the LEP data presented at this meeting therefore have implications not only for the photon-emission description, but also for the showering approach as a whole. One possibility is that, for the first time, we here can study experimentally the correct choice of Sudakov suppression, i.e. of 'time' ordering of emissions.

In this paper we have not discussed the additional photons appearing in multihadronic events, from initial-state radiation and, more importantly, in the decays of unstable particles produced in the fragmentation of the partonic state. These aspects are described in ref. [1], and have not been modified in recent years. Again, discrepancies are found between predictions and data. It is too early to say much about these, but conceivably we are here encountering a 'higher-twist' type of effect in the fragmentation process, where the fragmentation $p_\perp$ of a hadron is occasionally larger than predicted by the standard Gaussian parametrization used in the program.

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Prompt Photon Production in Showers

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Abstract

Prompt γ's provide important insight into the structure of parton-shower evolution. A few different topics are discussed in this paper, such as the cut-off scale for photon emission, the competition between photon and gluon radiation, and the absence of angular ordering. Unfortunately, it may be difficult to observe experimentally some of the more interesting aspects.

1 Introduction

The overwhelming fraction of all γ's observed in multihadronic events comes from the decay of unstable particles, such as π^0 and η. These photons are important in fragmentation studies, but mainly act as background for studies of perturbative phenomena. Since the decay photons are found almost exclusively inside jets, cuts can be used to isolate at least a fraction of the photons produced by perturbative processes, i.e. initial- and final-state radiation.

In the energy range of the previous generation of e^+e^- machines, PETRA, PEP, and TRISTAN, photon radiation off the e^+e^- initial state dominates over final-state photon radiation for two reasons: the electrons have a larger charge than the quarks, and the probability to emit a high-energy photon in the initial state is enhanced by the $1/s$ behaviour of the continuum annihilation cross-section. Furthermore, interference between initial- and final-state radiation is large, and not well understood, since the effects of confinement would have to be included in a complete treatment.

At LEP, initial-state radiation is strongly suppressed, since the emission of an energetic photon would bring the c.m. energy below the Z^0 mass. Interference should then also be small, although that issue is still not well understood (see ref. [1] for further details). LEP is therefore the ideal machine to study the production of photons as part of final-state showers.

Clearly, quarks have a choice between emitting photons and emitting gluons. The relative probability for the two is given by the ratio

$$\frac{\mathcal{P}_{q\to q\gamma}}{\mathcal{P}_{q\to qg}} = \frac{\alpha_{em}\left\langle e_q^2\right\rangle}{\alpha_S C_F} \approx \frac{\frac{1}{137}0.22}{0.25\,\frac{4}{3}} \approx \frac{1}{200}, \qquad (10)$$

where the average squared charge is given for the standard LEP flavour mixture and α_S is the typical first-order value used in QCD shower descriptions. The photon emission is therefore strongly affected by perturbative QCD effects, as we shall see.

When discussing prompt photon production in hadronic events, a number of issues can be raised, such as:

1. At what scale Q_0 is the photon emission cut off?
2. What is the correct 'time' ordering of the shower evolution, and how is this reflected in the competition between photon and gluon emission?
3. Are there any visible consequences of the lack of angular ordering in QED showers, as compared with the angularly ordered QCD ones?
4. Can one determine coupling constants of quarks from the photon rate?
5. How serious a background is prompt γ's to a number of potential rare processes, such as $Z^0 \to H^0 + \gamma$?

Here a few comments will be given concerning the first three points. The other two, as well as many others, are discussed in other contributions to these proceedings.

2 Cut-off Scale

For QCD parton-shower evolution, it is customary to introduce a cut-off scale Q_0 with a value at around 1 GeV. Above this scale, perturbation theory is assumed to hold, below it a non-perturbative phenomenological description of fragmentation is used. We have some evidence that scales much larger than 1 GeV are disfavoured by data [2], at least within our current understanding of fragmentation. Much lower values are excluded for reasons of consistency, e.g. to avoid that perturbative expressions blow up. In specific fragmentation models, such as the Lund one [3], one can also show that the effect of a gluon emitted below the typical mass scale of primary hadrons is strongly damped.

