LUND UNIVERSITY

The 2006 European School
of High-Energy Physics
Aronsborg, Sweden
26, 27 June 2006

Monte Carlo Generators

Torbjörn Sjöstrand

CERN and Lund University

(today)

Introduction and Overview;
Matrix Elements; Parton Showers

(tomorrow)

Matching Issues; Multiple Interactions;
Summary and Outlook

Apologies

These lectures are focussed on LHC/Tevatron applications,
not specifically LEP/HERA/ILC/. . . (although many common points).

Even so, they will *not* cover:

⋆ Heavy-ion physics:
- without quark-gluon plasma formation, or
- with quark-gluon plasma formation.

⋆ Specific physics studies for topics such as
- B production,
- Higgs discovery,
- SUSY phenomenology,
- other new physics discovery potential.

⋆ The modelling of elastic and diffractive topologies.

They *will* cover the "normal" physics that will be there
in (essentially) all LHC pp events, from QCD to exotics,
with special emphasis on parton showers and underlying events.

Read More

T.S., YETI'06 lectures, March 2006:
http://www.thep.lu.se/∼torbjorn/ and click on "Talks"

Steve Mrenna, CTEQ Summer School lectures, June 2004:
http://www.phys.psu.edu/∼cteq/schools/summer04/mrenna/mrenna.pdf

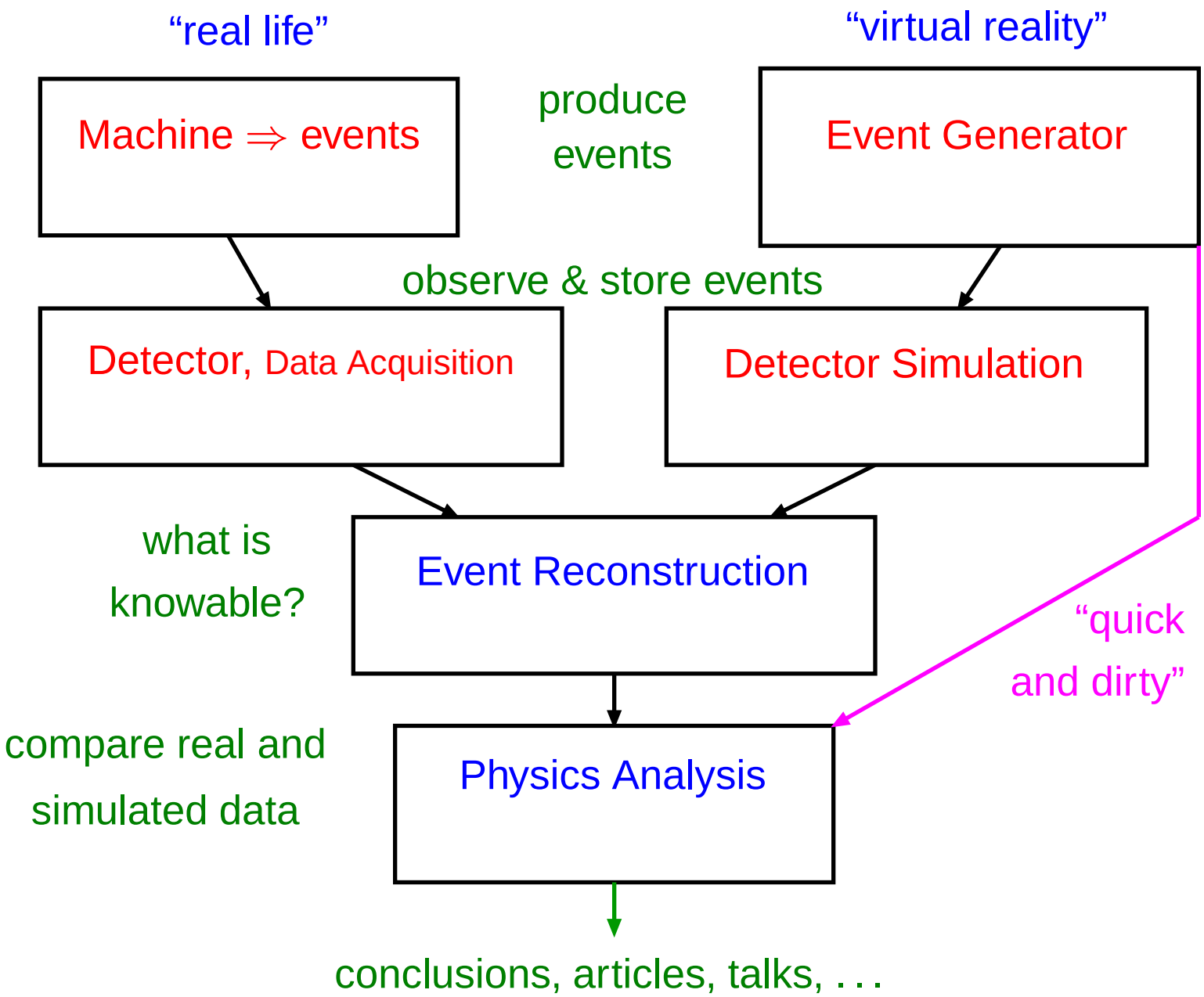
Mike Seymour, Academic Training lectures July 2003:
http://seymour.home.cern.ch/seymour/slides/CERNlectures.html

Bryan Webber, HERWIG lectures for CDF, October 2004:
http://www-cdf.fnal.gov/physics/lectures/herwig_Oct2004.html

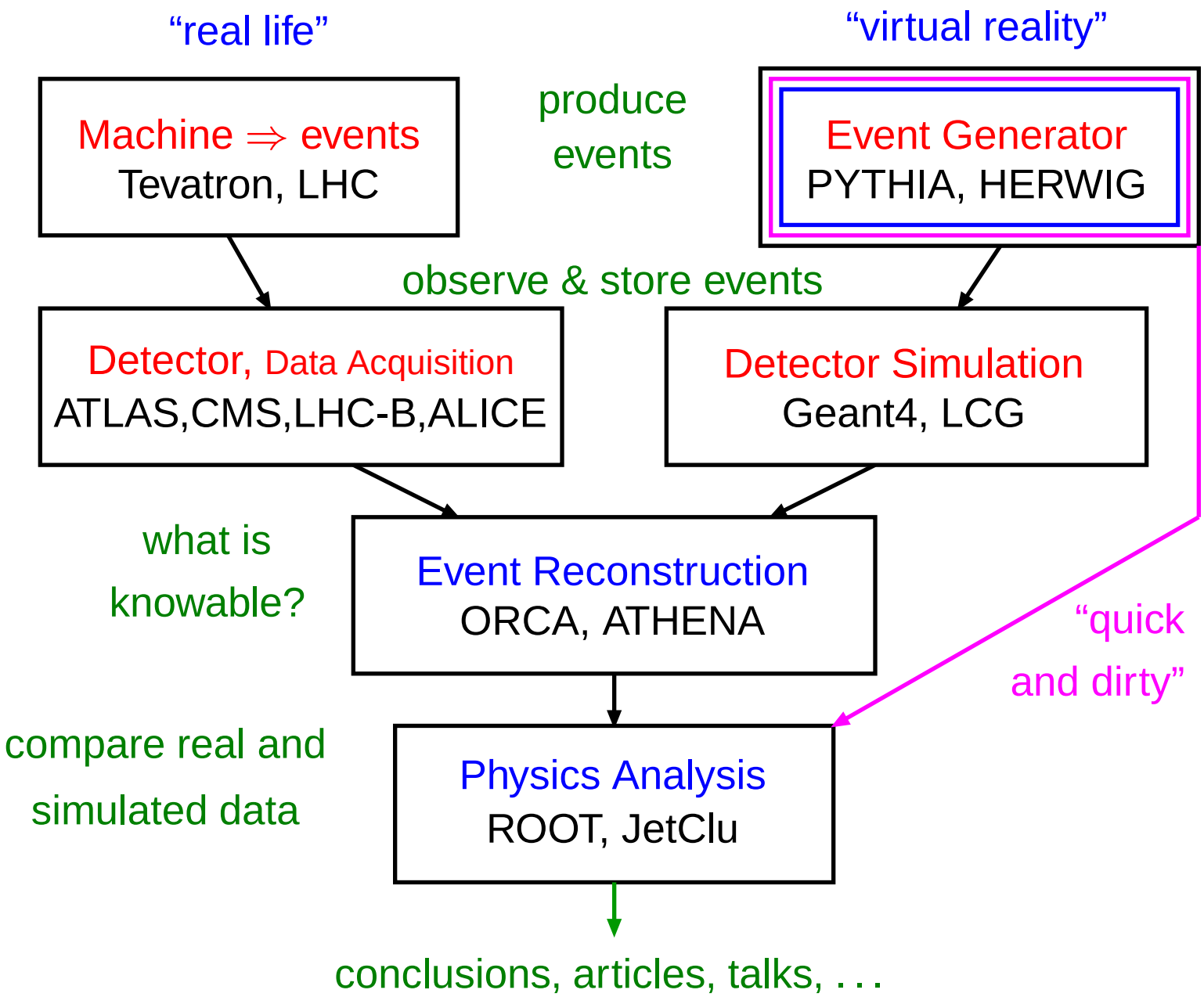
Michelangelo Mangano, KEK LHC simulations workshop, April 2004:
http://mlm.home.cern.ch/mlm/talks/kek04_mlm.pdf

The "Les Houches Guidebook to Monte Carlo Generators
for Hadron Collider Physics", hep-ph/0403045
http://arxiv.org/pdf/hep-ph/0403045

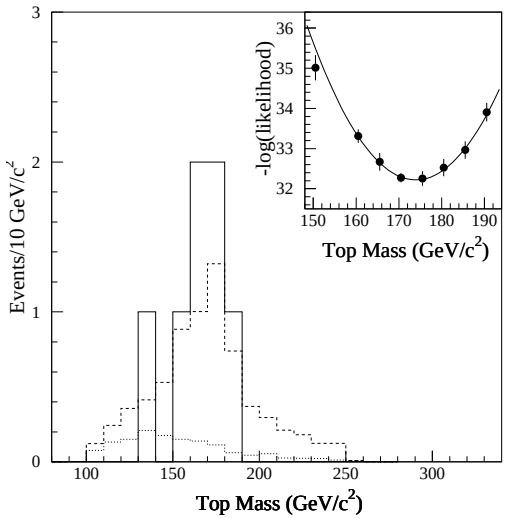
Event Generator Position



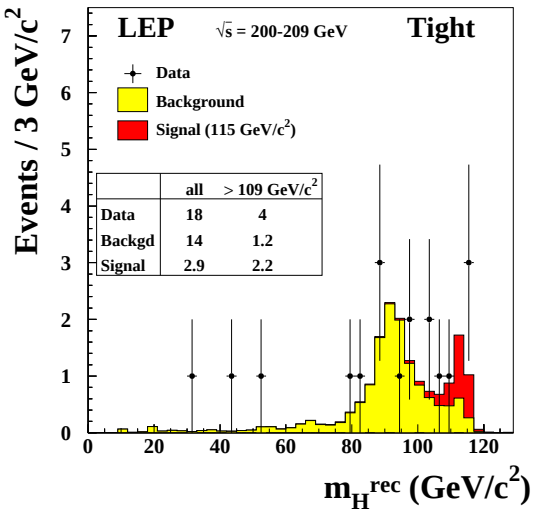
Event Generator Position



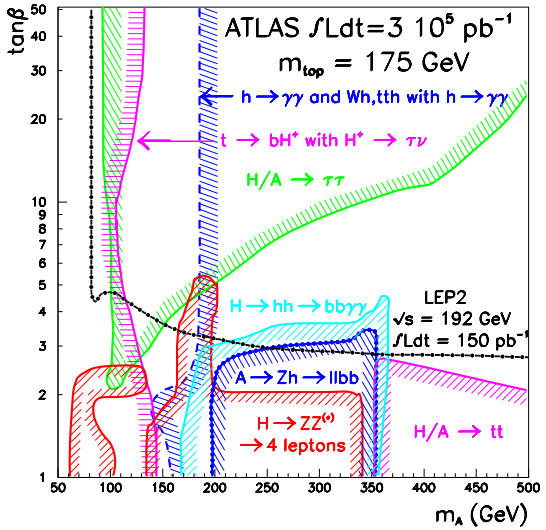
Why Generators? (I)



top discovery and mass determination



Higgs (non) discovery



Higgs and supersymmetry exploration

not feasible without generators

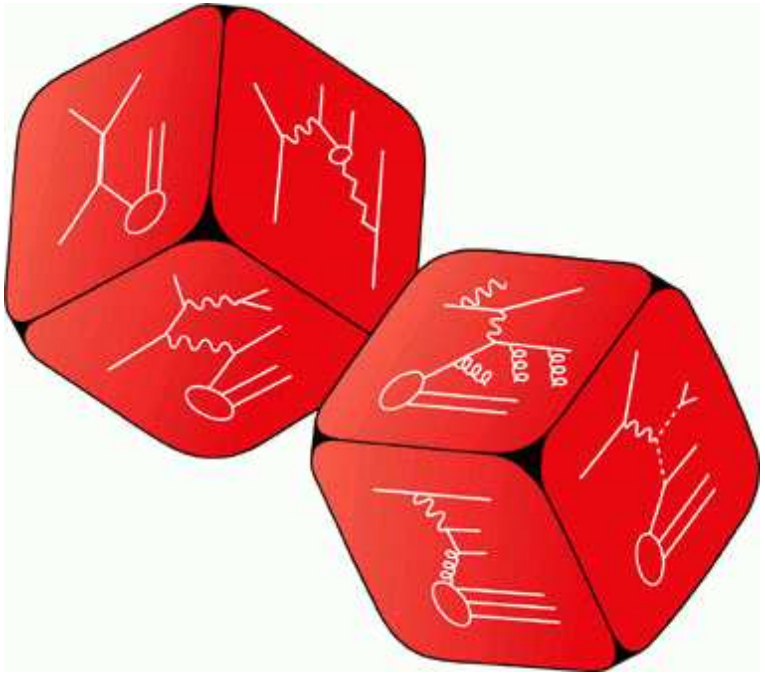
Why Generators? (II)

- Allow theoretical and experimental studies of *complex* multiparticle physics
- Large flexibility in physical quantities that can be addressed
- Vehicle of ideology to disseminate ideas from theorists to experimentalists

Can be used to

- predict event rates and topologies
  $\Rightarrow$ can estimate feasibility
- simulate possible backgrounds
  $\Rightarrow$ can devise analysis strategies
- study detector requirements
  $\Rightarrow$ can optimize detector/trigger design
- study detector imperfections
  $\Rightarrow$ can evaluate acceptance corrections

A tour to Monte Carlo



. . . because Einstein was wrong: God does throw dice!

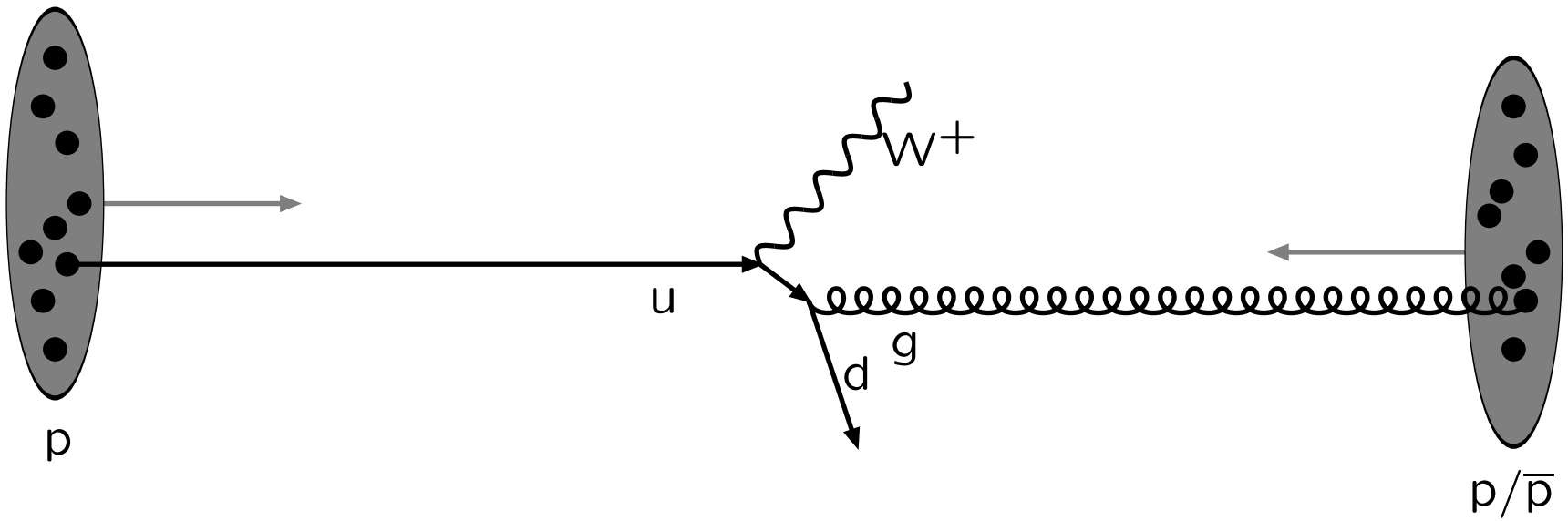
Quantum mechanics: amplitudes $\Longrightarrow$ probabilities

Anything that possibly can happen, will! (but more or less often)

