LUND UNIVERSITY

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Monte Carlo tools for the LHC
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Multiple Interactions
An Introduction

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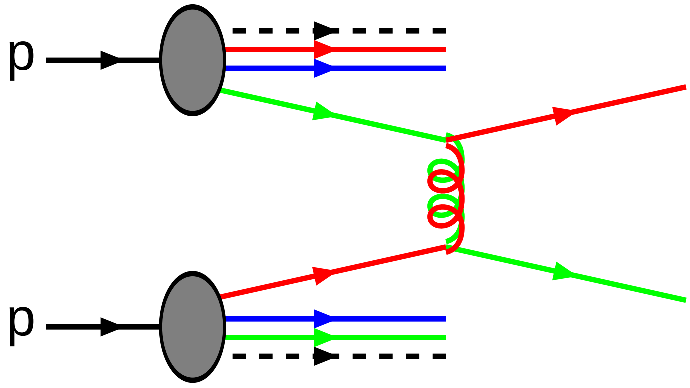
What is it and why care?
Models
Data and comparisons
Outlook

What is multiple interactions?

Cross section for $2 \to 2$ interactions,

$$\sigma = \sum_{i,j} \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\hat{t} \, f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$



$$\begin{aligned} \mathrm{q}\mathrm{q}' &\to \mathrm{q}\mathrm{q}' \\ \mathrm{q}\overline{\mathrm{q}} &\to \mathrm{q}'\overline{\mathrm{q}}' \\ \mathrm{q}\overline{\mathrm{q}} &\to \mathrm{g}\mathrm{g} \\ \mathrm{q}\mathrm{g} &\to \mathrm{q}\mathrm{g} \\ \mathrm{g}\mathrm{g} &\to \mathrm{g}\mathrm{g} \\ \mathrm{g}\mathrm{g} &\to \mathrm{q}\overline{\mathrm{q}} \end{aligned}$$

is dominated by t-channel gluon exchange,
so approximately (with $p_\perp^2 = \hat{t}\hat{u}/\hat{s} \approx |\hat{t}|$)

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp^2} \approx \iint \frac{\mathrm{d}x_1}{x_1} \frac{\mathrm{d}x_2}{x_2} F(x_1, p_\perp^2) \, F(x_2, p_\perp^2) \, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2}$$

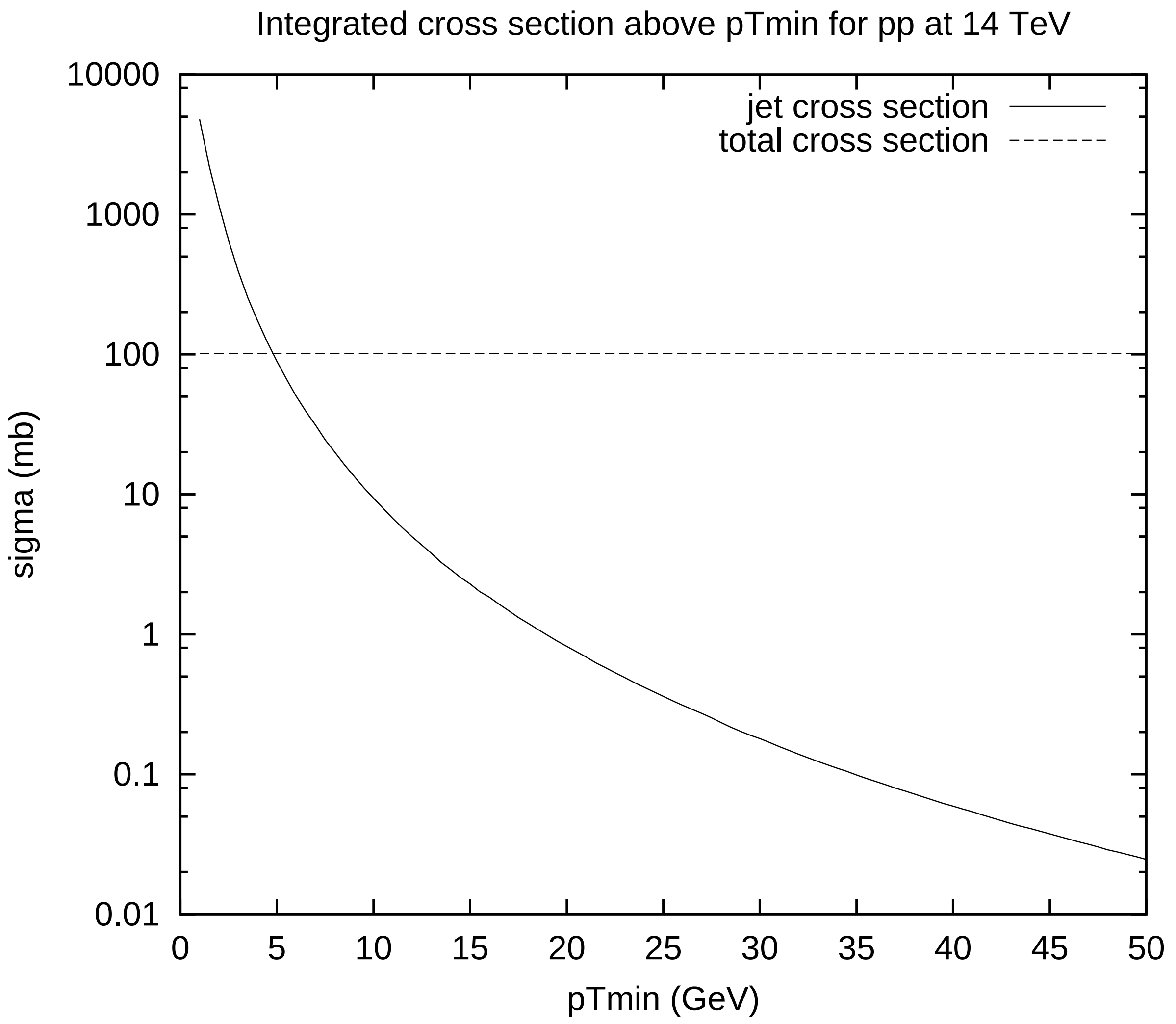
where $\displaystyle \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} = \frac{8\pi\alpha_\mathrm{s}^2(p_\perp^2)}{9p_\perp^4}$