These arguments do not hold for photon emission, since QED perturbation theory does not break down and photons are not affected by confinement forces. In principle, it would therefore be possible to have photons emitted down to current algebra quark masses of a few MeV. Alternatively, confinement forces could 'screen' the bare quarks, and provide an effective cut-off at one or a few GeV. A study of photon emission therefore gives us a glimpse of confinement at work.

This possibility was raised already ten years ago, in an evocative paper [4]. There the comment was made that one should study the two-dimensional distribution of photon energy, or $x = x_\gamma = 2E/E_{cm}$, versus $p_\perp = p_{\perp\gamma}$. For $p_\perp$ slices less than roughly $Q_0/2$, one would almost only find photons from decays, which have a very steeply dropping x spectrum, while the x spectrum at larger $p_\perp$ values would have a much larger tail out to large x values, as characteristic for a $1/x$ type bremsstrahlung spectrum.

This kind of slicing is shown in Fig. 1, comparing the spectrum of photons from fragmentation with that from final-state radiation. Two extreme choices of cut-off scale are used, one at 5 GeV, as proposed in ref. [4], and another at 0.01 GeV, about as low as one could possibly imagine. Results have been obtained with JETSET [5, 6], but qualitatively one would expect the same in any program.

We see that, for small $p_\perp$ values, the difference between the photon rate in the two shower alternatives is very large, but even with $Q_0 = 0.01$ GeV the prompt photon rate is several orders of magnitude below the fragmentation one, so that differences are unobservable. The