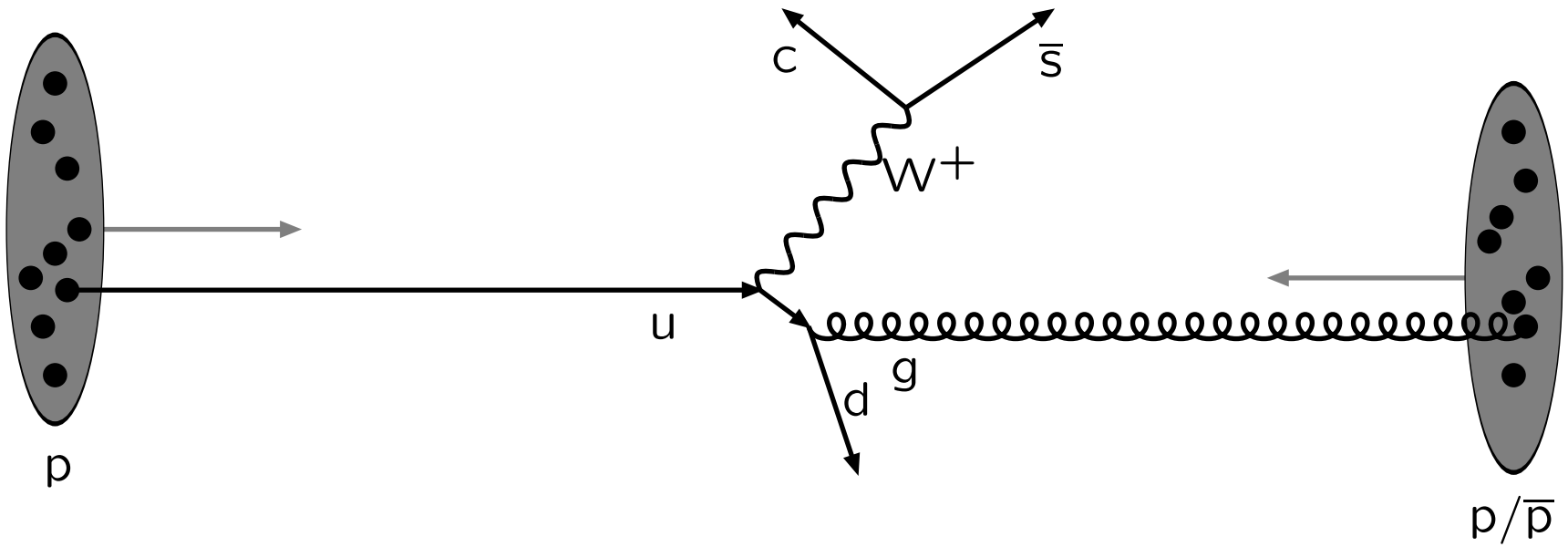
The structure of an event

Warning: schematic only, everything simplified, nothing to scale, . . .

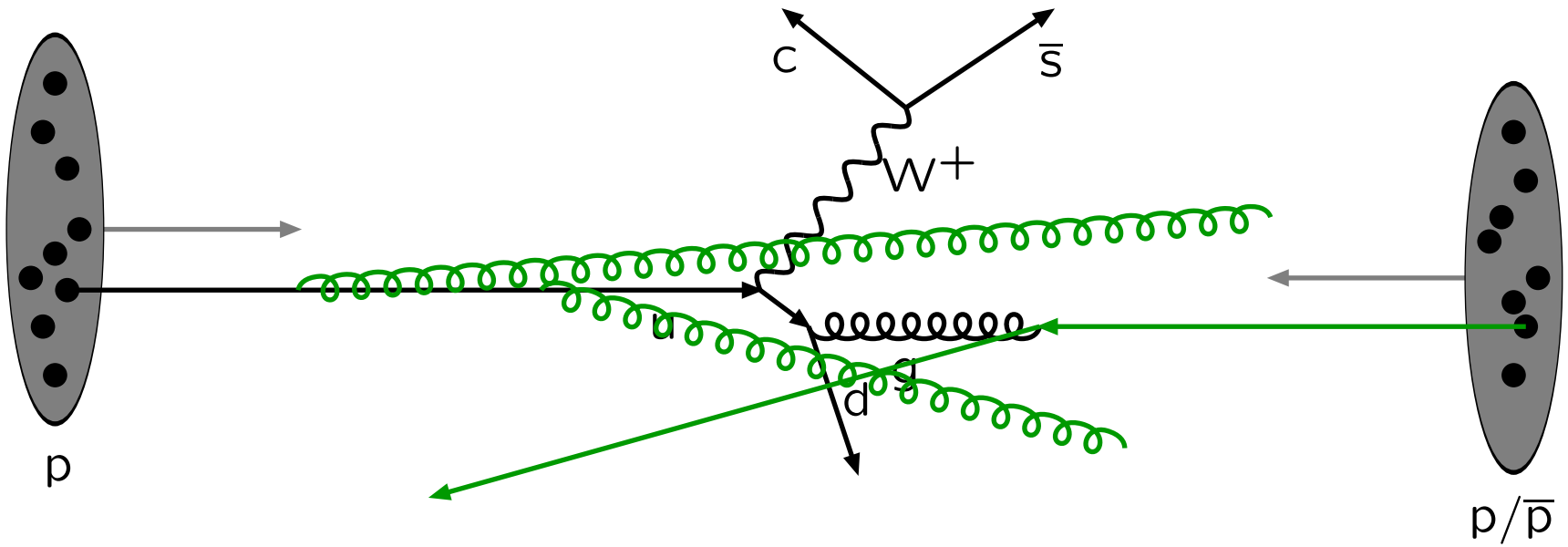
Incoming beams: parton densities



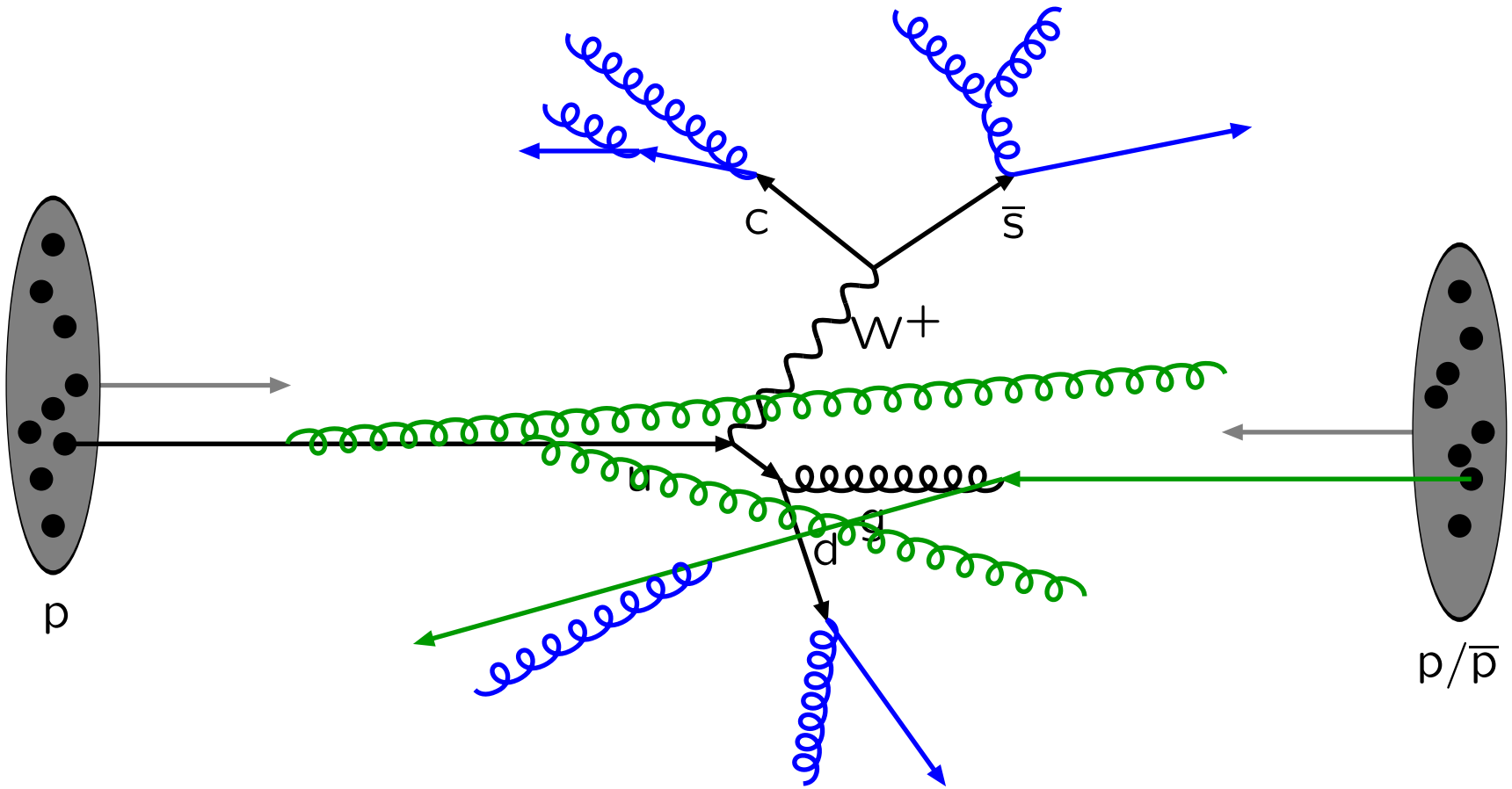
Hard subprocess: described by matrix elements



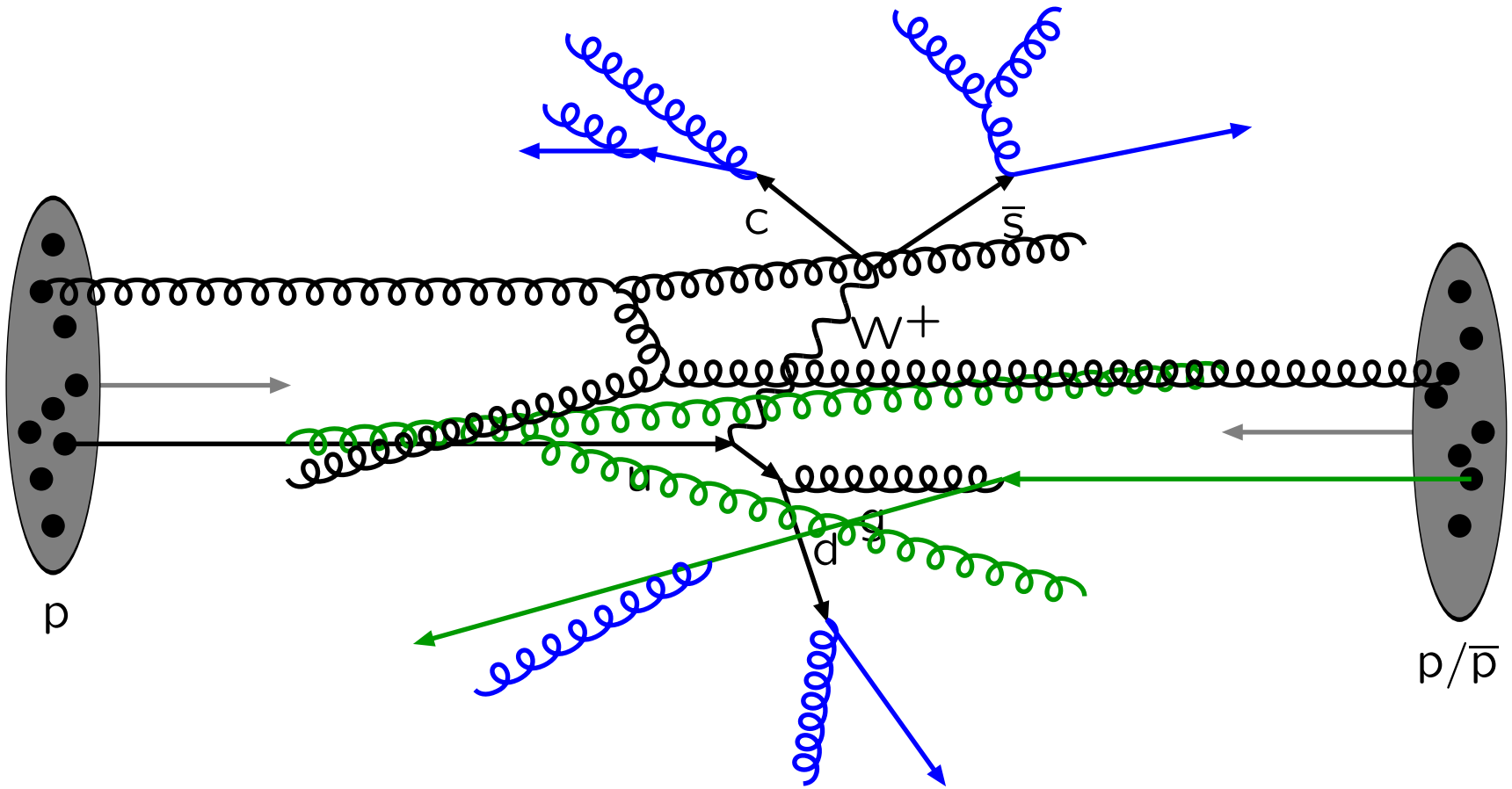
Resonance decays: correlated with hard subprocess



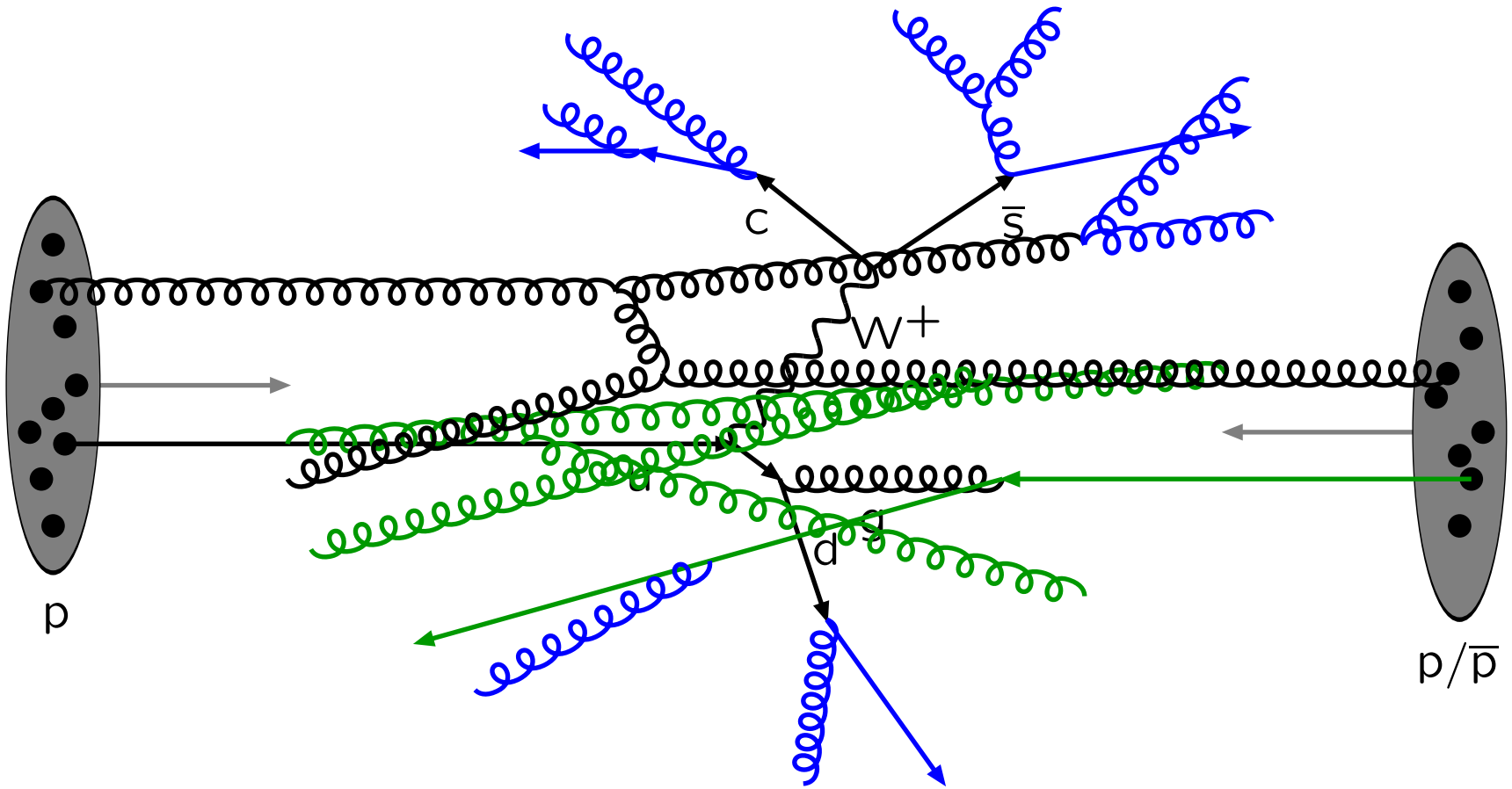
Initial-state radiation: spacelike parton showers