and
$$\begin{aligned} F(x, Q^2) &= \sum_q \left(x\,q(x, Q^2) + x\,\overline{q}(x, Q^2) \right) \\ &+ \frac{9}{4} x\, g(x, Q^2) \end{aligned}$$

Thus $\displaystyle \sigma_{\mathrm{int}}(p_{\perp\mathrm{min}}) = \int_{p_{\perp\mathrm{min}}}^{\sqrt{s}/2} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp} \mathrm{d}p_\perp \propto \frac{1}{p_{\perp\mathrm{min}}^2}$

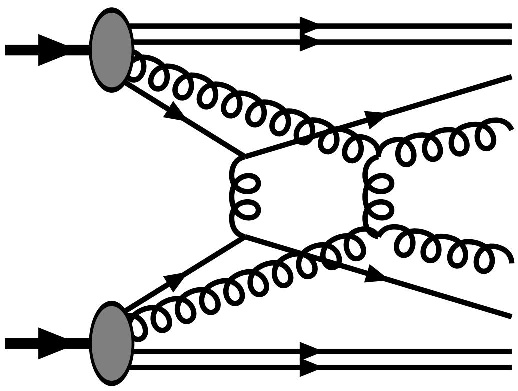
(for constant α_s and neglecting x integrals)

Numerically (for CTEQ 5L)



so $\sigma_{\mathrm{int}}(p_{\perp\mathrm{min}}) > \sigma_{\mathrm{tot}}$ for $p_{\perp\mathrm{min}} \lesssim 5$ GeV

Half a solution: many interactions per event



$$\sigma_{\mathrm{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\mathrm{int}} = \sum_{n=0}^{\infty} n\,\sigma_n$$