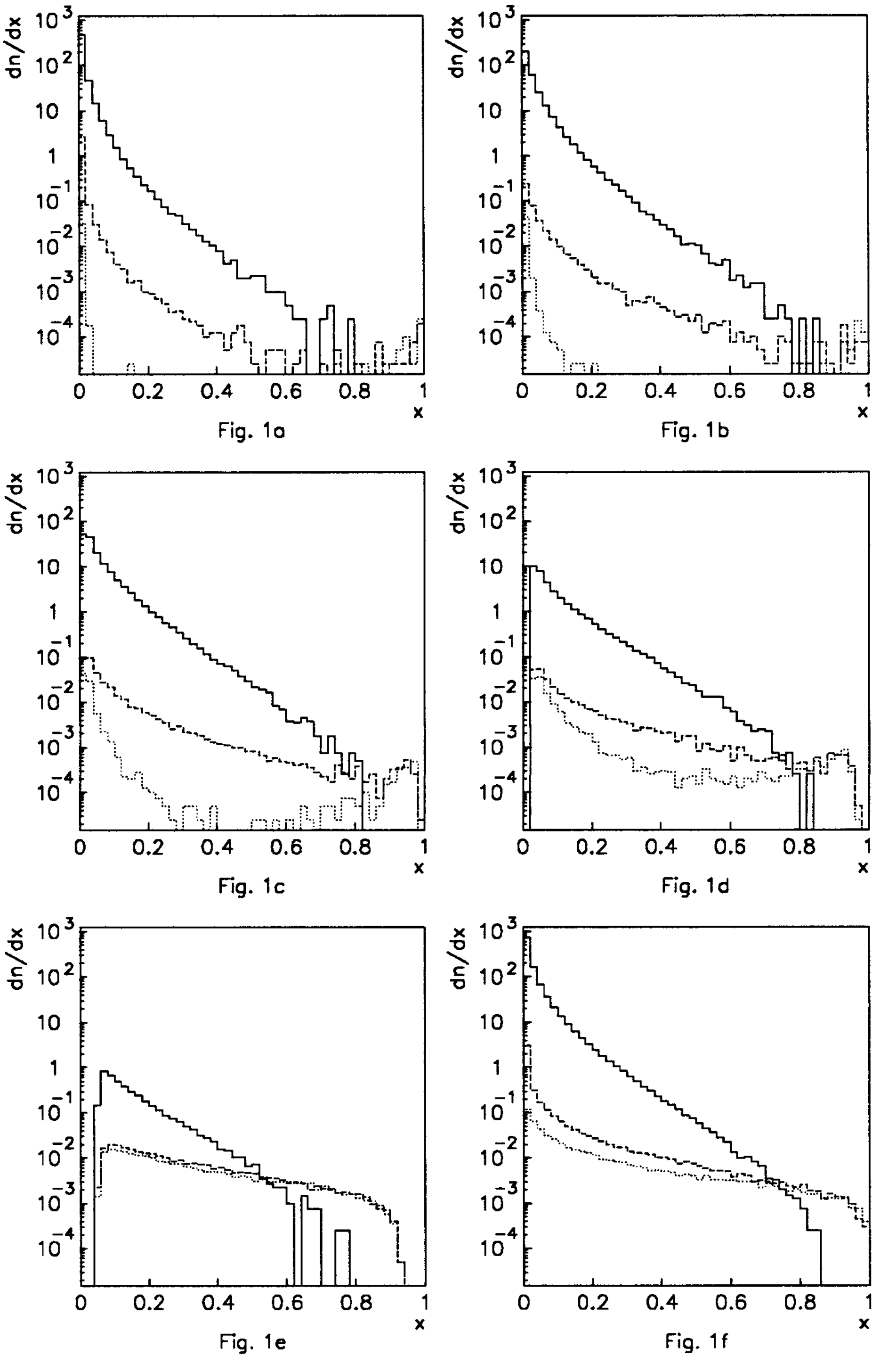

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 1d

Fig. 1e

Fig. 1f

Figure 1: Inclusive photon spectra dn/dx for different slices in $p_\perp$, (a) $p_\perp < 0.16$ GeV, (b) $0.16 < p_\perp < 0.4$, (c) $0.4 < p_\perp < 1.0$, (d) $1.0 < p_\perp < 2.5$, (e) $p_\perp > 2.5$, and (f) all $p_\perp$. The $p_\perp$ of a photon is defined with respect to the linearized sphericity axis (`LUSPHE` with `PARU(41) = 1.`), making use of all particles in the events, including the photon itself. The full histograms show the photons from fragmentation, the dashed (dotted) photons from a shower with cut-off $Q_0 = 0.01$ GeV (5 GeV). Results for LEP energies, $\sqrt{s} = 91.2$ GeV.

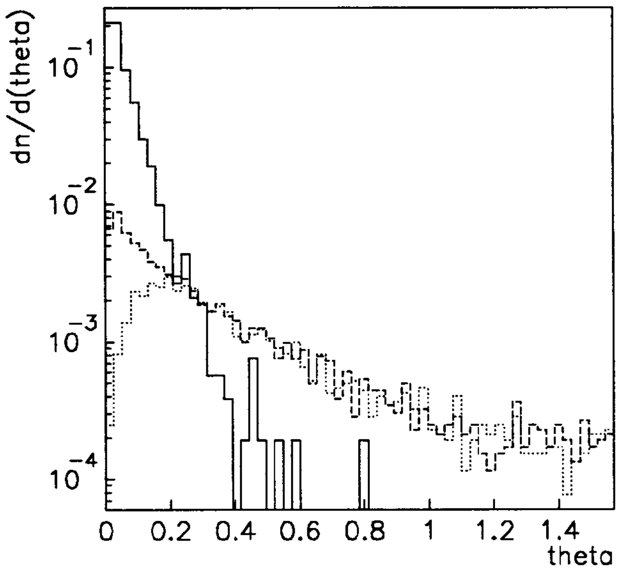

Figure 2: Photon spectrum $dn/d\theta$ for the slice $0.4 < x < 0.8$. The event axis is defined as in Fig. 1. The full histogram shows the photons from fragmentation, the dashed (dotted) one from shower with cut-off $Q_0 = 0.01$ GeV (5 GeV). Results for LEP energies, $\sqrt{s} = 91.2$ GeV.

exception is the region of large x values, where shower photons dominate. Here, however, the photons are predominantly emitted at large $p_\perp$ in the internal kinematical variables of the shower, and only appear at small $p_\perp$, owing to a tendency for the experimental event axis to align itself with any energetic photon (or jet). The rate of photons with low $p_\perp$ and high x is therefore comparable in the two shower alternatives.