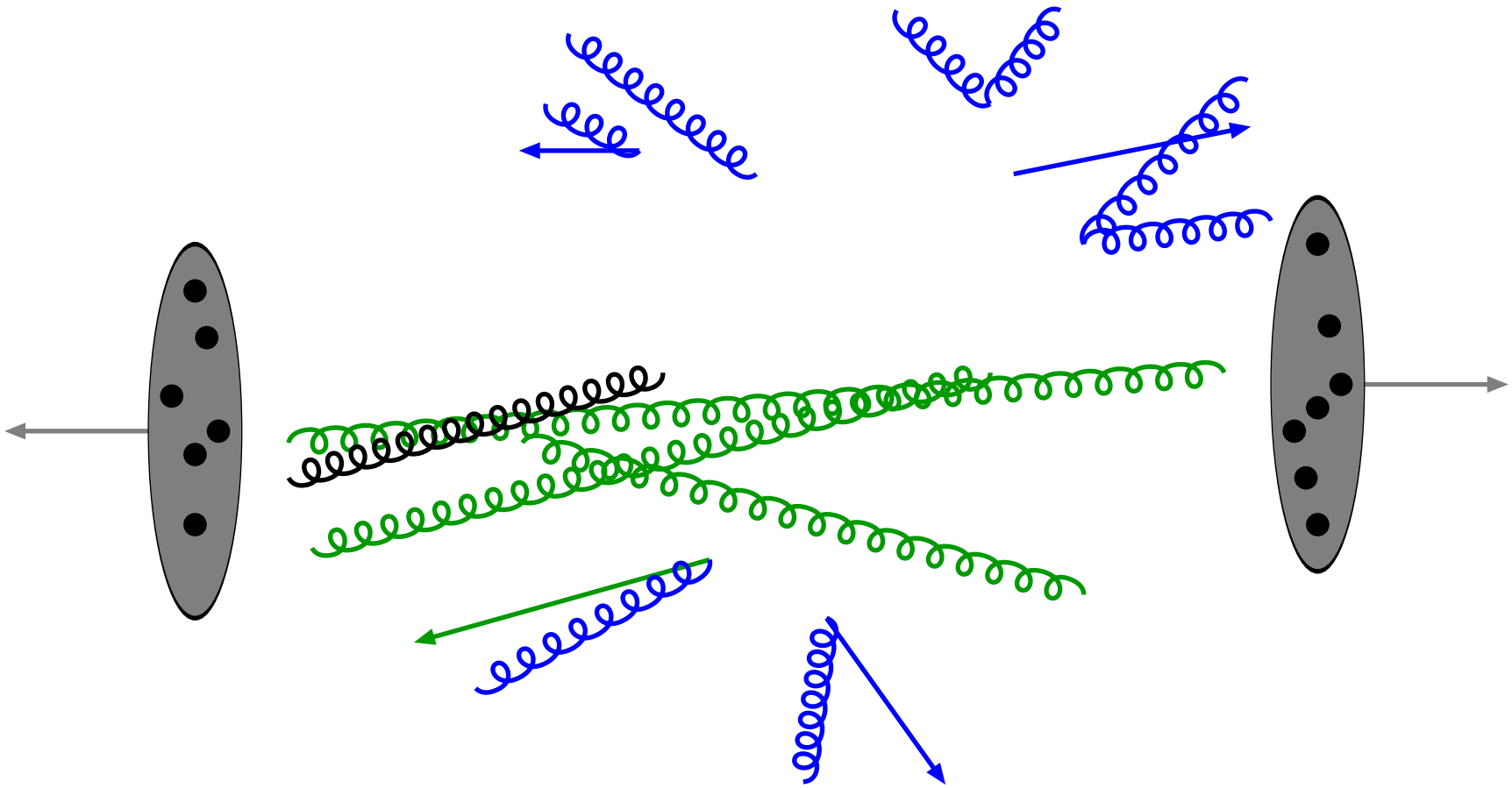
Final-state radiation: timelike parton showers



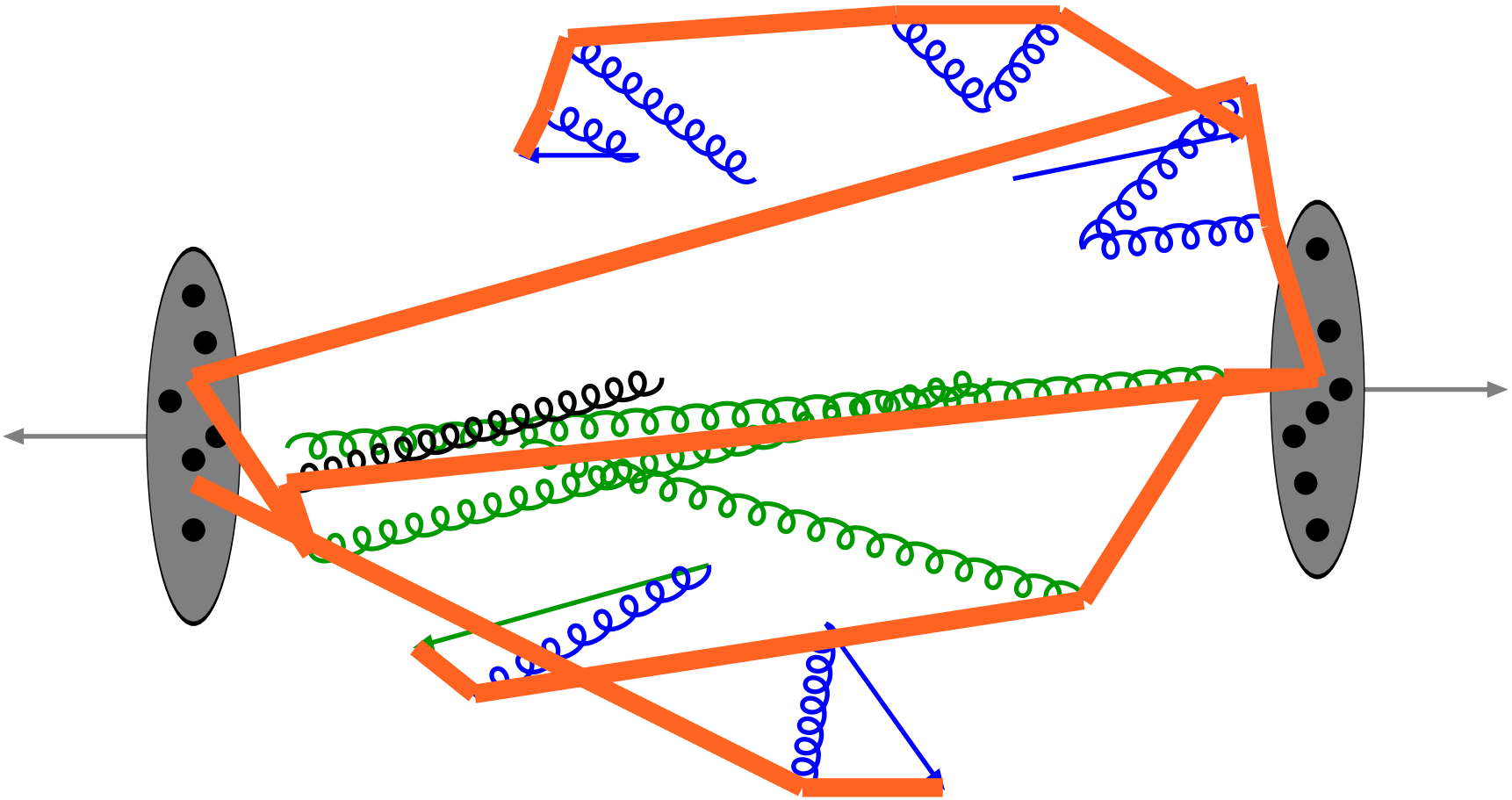
Multiple parton–parton interactions . . .



. . . with its initial- and final-state radiation

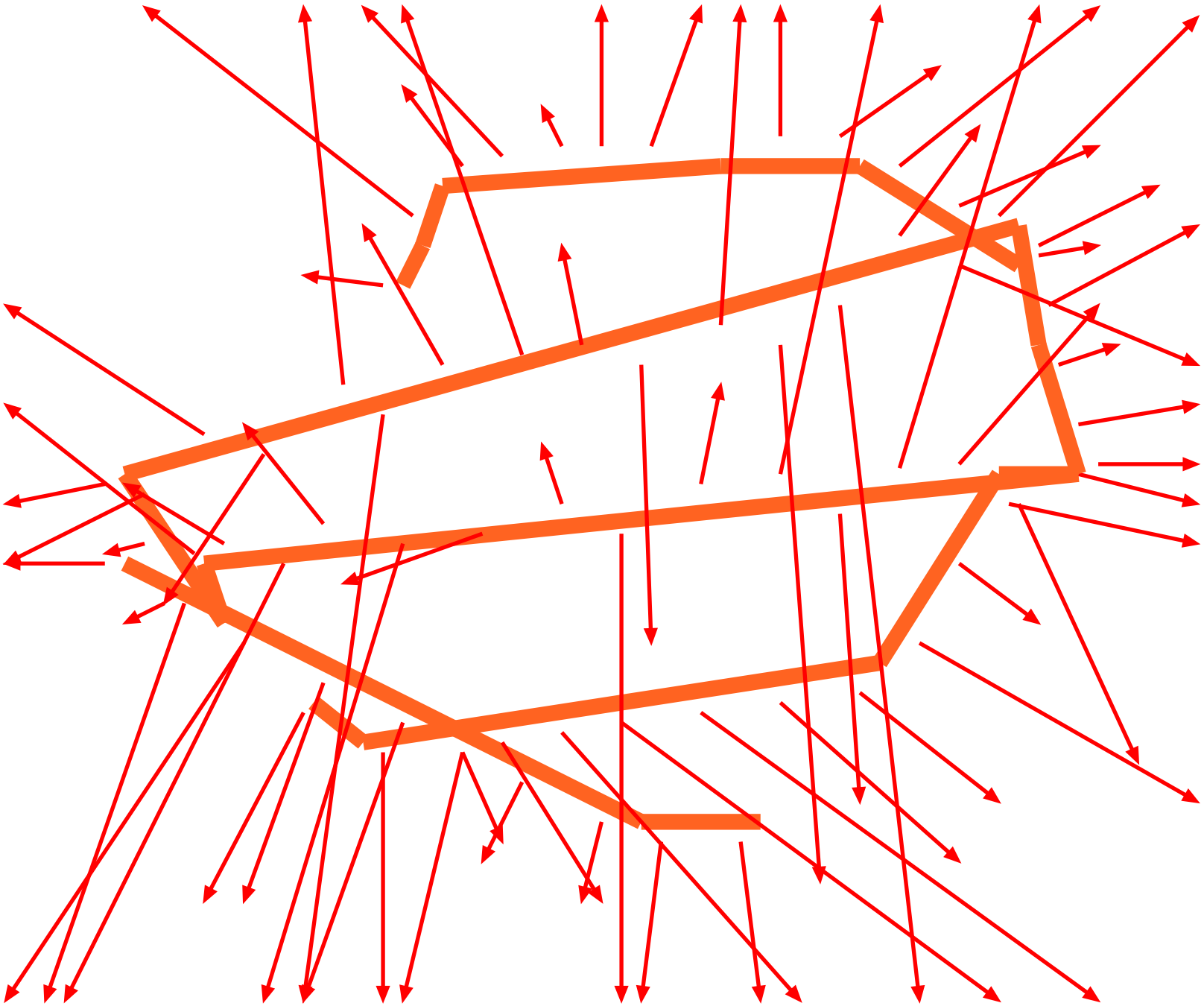


Beam remnants and other outgoing partons

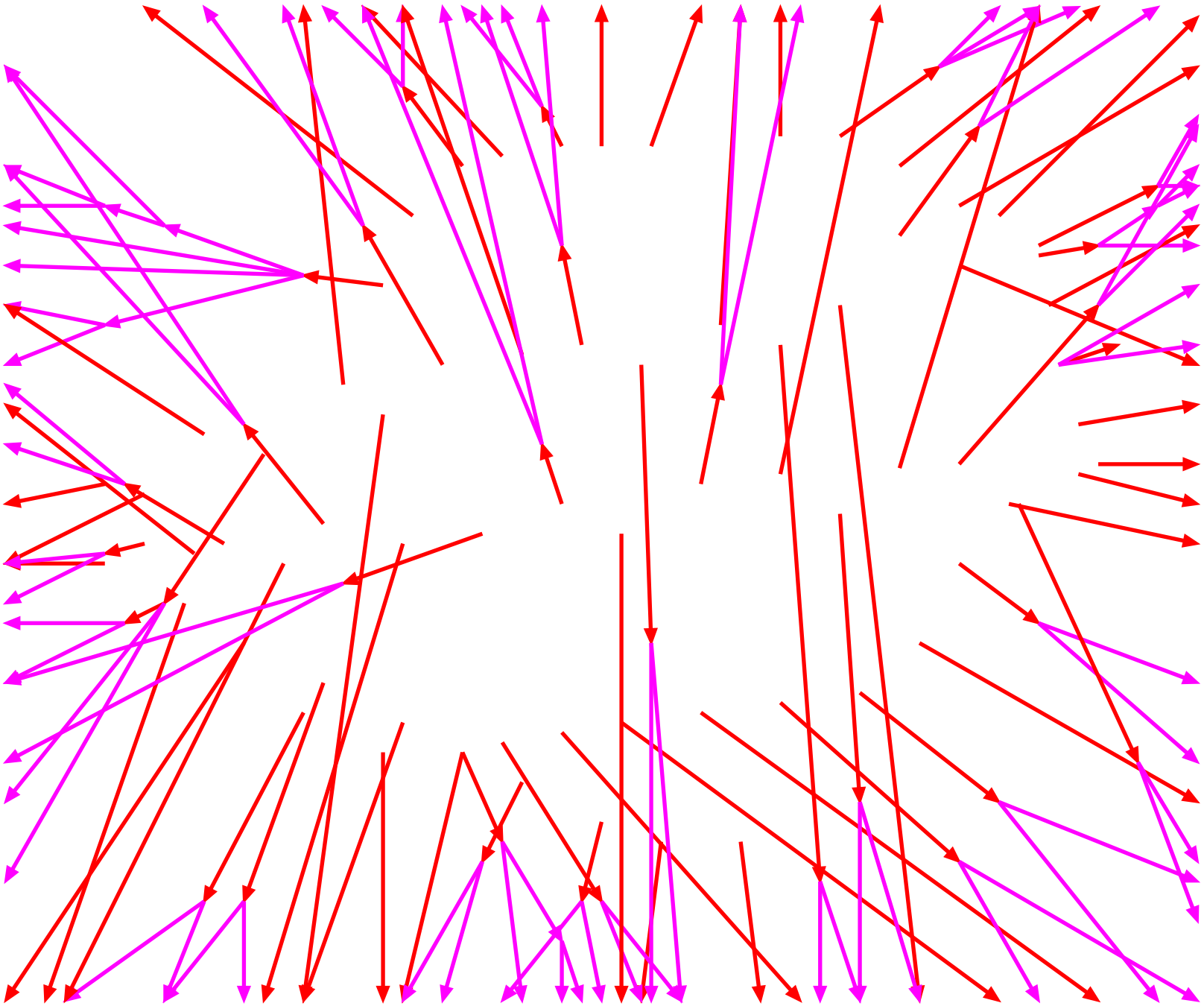


Everything is connected by colour confinement strings

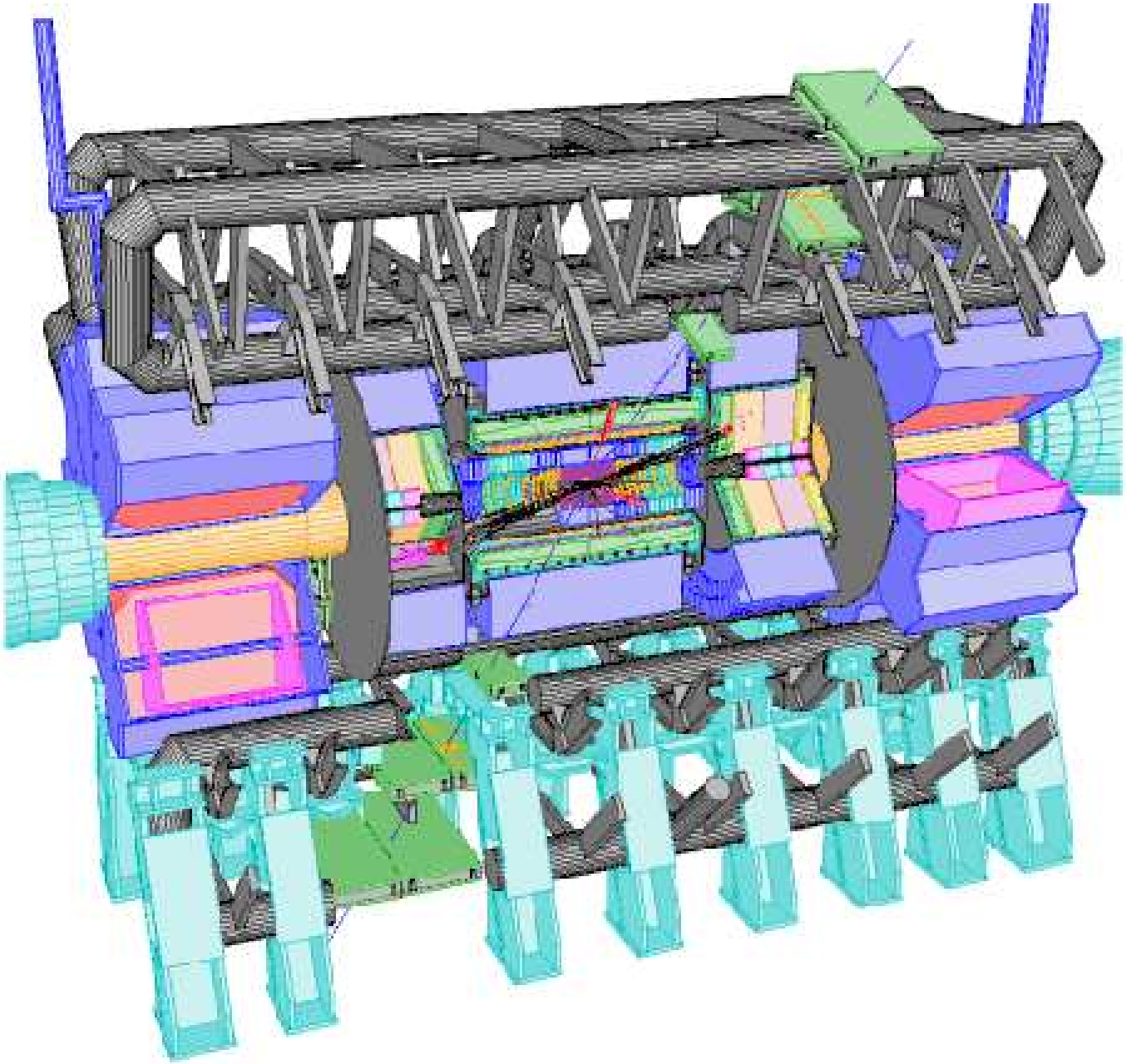
Recall! Not to scale: strings are of hadronic widths



The strings fragment to produce primary hadrons



Many hadrons are unstable and decay further



These are the particles that hit the detector

The Monte Carlo method

Want to generate events in as much detail as Mother Nature

$\Longrightarrow$ get average *and* fluctutations right

$\Longrightarrow$ make random choices, $\sim$ as in nature

$$\sigma_{\text{final state}} = \sigma_{\text{hard process}}\, \mathcal{P}_{\text{tot,hard process}\to\text{final state}}$$

(appropriately summed & integrated over non-distinguished final states)

where $\mathcal{P}_{\text{tot}} = \mathcal{P}_{\text{res}}\, \mathcal{P}_{\text{ISR}}\, \mathcal{P}_{\text{FSR}}\, \mathcal{P}_{\text{MI}} \mathcal{P}_{\text{remnants}}\, \mathcal{P}_{\text{hadronization}}\, \mathcal{P}_{\text{decays}}$

with $\mathcal{P}_i = \prod_j \mathcal{P}_{ij} = \prod_j \prod_k \mathcal{P}_{ijk} = \ldots$ in its turn

$\Longrightarrow$ **divide and conquer**

an event with n particles involves $\mathcal{O}(10n)$ random choices,

(flavour, mass, momentum, spin, production vertex, lifetime, ...)