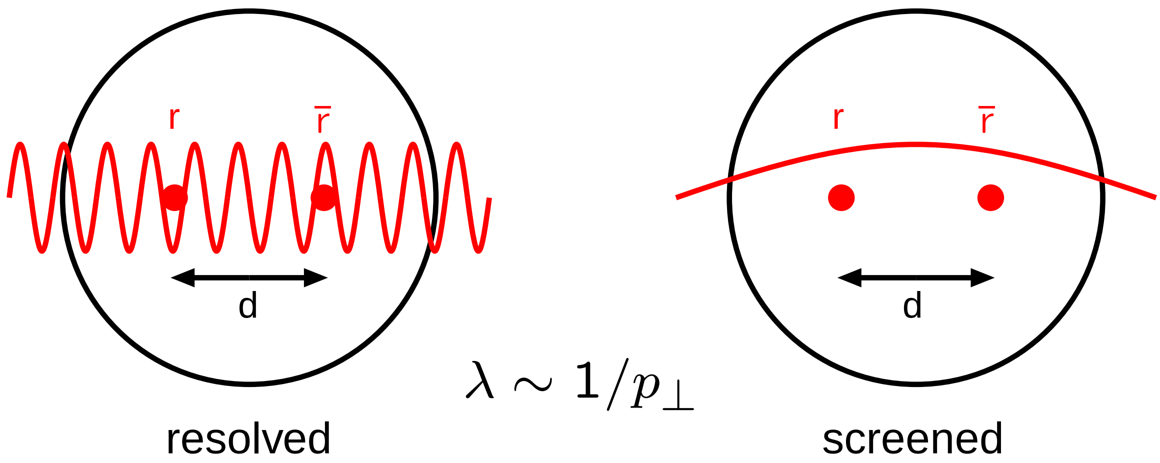
$$\sigma_{\mathrm{int}} > \sigma_{\mathrm{tot}} \Longleftrightarrow \langle n \rangle > 1$$

Other half of solution:
perturbative QCD not valid at small $p_\perp$
since q, g not asymptotic states.

Naively breakdown at

$$p_{\perp\mathrm{min}} \simeq \frac{\hbar}{r_\mathrm{p}} \approx \frac{0.2\ \mathrm{GeV}\cdot\mathrm{fm}}{0.7\ \mathrm{fm}} \approx 0.3\ \mathrm{GeV} \simeq \Lambda_\mathrm{QCD}$$

but better replace r_p by (unknown) colour screening length d in hadron



so modify

$$\begin{aligned}
\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} &\approx \frac{8\pi\alpha_\mathrm{s}^2(p_\perp^2)}{9p_\perp^4} \\
&\to \frac{8\pi\alpha_\mathrm{s}^2(p_\perp^2)}{9p_\perp^4}\,\theta\left(p_\perp - p_{\perp\mathrm{min}}\right) \qquad \text{(simpler)} \\
\text{or} &\to \frac{8\pi\alpha_\mathrm{s}^2(p_{\perp 0}^2 + p_\perp^2)}{9(p_{\perp 0}^2 + p_\perp^2)^2} \qquad \text{(more physical)}
\end{aligned}$$

where $p_{\perp\mathrm{min}}$ or $p_{\perp 0}$ are free parameters, empirically of order 2 GeV

Why care?

Energy	typical number of interactions per event
Tevatron	2 – 4
LHC	5 – 10

So multiple interactions are responsible for:

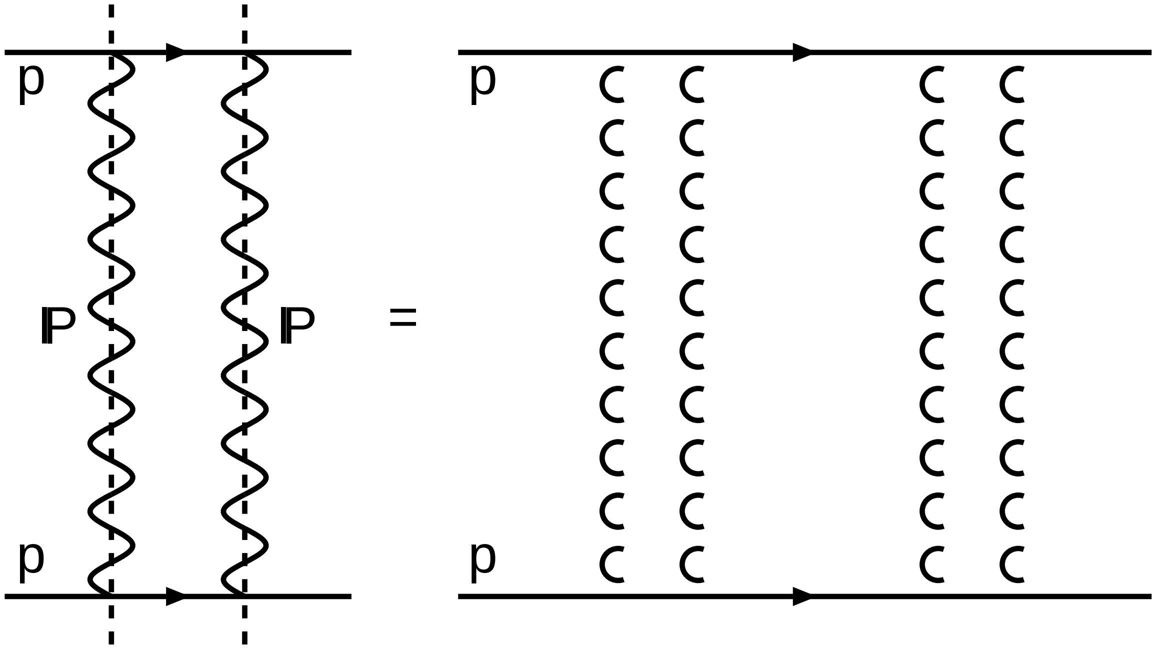
- large fraction of total multiplicity
- fluctuations to large multiplicities
- rapidity correlations in activity
- multiple (mini)jet production
- jet profile and jet pedestal
- shifts in jet energy scale