As slices with higher and higher $p_\perp$ are considered, the x range increases where the shower photon rate is a significant fraction of the total. At the same time, however, the difference between the two extreme choices of shower cut-off disappears, as it should. Taken together, it seems difficult to find a window where fragmentation background is negligible, and yet differences induced by a variation of shower cut-off scale are large. The same conclusion emerges from Fig. 2, where we have retained the most promising range of x values, $0.4 < x < 0.8$, and instead show the photon rate as a function of angle away from the event axis.

We emphasize that this does not necessarily prove that information on the cut-off scale is inaccessible. Photon isolation criteria and π^0 rejection capabilities could be used to improve the signal-to-background ratio. To assess this possibility it is necessary to make specific detector-dependent assumptions, wherefore we do not pursue it further here. On the other hand, the two alternatives we have compared are extremes; a more realistic task might be to distinguish $Q_0 = 0.3$ GeV from $Q_0 = 1$ GeV, between which differences are rather smaller.

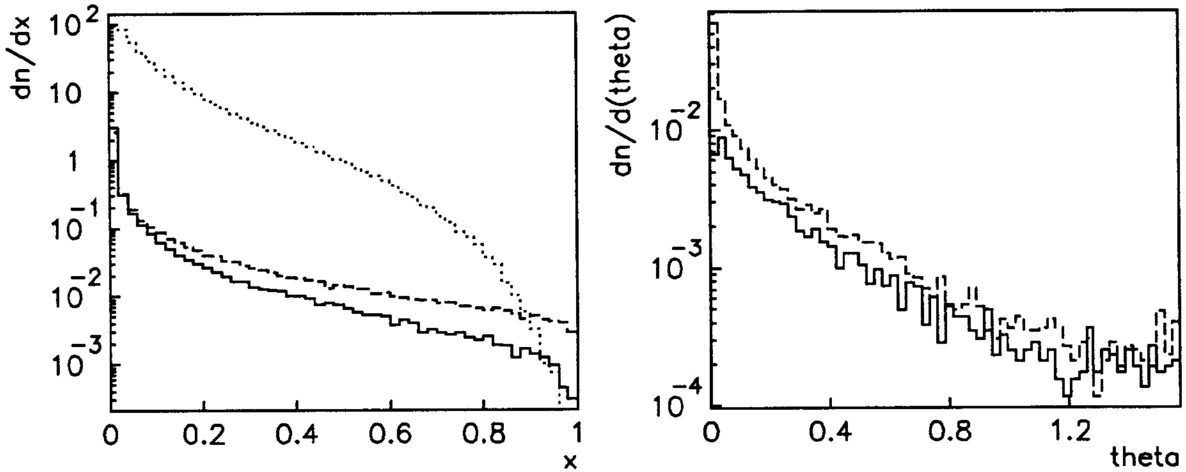

Fig. 3a

Fig. 3b

Figure 3: Effects of competition between photon and gluon emission. In (a) the x spectrum: the dashed (full) histogram for photons without (with) competition included; the dotted histogram is the gluon spectrum. In (b) the distribution in angle θ between photons and the event axis (defined as in Fig. 1) for photons in the range $0.4 < x < 0.8$: the dashed (full) histogram is for photons without (with) competition. Results for LEP energies, $\sqrt{s} = 91.2$ GeV.