LHC: ~ 100 charged and ~ 200 neutral ($+$ intermediate stages)

$\Longrightarrow$ several thousand choices

(of $\mathcal{O}(100)$ different kinds)

Generator Landscape

	General-Purpose	Specialized
Hard Processes	**HERWIG**	a lot
Resonance Decays		HDECAY, . . .
Parton Showers	**PYTHIA**	Ariadne/LDC, NLLjet
Underlying Event	ISAJET	DPMJET
Hadronization	SHERPA	none (?)
Ordinary Decays		TAUOLA, EvtGen

specialized often best at given task, but need General-Purpose core

Matrix Elements and Their Usage

$\mathcal{L}$

$\Rightarrow$ Feynman rules

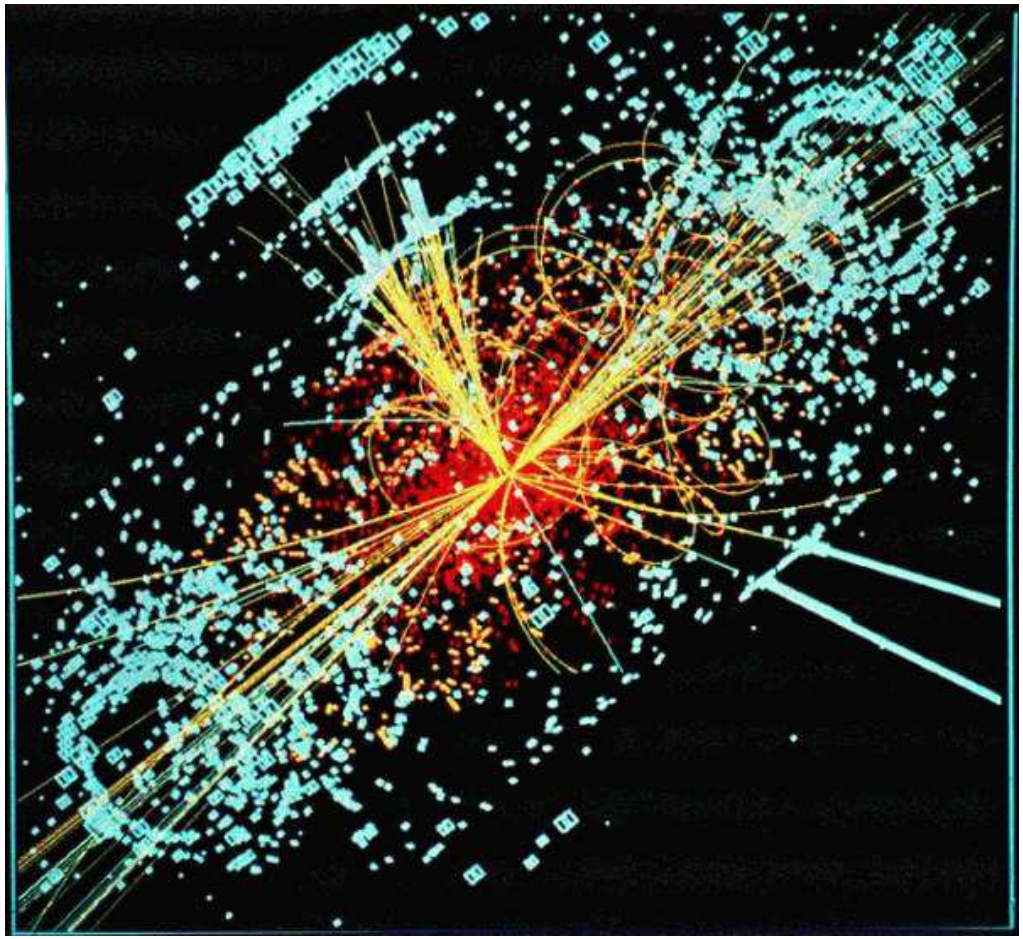
$\Rightarrow$ Matrix Elements

$\Rightarrow$ Cross Sections

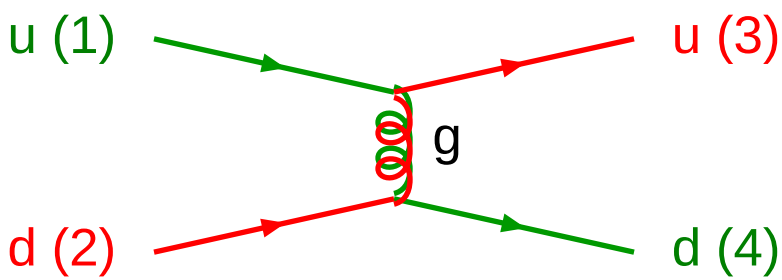
$+$ Kinematics

$\Rightarrow$ Processes

$\Rightarrow \ldots \Rightarrow$



Cross sections and kinematics

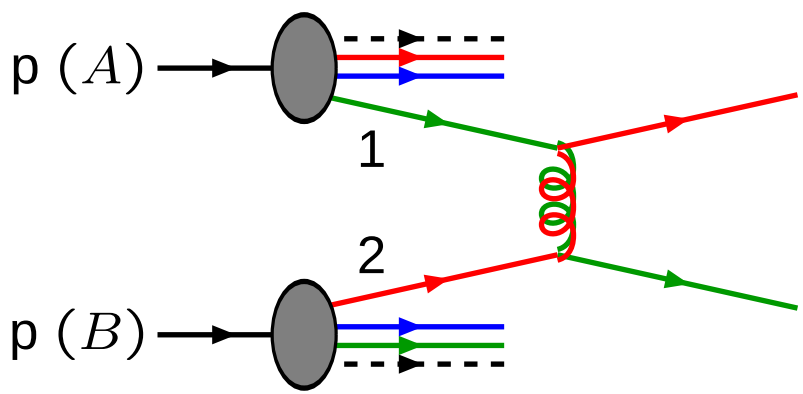


$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2 = -\hat{s}(1 - \cos\hat{\theta})/2$$

$$\hat{u} = (p_1 - p_4)^2 = -\hat{s}(1 + \cos\hat{\theta})/2$$

$$\mathrm{qq}' \to \mathrm{qq}' \quad : \quad \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} = \frac{\pi}{\hat{s}^2}\,\frac{4}{9}\,\alpha_s^2\,\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \qquad (\sim \text{Rutherford})$$



$$s = (p_A + p_B)^2$$

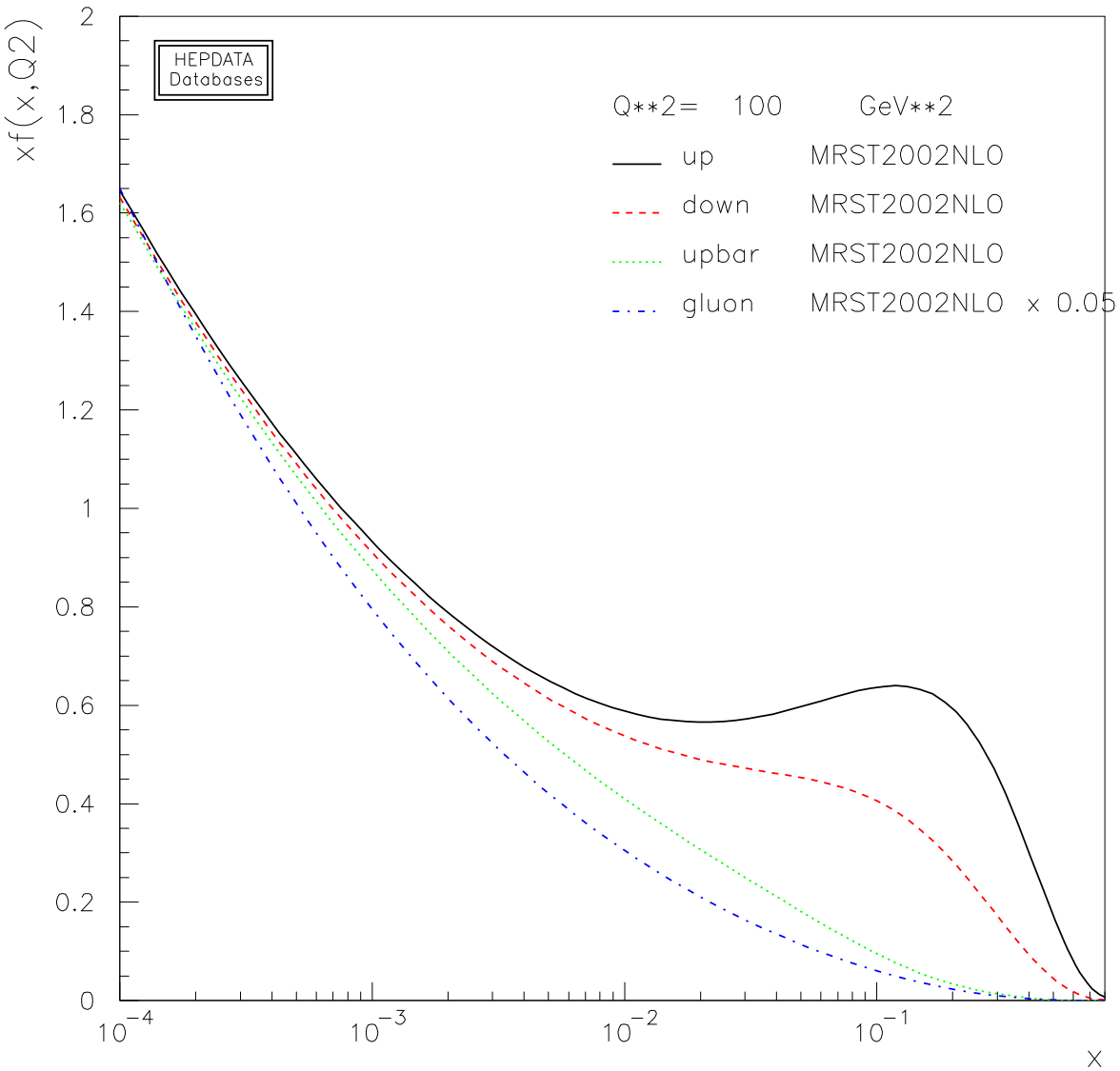
$$x_1 \approx E_1/E_A$$

$$x_2 \approx E_2/E_B$$

$$\hat{s} = x_1 x_2 s$$

$$\sigma = \sum_{i,j} \iiint \mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}\hat{t}\; f_i^{(A)}(x_1, Q^2)\; f_j^{(B)}(x_2, Q^2)\,\frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

Parton Distribution/Density Functions (PDF)



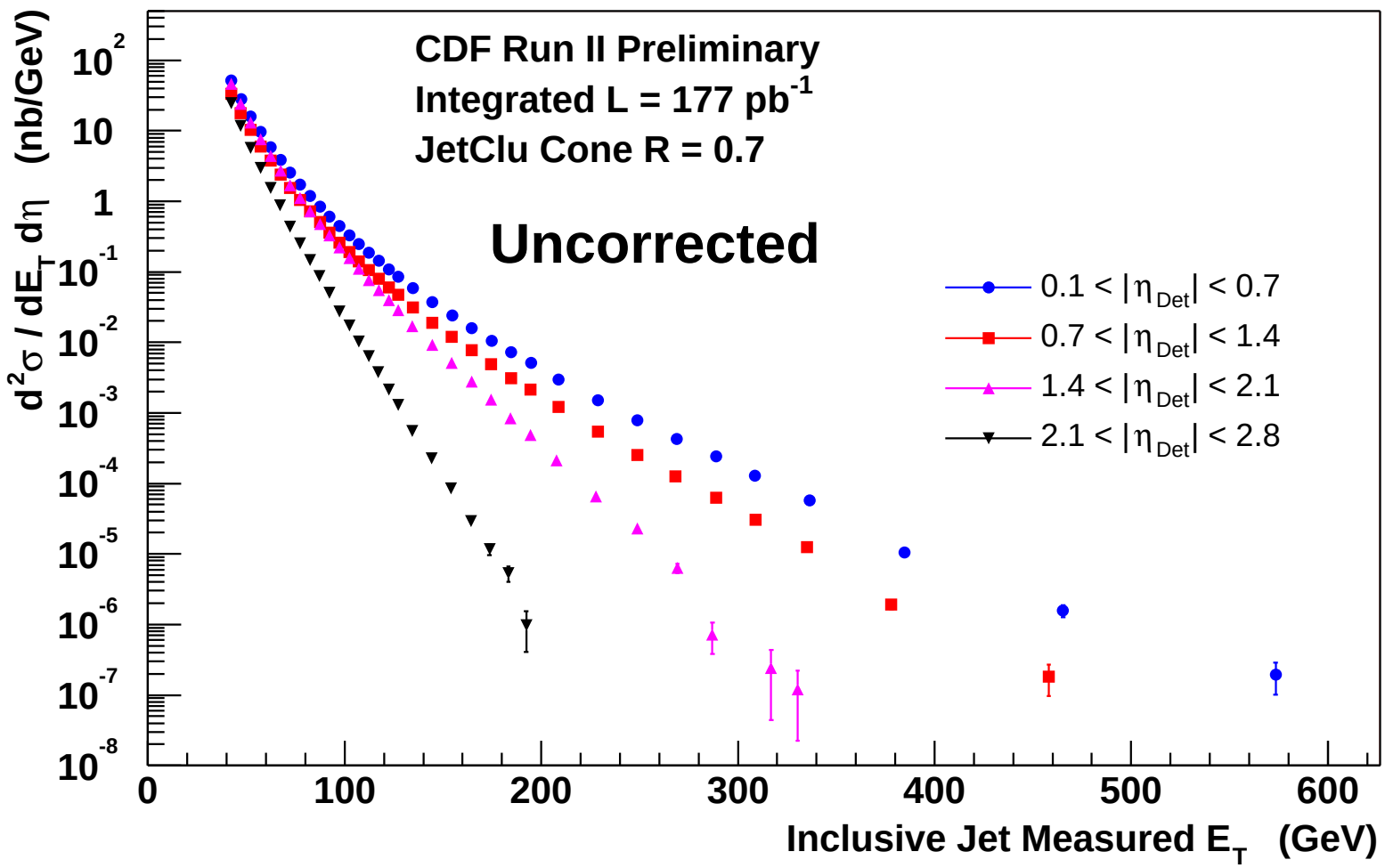
initial conditions nonpertubative

evolution pertubative (DGLAP)

http://durpdg.dur.ac.uk/hepdata/pdf.html

Peaking of PDF's at small x and of QCD ME's at low $p_\perp$

$\Longrightarrow$ most of the physics is at low transverse momenta . . .