Precision physics involving jets or underlying events impossible without understanding of multiple interactions

Nontrivial example: Higgs $\to \gamma\gamma$
$M_{\gamma\gamma}^2 = 2E_{\gamma 1}E_{\gamma 2}\,(1 - \cos\theta_{\gamma\gamma})$
where $\theta_{\gamma\gamma}$ reconstructing requires to select one primary vertex among $\sim$ 30 pp events.
Is the primary vertex of most or some special charged tracks also the $\gamma\gamma$ production vertex?

Models

Soft limit is cut pomeron language:



V.A. Abramovski, O.V. Kancheli, V.N. Gribov;
A. Capella, J. Tran Thanh Van (DTU);
P. Aurenche, F.W. Bopp, J. Ranft, R. Engel (DTUJET);
K. Fialkowski, A, Kotanski;
A.B. Kaidalov, K.A. Ter Martirosyan;
F. Paige (ISAJET); . . .

Implicit in eikonal models for pp cross sections:

T.T. Chou, C.N. Yang;
C. Bourrely, J. Soffer, T.T. Wu;
P. L'Heureux, B. Margolis, P. Valin;
L. Durand, H. Pi; . . .

Double parton scattering also studied:

P.V. Landshoff, J.C. Polkinghorne;
C. Goebel, D.M. Scott, F. Halzen;
N. Paver, D.Treleani;
B. Humpert, R. Odorico; . . .

T. Sjöstrand, M. van Zijl, PRD36 (1987) 2019:
first model(s) for event properties based on perturbative multiple interactions

(1) Simple scenario:

- Sharp cut-off at $p_{\perp\mathrm{min}}$ main free parameter
- Is only a model for inelastic nondiffractive events, representing $\sigma_{\mathrm{nd}} \simeq (2/3)\sigma_{\mathrm{tot}}$
- Average number of interactions per event is

$$\langle n \rangle = \frac{\sigma_{\mathrm{int}}(p_{\perp\mathrm{min}})}{\sigma_{\mathrm{nd}}} \qquad \text{(naively; see further)}$$

- Interactions occur independently

$$\Rightarrow \text{ Poissonian statistics } \mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}$$

with fraction $\mathcal{P}_0 = e^{-\langle n \rangle}$ pure low-$p_\perp$

- Interactions generated in ordered sequence $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > \ldots$ by “Sudakov” trick

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}p_{\perp i}} = \frac{1}{\sigma_{\mathrm{nd}}} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp} \exp\left[- \int_{p_\perp}^{p_{\perp(i-1)}} \frac{1}{\sigma_{\mathrm{nd}}} \frac{\mathrm{d}\sigma}{\mathrm{d}p'_\perp} \mathrm{d}p'_\perp \right]$$

- Impose momentum conservation in PDF’s
$\Rightarrow \mathcal{P}_n$ narrower than Poissonian
- Need to simplify after first interaction:
only gg or $\mathrm{q\overline{q}}$ final states, no showers, ...