3 Competition

The rate of photon emission off a final-state lepton pair, e.g. in the process $e^+e^- \rightarrow \mu^+\mu^-$, is well understood. As already mentioned, the issues of cut-offs and confinement effects in the low-mass region introduce complications when the QED results are applied to quarks. However, also for the region of hard and acollinear photons, quarks do not emit in the same way as leptons do. The reason is a competition between photon radiation, $q \rightarrow q\gamma$, and gluon radiation, $q \rightarrow qg$.

To first approximation, competition may be understood as a simple consequence of energy–momentum conservation, universal to all implementations of photon and gluon bremsstrahlung: a quark which has already radiated a fraction of its energy to gluons has a reduced phase space for photon emission. The energy radiated to gluons is not negligible — in JETSET typically half the original quark energy is radiated during the evolution of the shower — and therefore competition effects are quite significant.

In Fig. 3a the photon x spectrum is compared when gluon emission in JETSET is switched off and when it is allowed. The effects of competition are larger the larger the photon energy, since even a small amount of energy lost to gluon radiation is enough to close the phase space near $x = 1$. For typical experimental cuts at LEP, competition gives about a factor of 2 suppression in photon rate compared with naive predictions. For comparison, the gluon spectrum is also shown in Fig. 3a. This spectrum lies two or three orders of magnitude above the photon one, owing in part to the difference in couplings, eq. (10), in part to the multiplication of gluons in branchings $g \rightarrow gg$. This also explains why the gluon spectrum drops faster than the photon one at large x: an energetic gluon is likely to branch into several softer ones.

Gluon radiation may also affect the shape of many photon distributions, not only the normalization. An example is given in Fig. 3b, where the peak of almost collinear photon emission is smeared out, since the experimentally determined event axis and the axis of the radiating quark are both shifted around by gluon emission.

If the bulk of the competition effects come from trivial kinematical considerations, there is still some uncertainty left, which relates to the details of the parton shower, specifically to the order in which emissions are considered. Clearly the 'first' emission can take place within a larger phase space than subsequent ones; the details are discussed in refs. [6, 7]. If competition as such can give a factor of two suppression in photon rate, the choice of the 'time' order in which this competition is played out seems to introduce up to 20% variations in results. This is based on the experience reported by OPAL [8] and other collaborations at this meeting, comparing JETSET and Ariadne [9] predictions. Continued studies may help to improve our understanding not only of photon emission but also of the shower evolution structure as a whole; examples of further tests are given in ref. [10].

4 Angular Ordering

In QCD, both quarks and gluons carry colour charge and can radiate additional gluons. If an initial quark has 'already' radiated a number of reasonably hard gluons, the probability to radiate an additional softer gluon receives a contribution from each of the existing partons, from branchings $q \rightarrow qg$ and $g \rightarrow gg$. When this soft gluon is radiated at a large angle with respect to all of the other partons, the emission rate is overestimated if the individual branching probabilities are added incoherently: interference terms are mainly destructive. Simply put, a soft gluon of large wavelength is not able to resolve the individual colour charges, but only observes the net charge, which equals the original q charge. It turns out that a probabilistic picture can be preserved if emissions are ordered in terms of a decreasing opening angle between the two daughter partons of each branching [11], i.e. if the phase space for allowed branchings is restricted. Specifically, each gluon radiated from the quark should be restricted to have a smaller opening angle than the preceding one.

Here QED is different, since photons do not carry any charge, i.e. only the original q itself radiates. The phenomenon of destructive interference is thus absent, except in trivial forms such as that between the original q and $\overline{q}$, or between a $q\overline{q}$ pair produced in a shower branching $g \rightarrow q\overline{q}$. Therefore the emission of different photons off a $q\overline{q}$ pair factorizes, up to some small effects of energy–momentum conservation. In particular, there is no requirement of decreasing emission angles in the shower evolution.