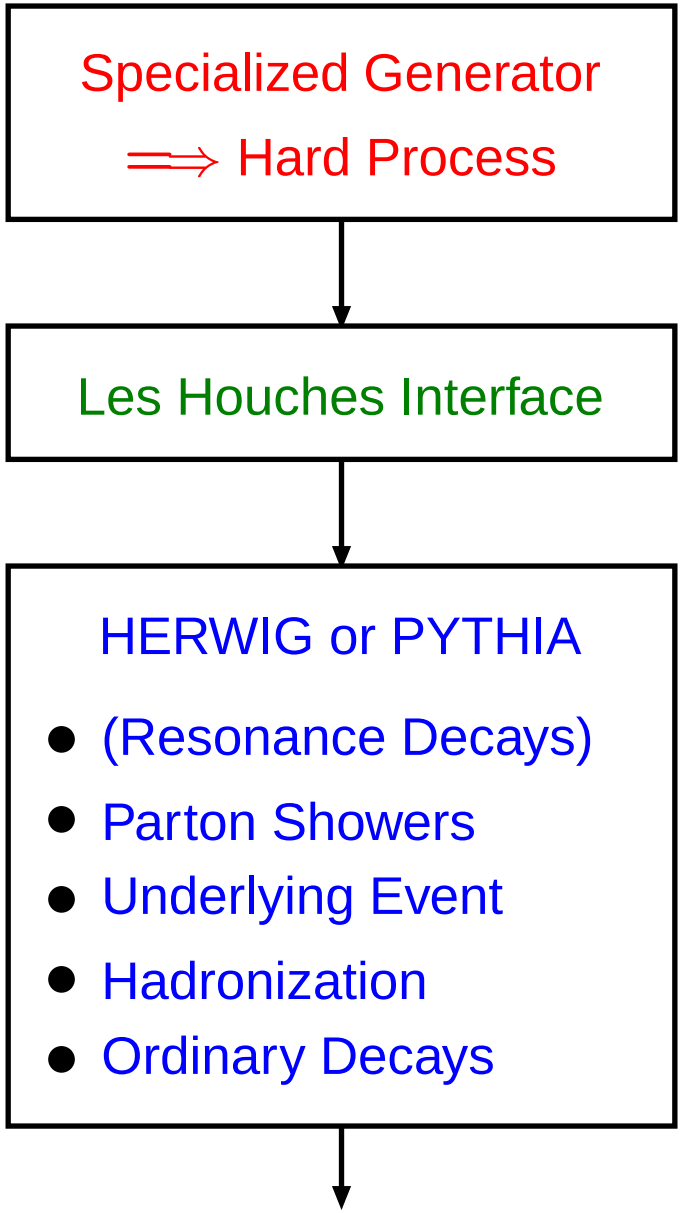


. . . but New Physics likely to show up at large masses/$p_\perp$'s

The Smaller Picture: Subprocess Survey

Kind	Process	PYT	HER	ISA
QCD & related	Soft QCD	$\star$	$\star$	$\star$
	Hard QCD	$\star$	$\star$	$\star$
	Heavy flavour	$\star$	$\star$	$\star$
Electroweak SM	Single $\gamma^*/Z^0/W^\pm$	$\star$	$\star$	$\star$
	$(\gamma/\gamma^*/Z^0/W^\pm/f/g)^2$	$\star$	$\star$	$\star$
	Light SM Higgs	$\star$	$\star$	$\star$
	Heavy SM Higgs	$\star$	$\star$	$\star$
SUSY BSM	$h^0/H^0/A^0/H^\pm$	$\star$	$\star$	$\star$
	SUSY	$\star$	$\star$	$\star$
	$\not{R}$ SUSY	$\star$	$\star$	—
Other BSM	Technicolor	$\star$	—	$(\star)$
	New gauge bosons	$\star$	—	—
	Compositeness	$\star$	—	—
	Leptoquarks	$\star$	—	—
	$H^{\pm\pm}$ (from LR-sym.)	$\star$	—	—
	Extra dimensions	$(\star)$	$(\star)$	$(\star)$

The Les Houches Accord

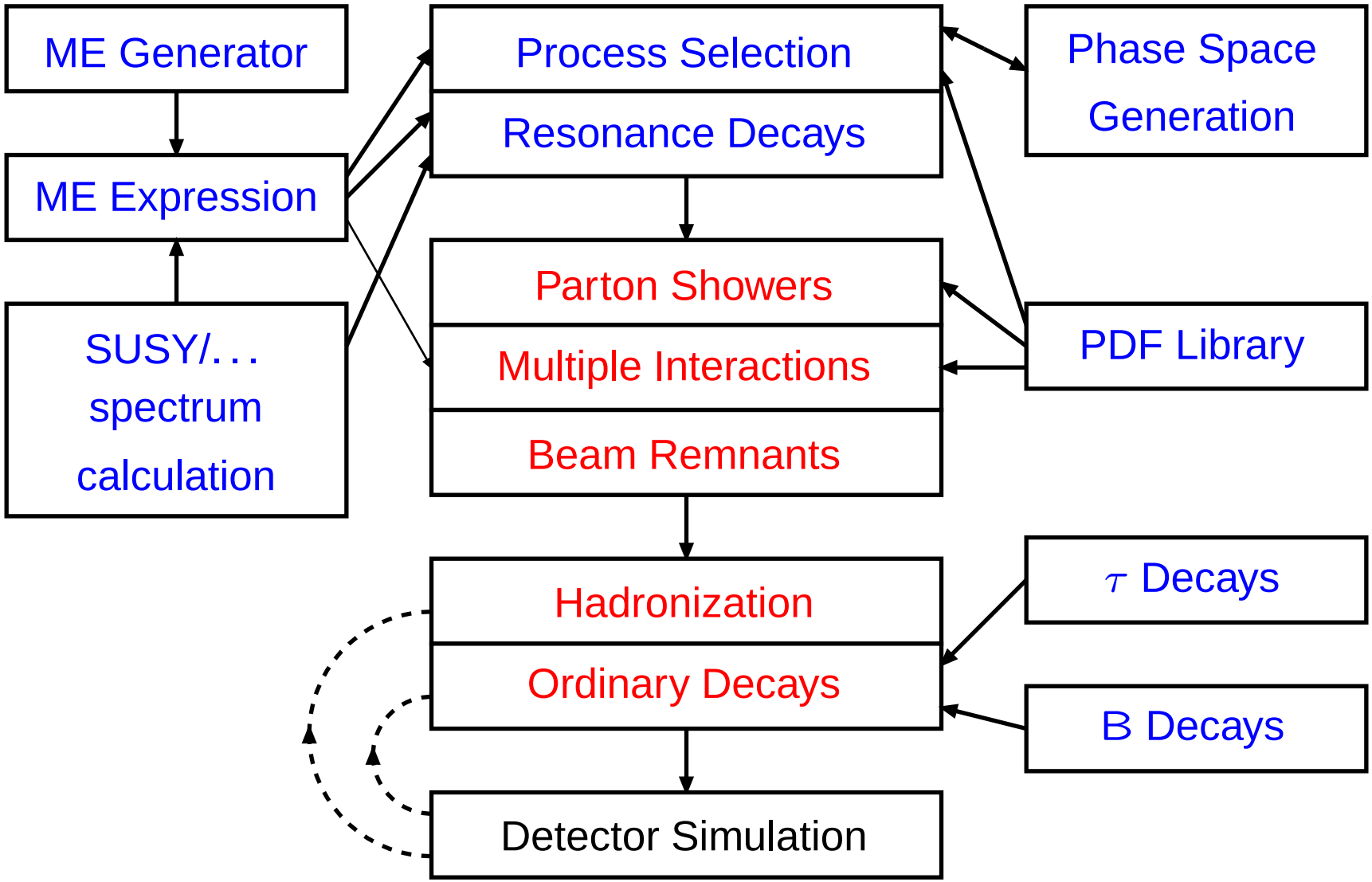


Some Specialized Generators:

- AcerMC: $t\bar{t}b\bar{b}$, ...
- ALPGEN: $W/Z+ \leq 6j$, $nW + mZ + kH+ \leq 3j$, ...
- AMEGIC++: generic LO
- CompHEP: generic LO
- GRACE+Bases/Spring: generic LO+ some NLO loops
- GR@PPA: $b\bar{b}b\bar{b}$
- MadCUP: $W/Z+ \leq 3j, t\bar{t}b\bar{b}$
- MadGraph+HELAS: generic LO
- MCFM: NLO $W/Z+ \leq 2j$, WZ, WH, $H+ \leq 1j$
- O'Mega+WHIZARD: generic LO
- VECBOS: $W/Z+ \leq 4j$

Apologies for all unlisted programs

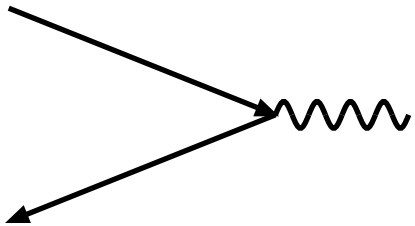
The Bigger Picture

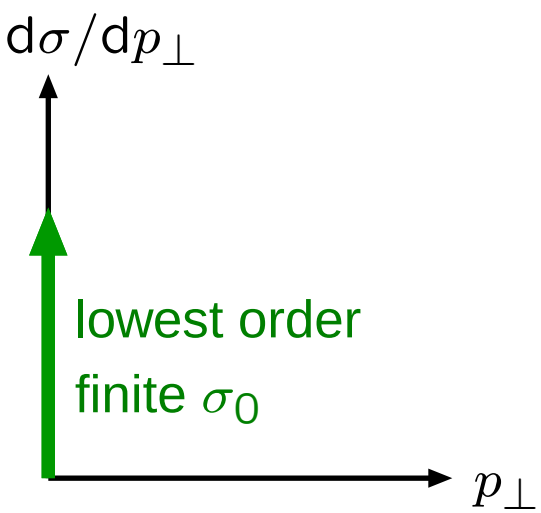


$\Longrightarrow$ need standardized interfaces (LHAPDF, SUSY LHA, . . .)

Next-to-leading order (NLO) calculations

I. Lowest order,
$\mathcal{O}(\alpha_{\mathrm{em}})$:
$q\bar{q} \to Z^0$





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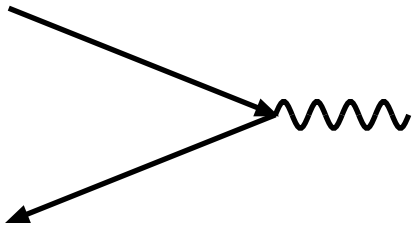
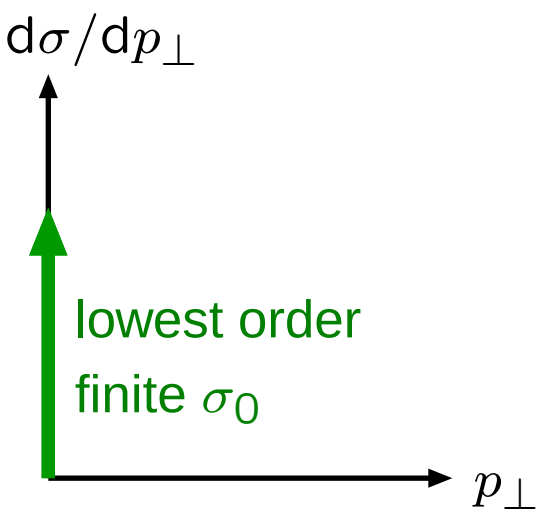
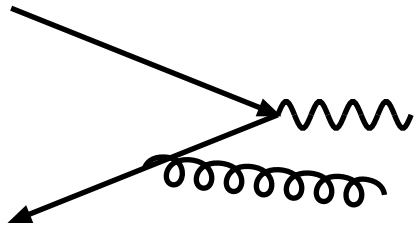
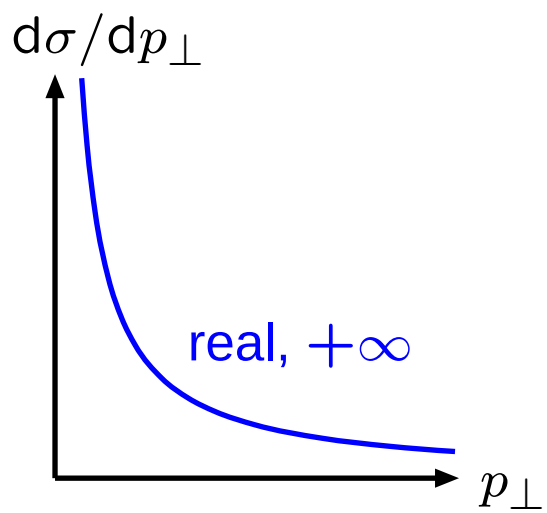




II. First-order real,
$\mathcal{O}(\alpha_{\mathrm{em}}\alpha_{\mathrm{s}})$:
$q\bar{q} \to Z^0 g$ etc.





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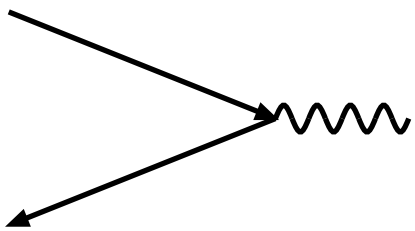

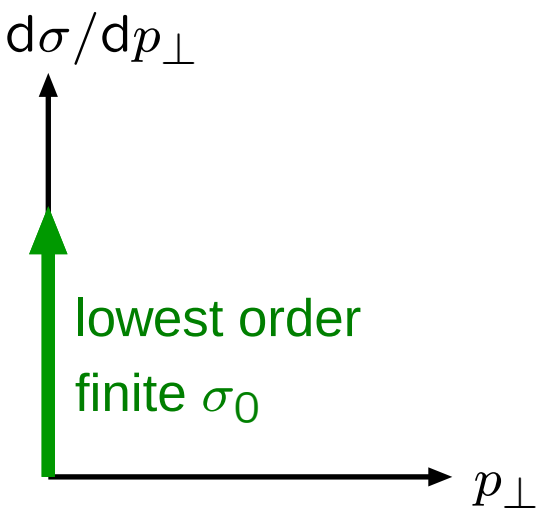

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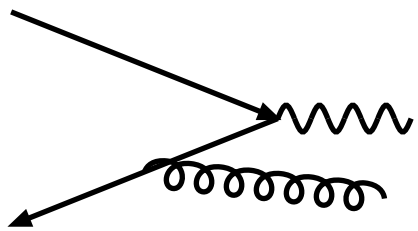
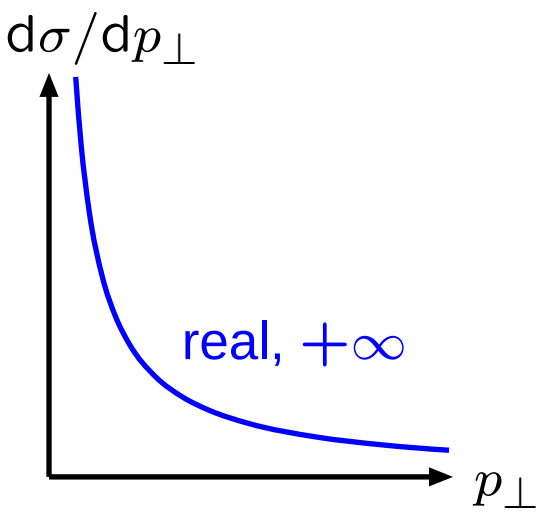
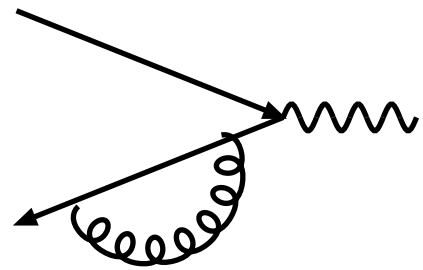
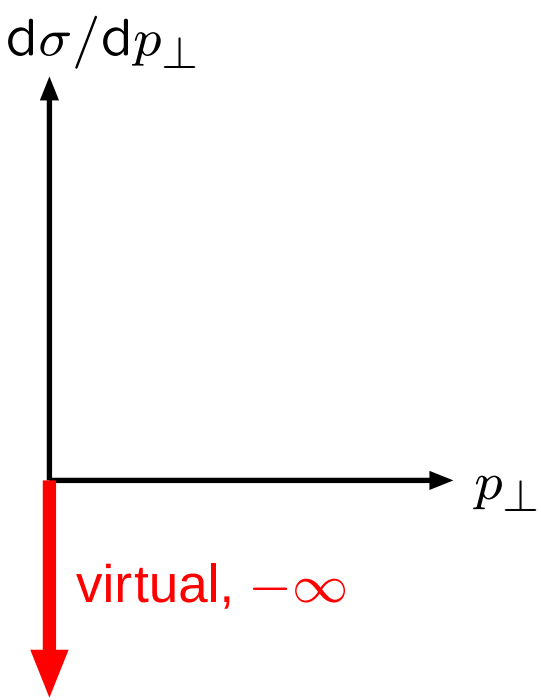