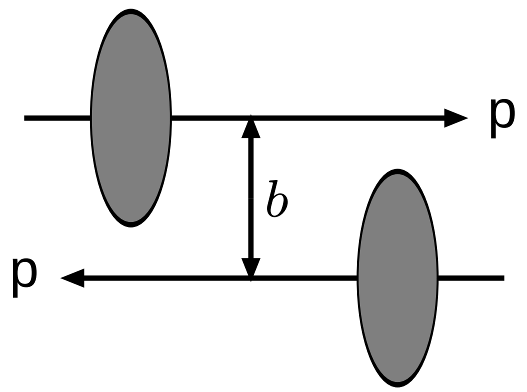
(2) More sophisticated scenario:

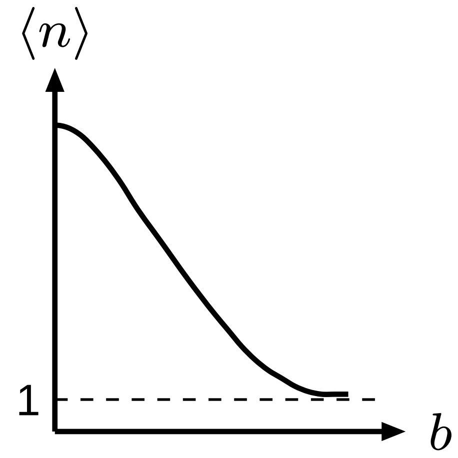
- Smooth turn-off at $p_{\perp 0}$ scale
- Require ≥ 1 interaction in an event
- Hadrons are extended:

$$\rho_{\text{matter}}(r) = N_1 \exp\left(-\frac{r^2}{r_1^2}\right) + N_2 \exp\left(-\frac{r^2}{r_2^2}\right)$$

where $r_2/r_1 \neq 1$ represents "hot spots"

- Events are distributed in impact parameter b





- Central collisions normally are more active

$\Rightarrow \mathcal{P}_n$ broader than Poissonian

- More time-consuming $(b, p_\perp)$ generation
- "Trigger bias": hard jet $\Rightarrow$ central collision

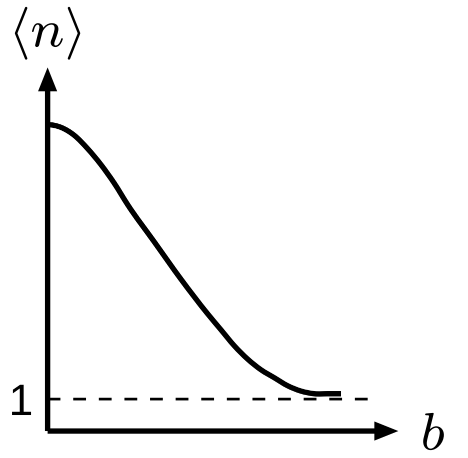
$\Rightarrow$ above-average active underlying event

- Need to simplify after first interaction, as before

The pedestal effect

Events with high-$p_\perp$ jet or W/Z or ...
have more underlying activity

"trigger bias":
high-$p_\perp$ jet
$\Rightarrow$ central collision
$\Rightarrow$ more interactions
$\Rightarrow$ larger activity
or



events with n interactions have
n chances that one of them is hard

Define matter overlap of collision

$$\mathcal{O}(b) \propto \iint \mathrm{d}^3x\,\mathrm{d}t\,\rho\left(x-\frac{b}{2},y,z-vt\right)\,\rho\left(x+\frac{b}{2},y,z+vt\right)$$

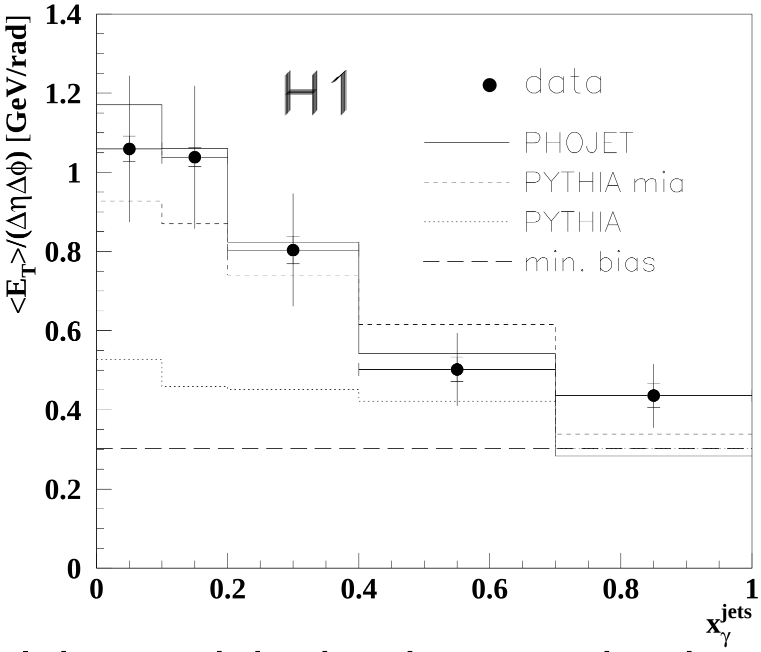
with $\rho = \rho_{\mathrm{matter,boosted}}$, such that
$\langle \mathcal{O}(b)\rangle = 1$ (kind of). Then