The very first branching of a shower, either $q \rightarrow qg$ or $q \rightarrow q\gamma$, is constrained to agree with first-order matrix elements, and therefore no difference is expected between gluon and photon emission. In subsequent branchings, however, one expects a smaller average angle $\langle\theta_{qg}\rangle$ than $\langle\theta_{q\gamma}\rangle$. Results for the JETSET program are shown in Fig. 4a, where the average emission angle is plotted as a function of the mass of the branching quark. Unfortunately, the differences between the g and the γ results are rather small: at large masses the behaviour is dominated by the first branchings, and at small masses trivial kinematics constraints impose a similar behaviour for $q \rightarrow qg$ and $q \rightarrow q\gamma$. If first branchings are removed, Fig. 4b,

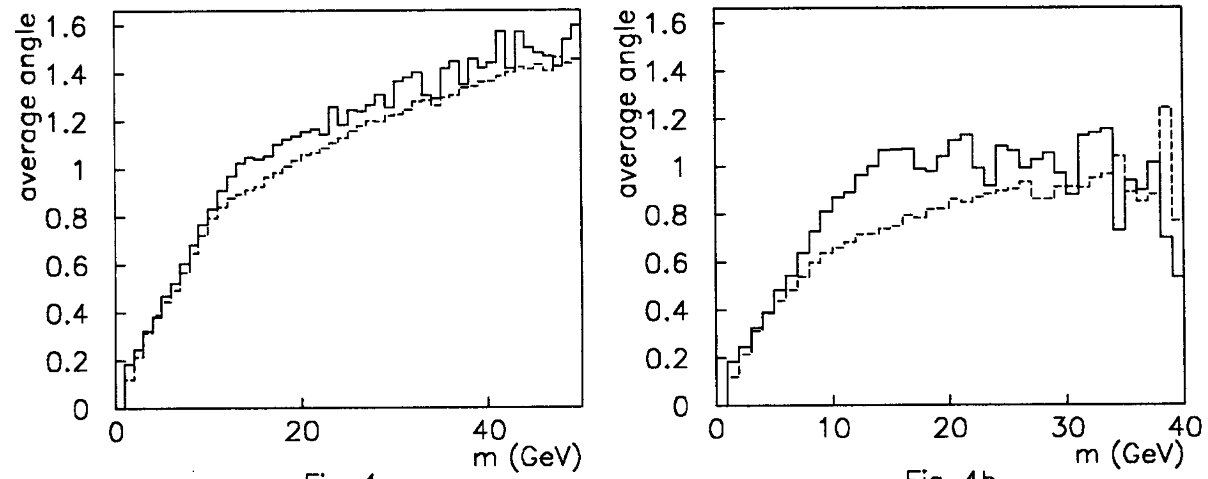

Fig. 4a

Fig. 4b

Figure 4: Average angle between the direction of the emitting quark and the emitted gluon or photon, as a function of the mass of the emitting quark. In (a) all quark branchings are included, in (b) the first branching of the q and $\bar{q}$ have been removed. Full histograms are for photon emission and dashed ones for gluon emission. Results for LEP energies, $\sqrt{s} = 91.2$ GeV.

differences become somewhat more visible.

It would be very nice with a direct experimental observation of the lack of angular ordering in photon emission, and in principle this would be possible by a suitable reconstruction of jets and an ordering of jet branchings by mass (cf. ref. [2]). However, the picture is complicated by a number of effects, such as the presence of branchings $g \to gg$, the difference in kinematics betweeen the emission of photons on the mass shell (almost always) and gluons with large masses (usually), the running of α_S (which implicitly depends on the opening angle), etc. It therefore seems unlikely that this particular kind of studies are possible at LEP.

5 Summary

At LEP energies, studies of prompt photon production may give a new insight. We have discussed, in this paper, a few of the more important aspects of photon emission in QCD showers, and suggested tests. Some of these may not be experimentally feasible, but at least they may stimulate continued efforts to find interesting observables. Already today the importance of competition between photon and gluon emission is well established, and we are starting to learn about the correct 'time' ordering choice for parton-shower evolution in general. Higher statistics and more precise tests are likely to provide us with further important information, maybe not accessible any other way.

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