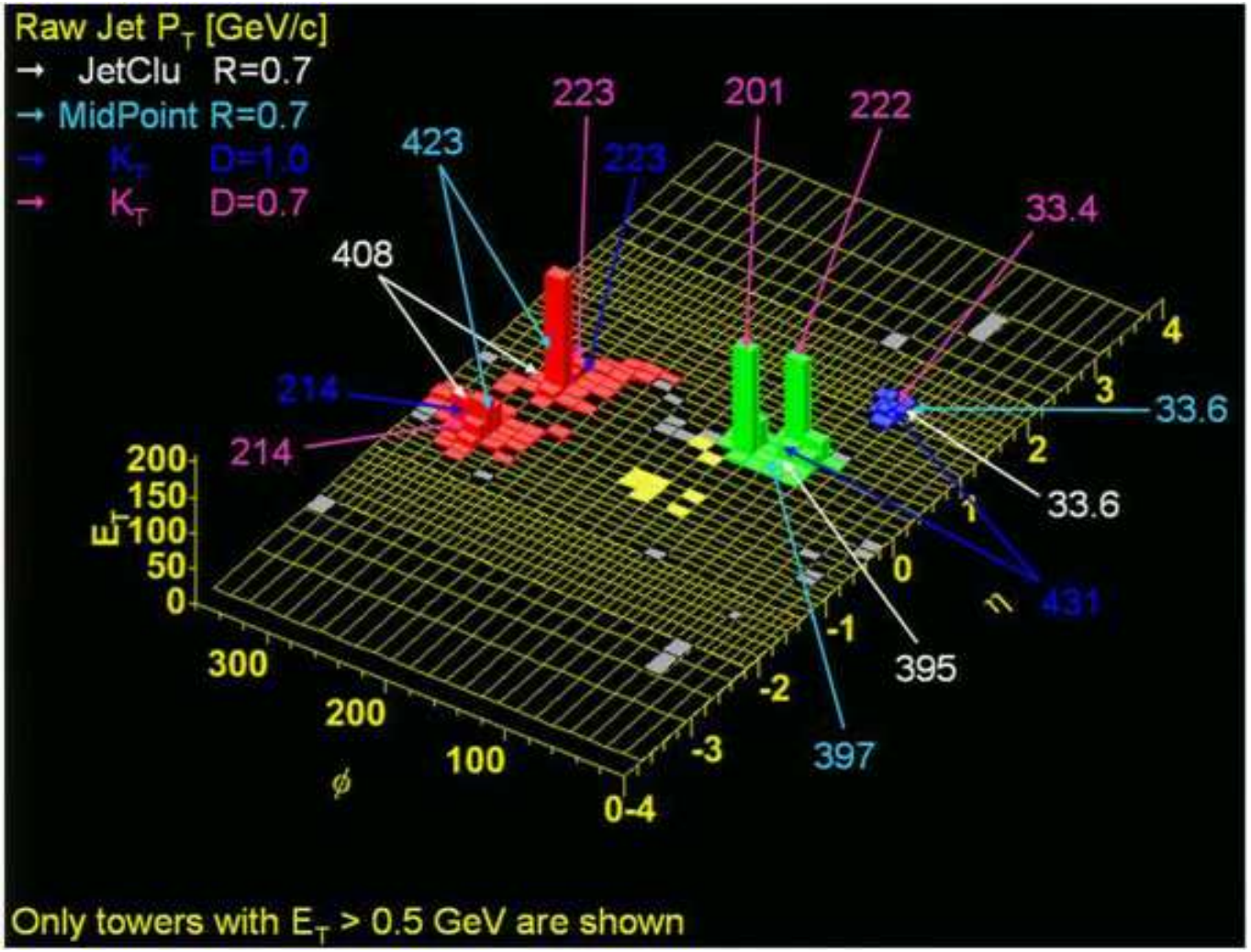


III. First-order virtual,
$\mathcal{O}(\alpha_{\mathrm{em}}\alpha_s)$:
$q\bar{q} \to Z^0$ with loops





Parton Showers



- Final-State (Timelike) Showers
- Initial-State (Spacelike) Showers
- Matching to Matrix Elements

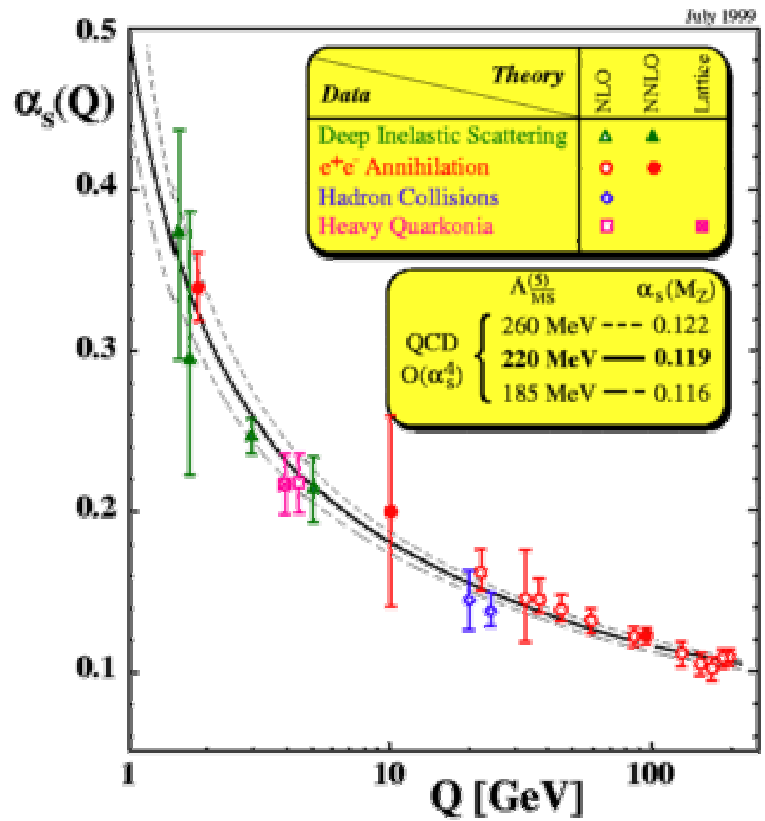
Divergences

Emission rate q $\to$ qg diverges when

- collinear: opening angle $\theta_{\mathrm{qg}} \to 0$
- soft: gluon energy $E_{\mathrm{g}} \to 0$

Almost identical to e $\to$ eγ
("bremsstrahlung"),
but QCD is non-Abelian so additionally

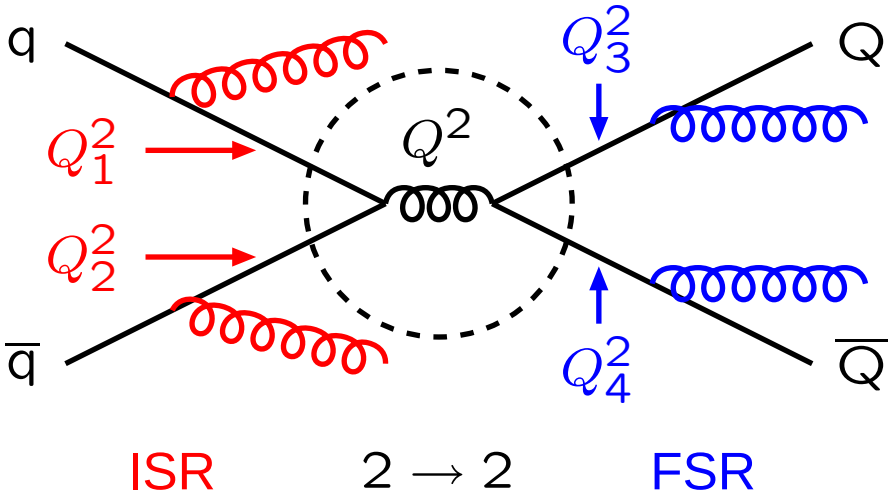
- g $\to$ gg similarly divergent
- $\alpha_{\mathrm{s}}(Q^2)$ diverges for $Q^2 \to 0$
  (actually for $Q^2 \to \Lambda^2_{\mathrm{QCD}}$)



Big probability for one emission $\Longrightarrow$ also big for several
$\Longrightarrow$ with ME's need to calculate to high order **and** with many loops
$\Longrightarrow$ extremely demanding technically (not solved!), and
involving big cancellations between positive and negative contributions.
Alternative approach: **parton showers**

The Parton-Shower Approach

$$2 \to n \;=\; (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$



FSR = Final-State Rad.;
timelike shower
$Q_i^2 \sim m^2 > 0$ decreasing

ISR = Initial-State Rad.;
spacelike shower
$Q_i^2 \sim -m^2 > 0$ increasing

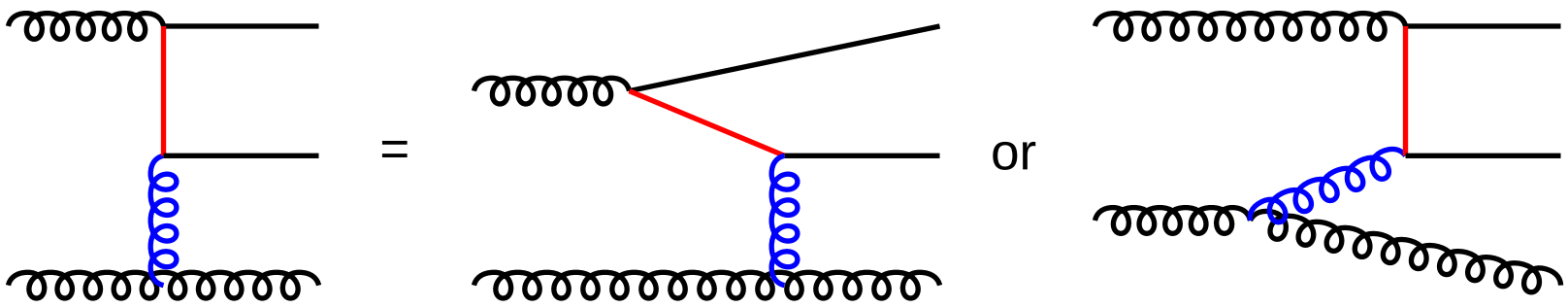
$2 \to 2$ = hard scattering (on-shell):

$$\sigma = \iiint \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}\hat{t}\; f_i(x_1, Q^2)\, f_j(x_2, Q^2)\, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

Shower evolution is viewed as a probabilistic process,
which occurs with unit total probability:
the cross section is not directly affected,
but indirectly it is, via the changed event shape

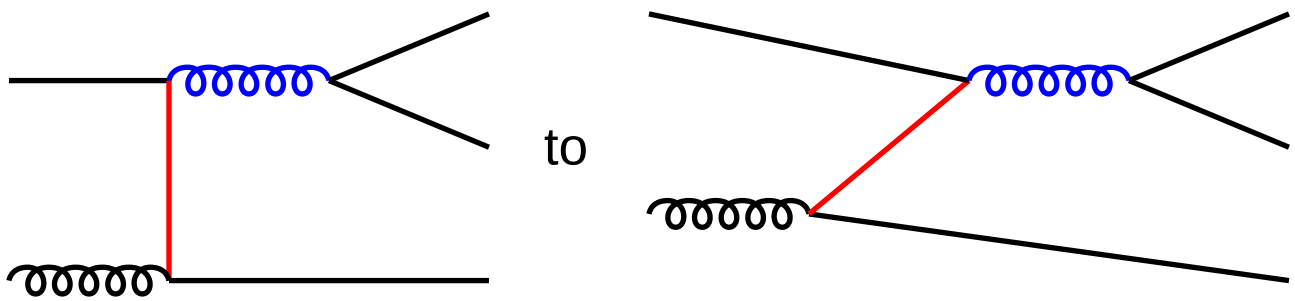
Doublecounting

A $2 \to n$ graph can be "simplified" to $2 \to 2$ in different ways:



$g \to q\bar{q} \oplus qg \to qg$ $\qquad$ $g \to gg \oplus gg \to q\bar{q}$

or deform



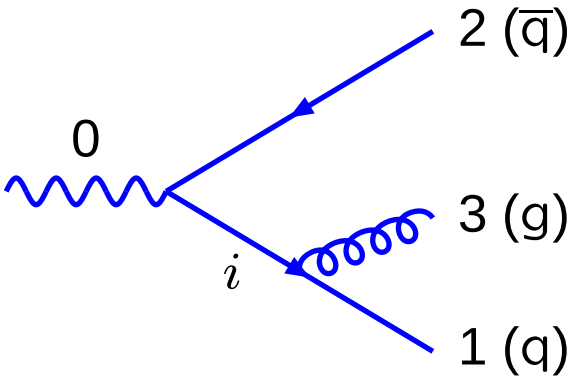
to

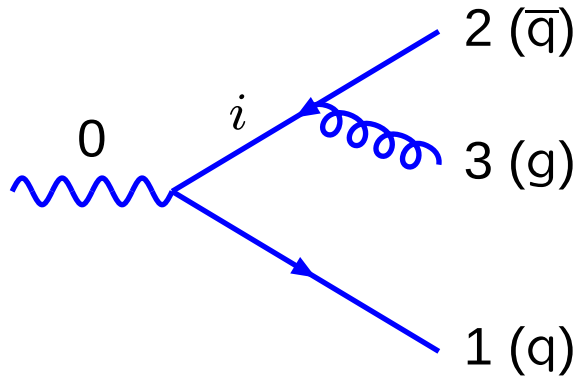
FSR $\qquad$ ISR

Do not doublecount: $2 \to 2$ = most virtual = shortest distance

Conflict: theory derivations often assume virtualities strongly ordered; interesting physics often in regions where this is not true!

From Matrix Elements to Parton Showers





$\boxed{\mathrm{e^+e^-} \to \mathrm{q\overline{q}g}}$

$x_j = 2E_j/E_{\mathrm{cm}} \Rightarrow$

$x_1 + x_2 + x_3 = 2$

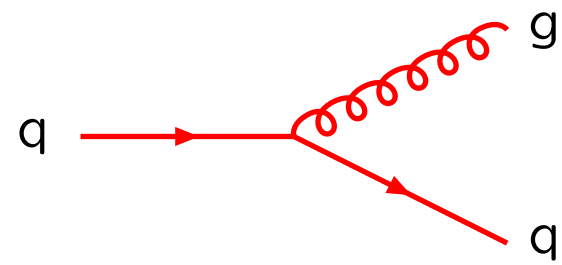
$$m_{\mathrm{q}} = 0: \quad \frac{\mathrm{d}\sigma_{\mathrm{ME}}}{\sigma_0} = \frac{\alpha_{\mathrm{s}}}{2\pi}\frac{4}{3}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\,\mathrm{d}x_1\,\mathrm{d}x_2$$

Rewrite for $x_2 \to 1$, i.e. q–g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\mathrm{cm}}^2} = \frac{Q^2}{E_{\mathrm{cm}}^2} \quad \Rightarrow \quad \mathrm{d}x_2 = \frac{\mathrm{d}Q^2}{E_{\mathrm{cm}}^2}$$

$$x_1 \approx z \quad \Rightarrow \quad \mathrm{d}x_1 \approx \mathrm{d}z$$

$$x_3 \approx 1 - z$$



$$\Rightarrow \mathrm{d}\mathcal{P} = \frac{\mathrm{d}\sigma}{\sigma_0} = \frac{\alpha_{\mathrm{s}}}{2\pi}\frac{\mathrm{d}x_2}{(1-x_2)}\frac{4}{3}\frac{x_2^2 + x_1^2}{(1-x_1)}\,\mathrm{d}x_1 \approx \frac{\alpha_{\mathrm{s}}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}\frac{4}{3}\frac{1+z^2}{1-z}\,\mathrm{d}z$$

Generalizes to DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

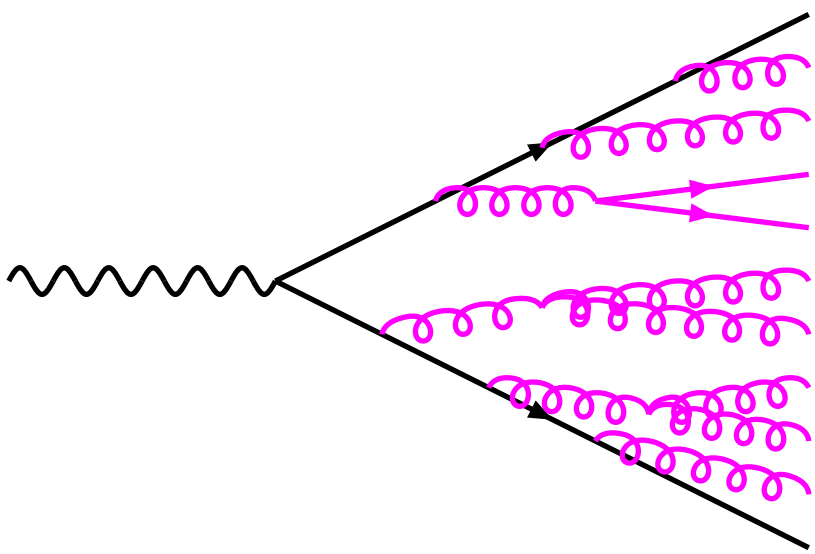
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\,\mathrm{d}z$$

$$P_{\mathrm{q}\to\mathrm{qg}} = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$P_{\mathrm{g}\to\mathrm{gg}} = 3\,\frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{\mathrm{g}\to\mathrm{q}\overline{\mathrm{q}}} = \frac{n_f}{2}(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})$$

Iteration gives final-state parton showers



Need soft/collinear cut-offs to stay away from nonperturbative physics.

Details model-dependent, e.g.

$Q > m_0 = \min(m_{ij}) \approx 1$ GeV,

$z_\mathrm{min}(E,Q) < z < z_\mathrm{max}(E,Q)$

or $p_\perp > p_{\perp\mathrm{min}} \approx 0.5$ GeV

The Sudakov Form Factor

Conservation of total probability:

$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$

"multiplicativeness" in "time" evolution:

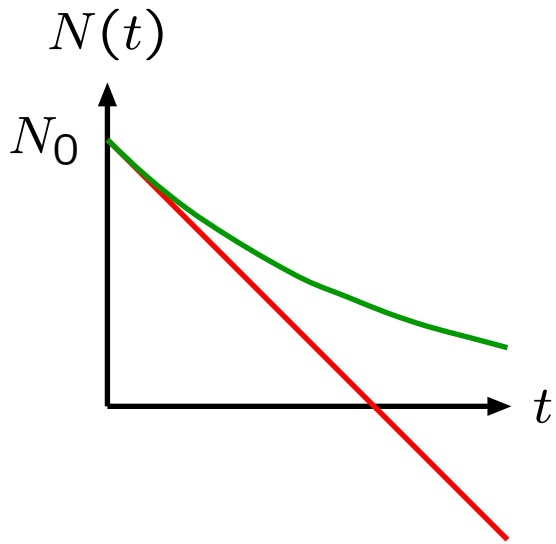
$\mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1)\, \mathcal{P}_{\text{nothing}}(T_1 < t \leq T)$

Subdivide further, with $T_i = (i/n)T$, $0 \leq i \leq n$:

$$\begin{aligned}
\mathcal{P}_{\text{nothing}}(0 < t \leq T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} \Big(1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})\Big) \\
&= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1})\right) \\
&= \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
\end{aligned}$$

$$\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T)\, \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

Example: radioactive decay of nucleus



naively: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN_0 \Rightarrow N(t) = N_0\,(1 - ct)$

depletion: a given nucleus can only decay once

correctly: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN(t) \Rightarrow N(t) = N_0\ \exp(-ct)$

generalizes to: $N(t) = N_0\ \exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

or: $\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -c(t)\ N_0\ \exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

sequence allowed: nucleus$_1 \to$ nucleus$_2 \to$ nucleus$_3 \to \ldots$

Correspondingly, with $Q \sim 1/t$ (Heisenberg)

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z\ \exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_\mathrm{max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int\frac{\alpha_\mathrm{s}}{2\pi}\,P_{a\to bc}(z')\,\mathrm{d}z'\right)$$

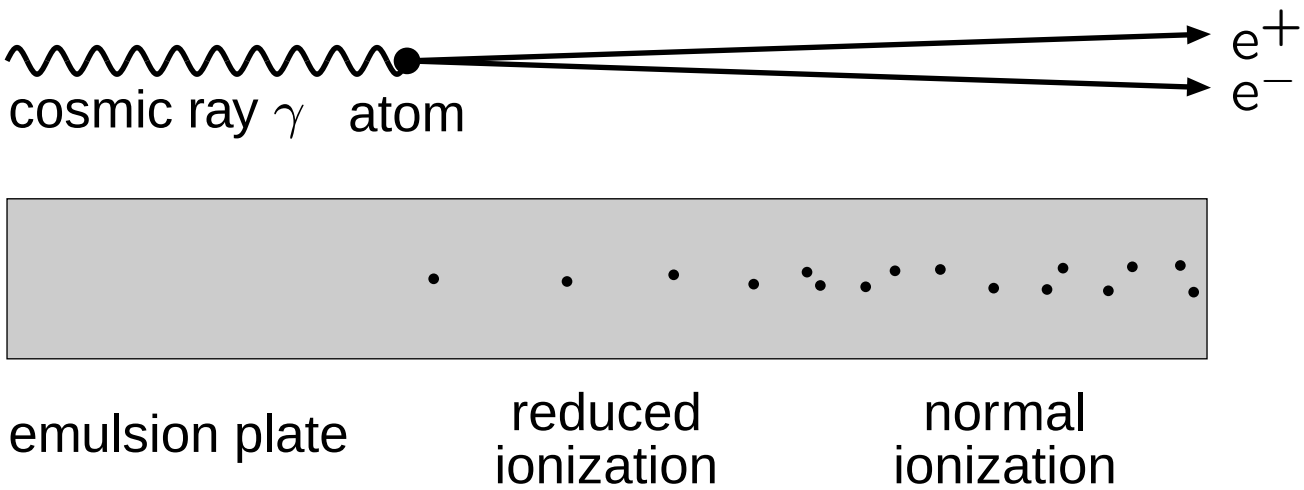
where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

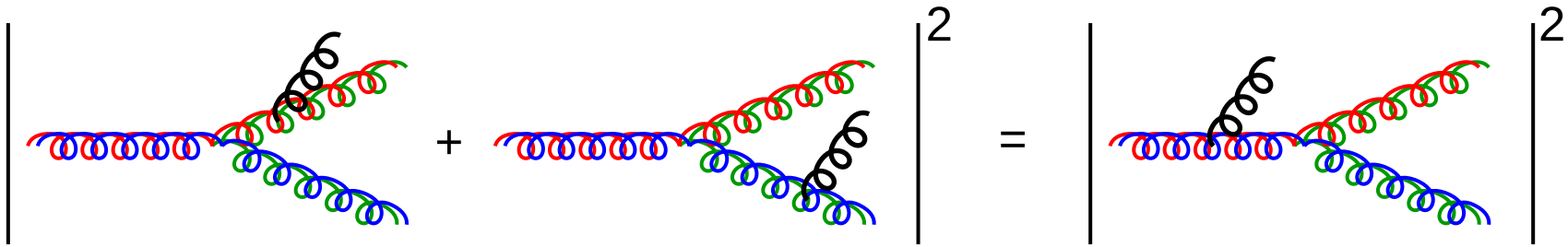
Note that $\sum_{b,c}\int \mathrm{d}Q^2 \int dz\ \mathrm{d}\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo*
($\equiv 1$ if extended over whole phase space, else possibly nothing happens)

Coherence

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission



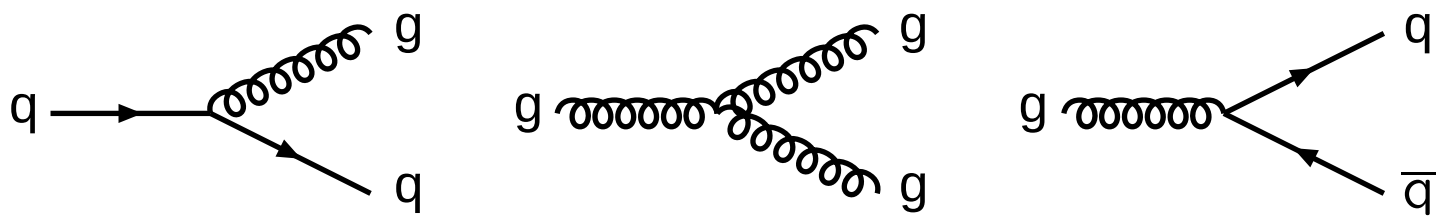
solved by
- requiring emission angles to be decreasing

or
- requiring transverse momenta to be decreasing

The Common Showering Algorithms

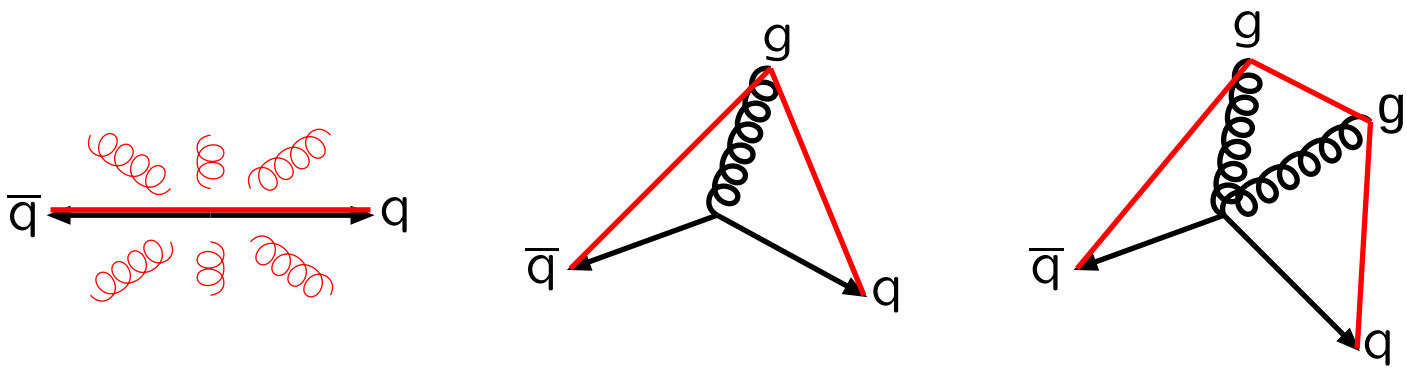
Three main approaches to showering in common use:

Two are based on the standard shower language
of $a \to bc$ successive branchings:



HERWIG: $Q^2 \approx E^2(1-\cos\theta) \approx E^2\theta^2/2$
PYTHIA: $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

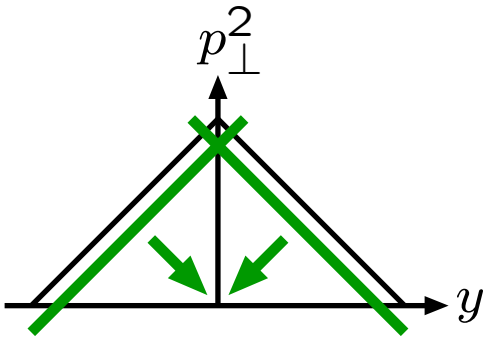
One is based on a picture of dipole emission $ab \to cde$:



ARIADNE: $Q^2 = p_\perp^2$; FSR mainly, ISR is primitive;
there instead LDCMC: sophisticated but complicated

Ordering variables in final-state radiation

PYTHIA: $Q^2 = m^2$



large mass first

$\Rightarrow$ "hardness" ordered

coherence brute force

covers phase space

ME merging simple

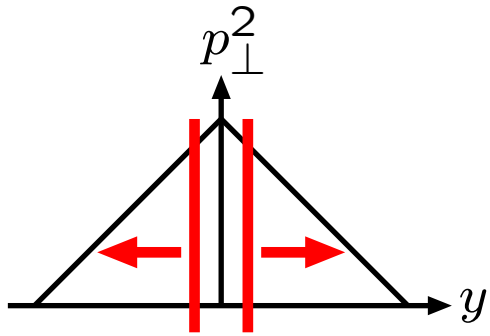
g $\longrightarrow$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $m^2 \longrightarrow -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first

$\Rightarrow$ **hardness not ordered**

coherence inherent

gaps in coverage

ME merging messy

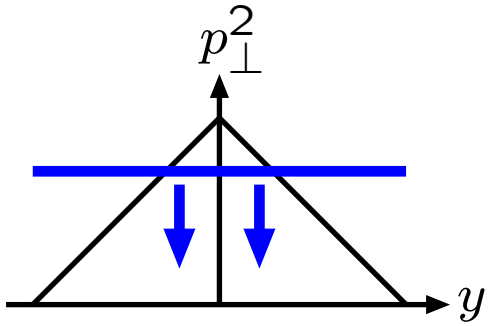
g $\longrightarrow$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $\theta \longrightarrow \theta$

ARIADNE: $Q^2 = p_\perp^2$



large $p_\perp$ first

$\Rightarrow$ "hardness" ordered

coherence inherent

covers phase space

ME merging simple

g $\longrightarrow$ q$\overline{\text{q}}$ **messy**

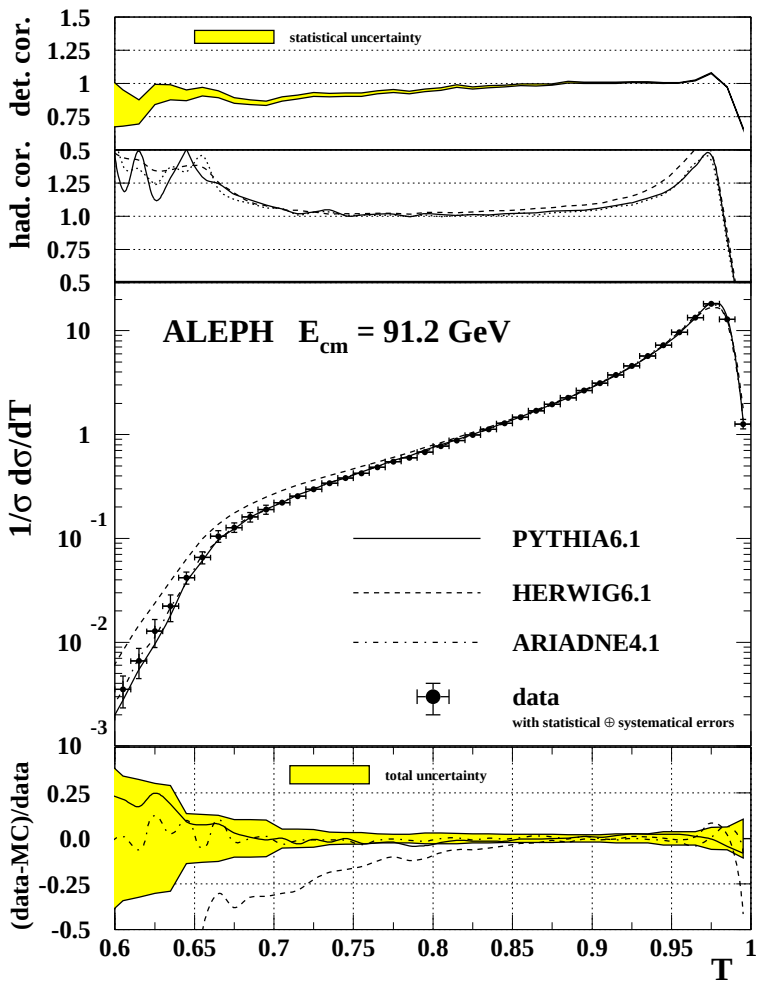
Lorentz invariant

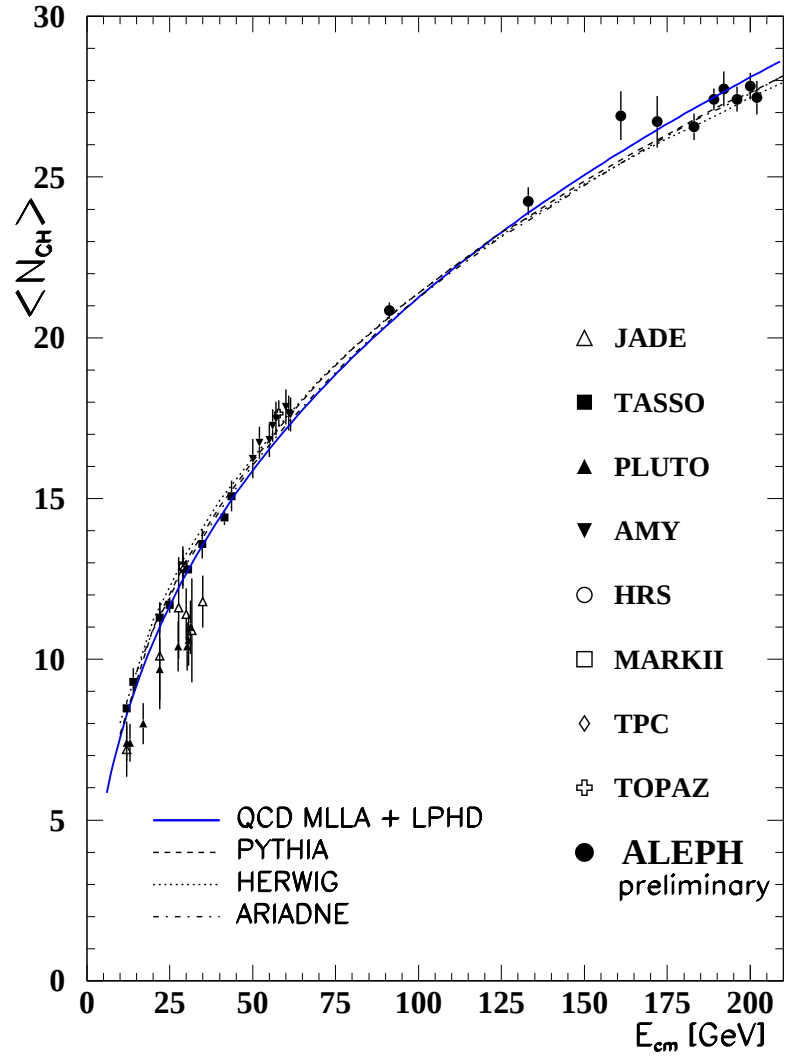
can stop/restart

ISR: more messy

Data comparisons

All three algorithms do a reasonable job of describing LEP data, but typically ARIADNE ($p_\perp^2$) $>$ PYTHIA (m^2) $>$ HERWIG (θ)





. . . and programs evolve to do even better . . .

Leading Log and Beyond

Neglecting Sudakovs, rate of one emission is:

$$\begin{aligned}\mathcal{P}_{\mathsf{q}\to\mathsf{qg}} &\approx \int \frac{\mathrm{d}Q^2}{Q^2} \int \mathrm{d}z\, \frac{\alpha_\mathsf{s}}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z} \\ &\approx \alpha_\mathsf{s}\, \ln\left(\frac{Q^2_{\mathsf{max}}}{Q^2_{\mathsf{min}}}\right) \frac{8}{3} \ln\left(\frac{1-z_{\mathsf{min}}}{1-z_{\mathsf{max}}}\right) \sim \alpha_\mathsf{s}\, \ln^2\end{aligned}$$

Rate for n emissions is of form:

$$\mathcal{P}_{\mathsf{q}\to\mathsf{q}n\mathsf{g}} \sim (\mathcal{P}_{\mathsf{q}\to\mathsf{qg}})^n \sim \alpha_\mathsf{s}^n\, \ln^{2n}$$

Next-to-leading log (NLL): inclusion of *all* corrections of type $\alpha_\mathsf{s}^n\, \ln^{2n-1}$

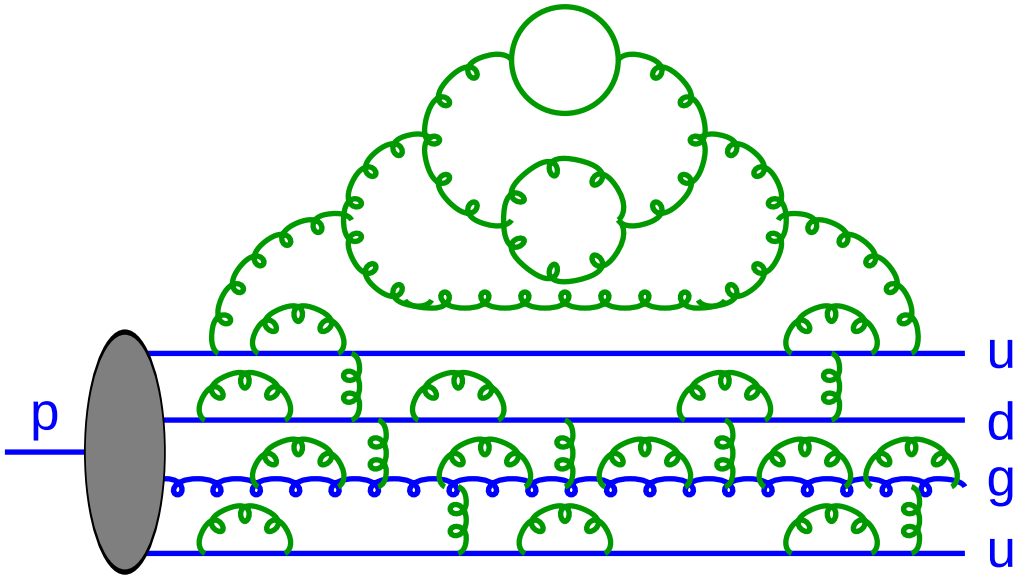
No existing generator completely NLL (NLLJET?), but

- energy-momentum conservation (and “recoil” effects)
- coherence
- $2/(1-z) \to (1+z^2)/(1-z)$
- scale choice $\alpha_\mathsf{s}(p_\perp^2)$ absorbs singular terms $\propto \ln z, \ln(1-z)$ in $\mathcal{O}(\alpha_\mathsf{s}^2)$ splitting kernels $P_{\mathsf{q}\to\mathsf{qg}}$ and $P_{\mathsf{g}\to\mathsf{gg}}$
- ...

$\Rightarrow$ far better than naive, analytical LL

Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2) =$ number density of partons i
at momentum fraction x and probing scale Q^2.

Linguistics (example):

$$F_2(x, Q^2) \;=\; \sum_i e_i^2 \, x f_i(x, Q^2)$$

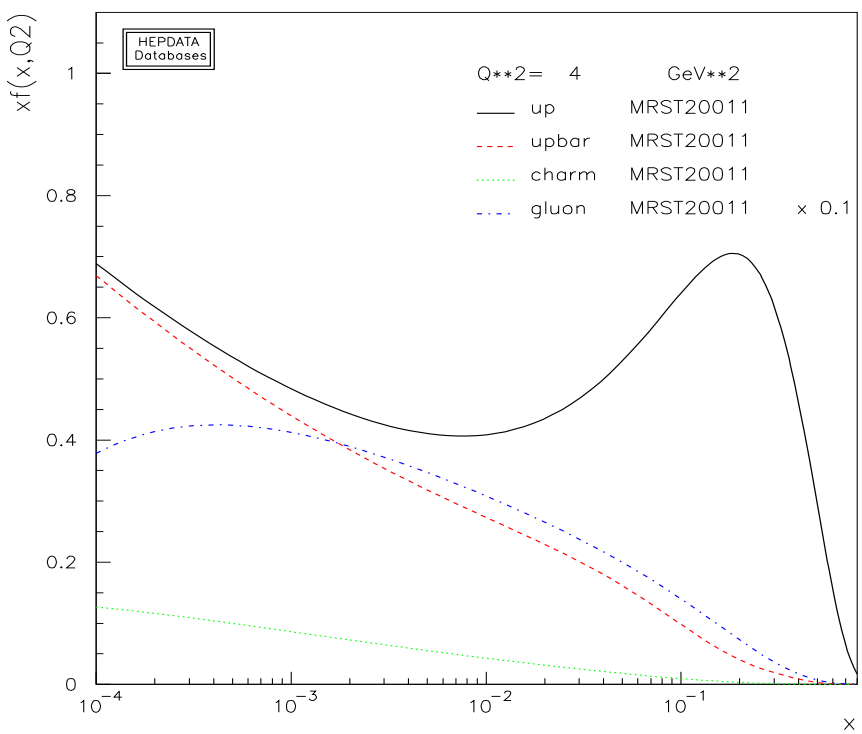
structure function parton distributions

Absolute normalization at small Q_0^2 unknown.

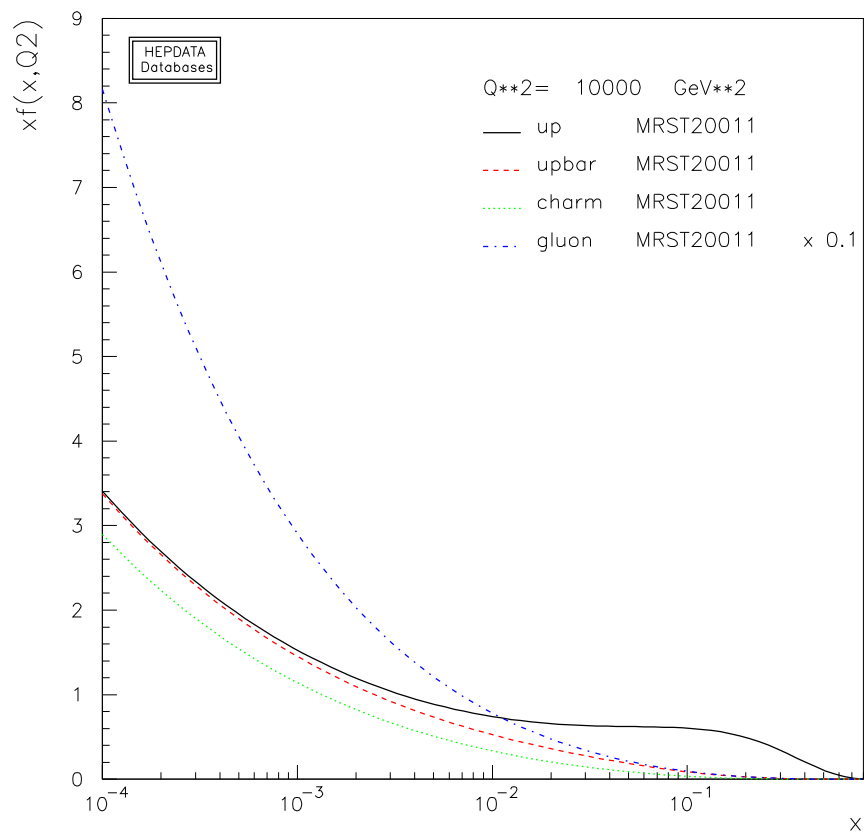
Resolution dependence by DGLAP:

$$\frac{\mathrm{d} f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z} f_a(x',Q^2) \frac{\alpha_\mathrm{s}}{2\pi} P_{a\to bc}\left(z = \frac{x}{x'}\right)$$

$Q^2 = 4$ GeV^2

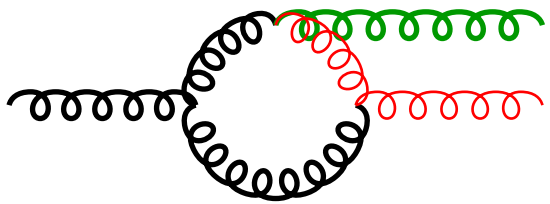


$Q^2 = 10000$ GeV^2

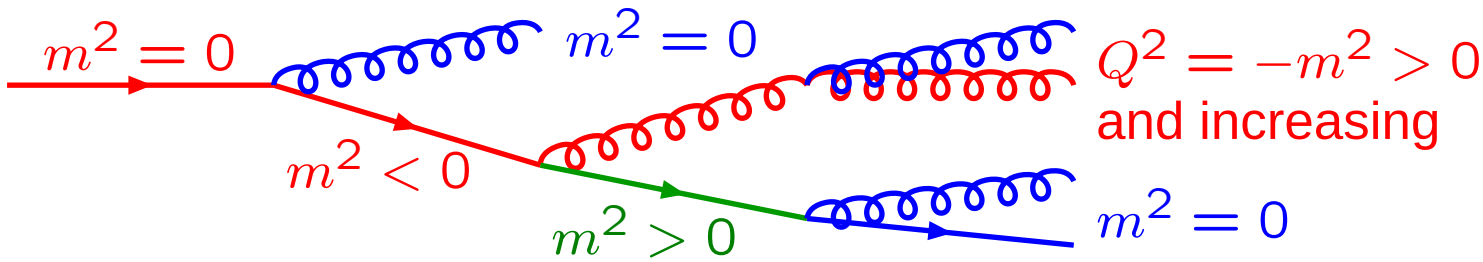


Initial-State Shower Basics

- Parton cascades in p are continuously born and recombined.
- Structure at Q is resolved at a time $t \sim 1/Q$ *before* collision.
- A hard scattering at Q^2 probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



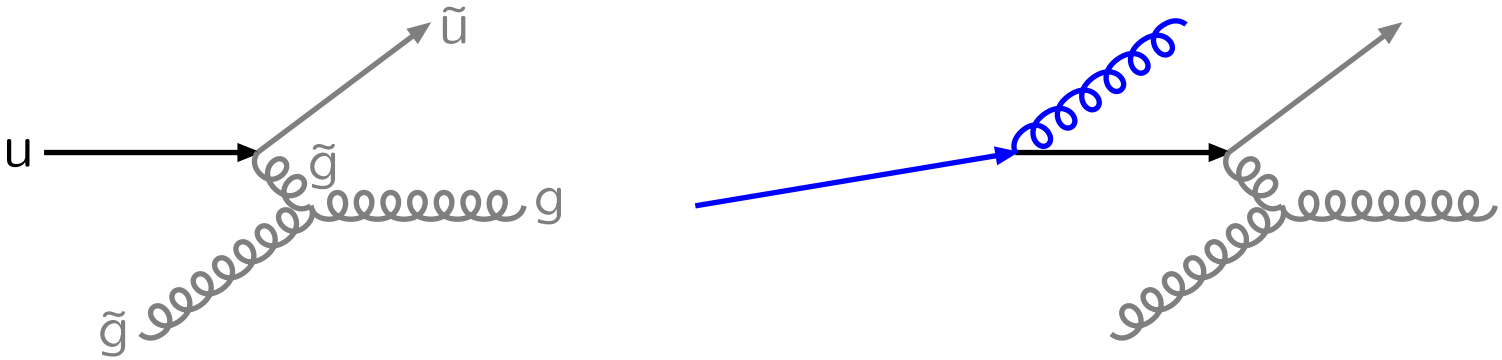
- Convenient reinterpretation:



Event generation could be addressed by **forwards evolution**:
pick a complete partonic set at low Q_0 and evolve, see what happens.
Inefficient:
1) have to evolve and check for *all* potential collisions, but 99.9. . . % inert
2) impossible to steer the production e.g. of a narrow resonance (Higgs)

Backwards evolution

Backwards evolution is viable and ∼equivalent alternative:
start at hard interaction and trace what happened "before"



Monte Carlo approach, based on *conditional probability*: recast

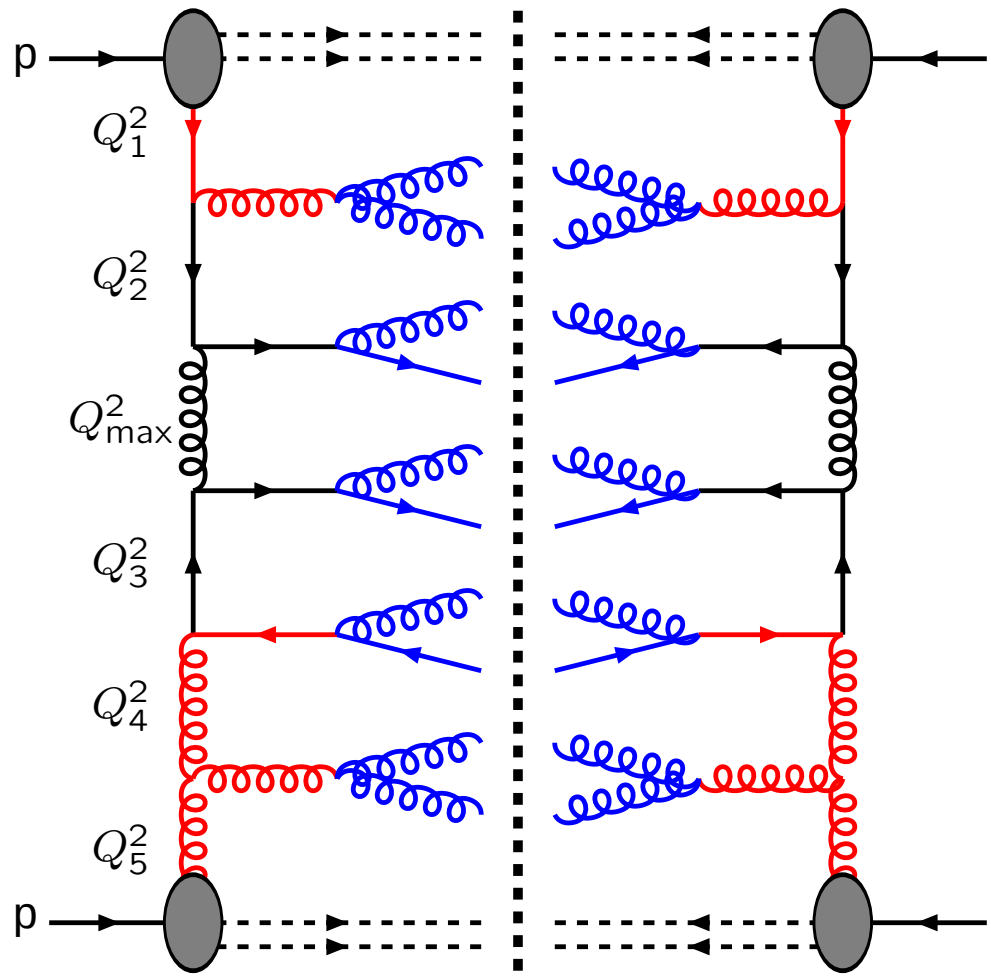
$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z}\, f_a(x',Q^2)\, \frac{\alpha_\mathrm{s}}{2\pi}\, P_{a\to bc}(z)$$

with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to

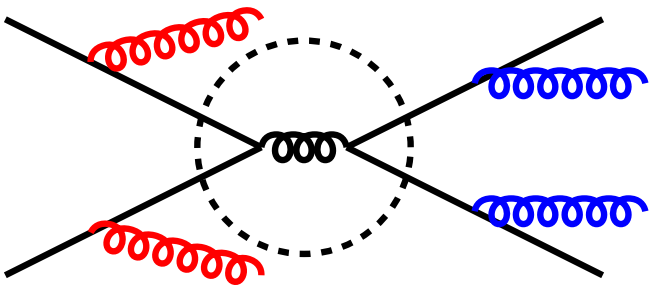
$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z\, \frac{x' f_a(x',t)}{x f_b(x,t)}\, \frac{\alpha_\mathrm{s}}{2\pi}\, P_{a\to bc}(z)$$

then solve for *de*creasing t, i.e. backwards in time,
starting at high Q^2 and moving towards lower,
with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

Ladder representation combines whole event:



cf. previously:



One possible Monte Carlo order:

1) Hard scattering

2) Initial-state shower from center outwards

3) Final-state showers

DGLAP: $Q^2_{max} > Q^2_1 > Q^2_2 \sim Q^2_0$
$Q^2_{max} > Q^2_3 > Q^2_4 > Q^2_5 \sim Q^2_0$

BFKL/CCFM: go beyond Q^2 ordering; important at small x and Q^2

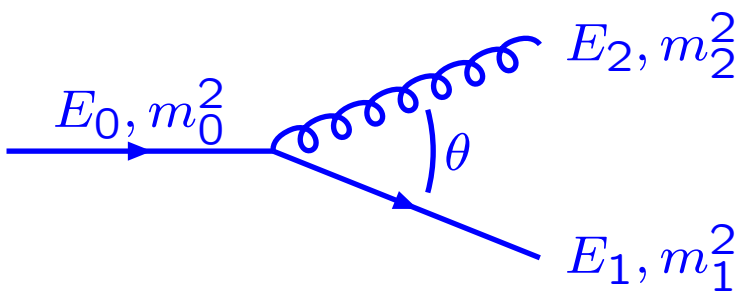
Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \;\cdot\; (\text{Sudakov})$$

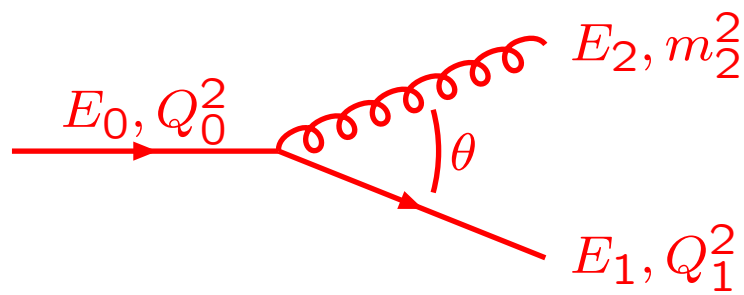
but

Final-state showers:
Q^2 timelike ($\sim m^2$)



decreasing E, m^2, θ
both daughters $m^2 \geq 0$
physics relatively simple
$\Rightarrow$ "minor" variations:
Q^2, shower vs. dipole, ...

Initial-state showers:
Q^2 spacelike ($\approx -m^2$)



decreasing E, increasing Q^2, θ
one daughter $m^2 \geq 0$, one $m^2 < 0$
physics more complicated
$\Rightarrow$ more formalisms:
DGLAP, BFKL, CCFM, GLR, ...