$$\begin{aligned}\frac{\mathrm{d}\mathcal{P}_{\mathrm{hardest}}}{\mathrm{d}^2b\,\mathrm{d}p_\perp} &= \mathcal{O}(b)\frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp}\exp\left[-\mathcal{O}(b)\int_{p_\perp}^{\sqrt{s}/2}\frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}\sigma}{\mathrm{d}p'_\perp}\mathrm{d}p'_\perp\right] \\ &\to \mathcal{O}(b)\frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp} \quad (p_\perp \text{ and } b \text{ decouples}) \\ &\quad \text{when } \int_{p_\perp}^{\sqrt{s}/2}\frac{\mathrm{d}\sigma}{\mathrm{d}p'_\perp}\mathrm{d}p'_\perp \ll \sigma_{\mathrm{nd}}\end{aligned}$$

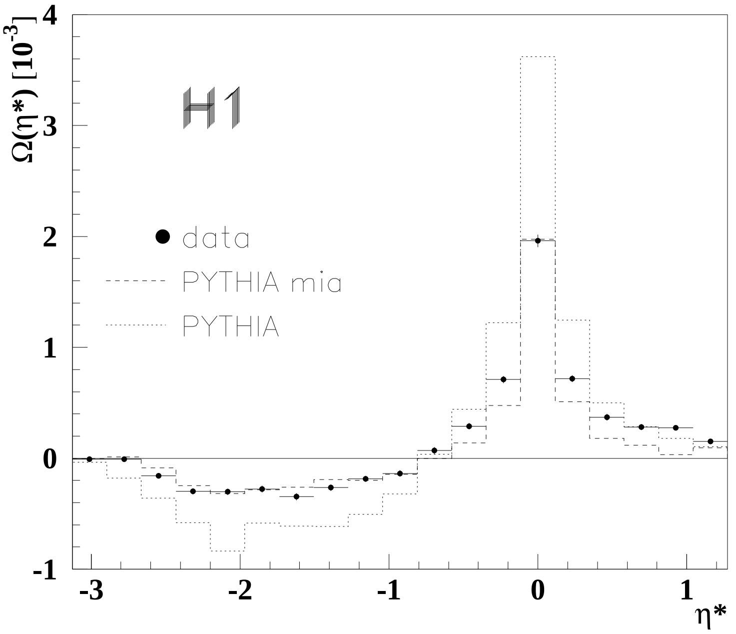
$\Longrightarrow$ centrality effect saturates at
$p_{\perp\mathrm{hard}} \sim 10$ GeV

Evidence for multiple interactions

- Width of multiplicity distribution: UA5, E735
- Forward–backward correlations: UA5
- Jet pedestal effect: UA1, H1, CDF



underlying activity in photoproduction vs. DIS



(anti)correlations in energy flow around jet

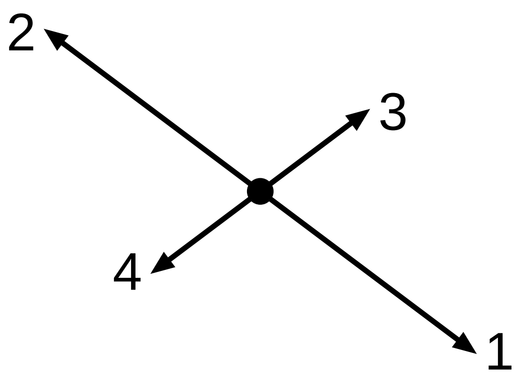
- Minijet rates: UA1

No. jets	UA1 (%)	no MI	simple	double Gaussian
1	9.96	14.30	11.51	8.88
2	3.45	2.45	2.45	2.67
3	1.12	0.22	0.32	0.74
4	0.22	0.01	0.04	0.25
5	0.05	0.00	0.00	0.07

- Direct observation: AFS, (UA2,) CDF

Order jets $\mathbf{p}_{\perp 1} > \mathbf{p}_{\perp 2} > \mathbf{p}_{\perp 3} > \mathbf{p}_{\perp 4}$ and define φ as angle between $\mathbf{p}_{\perp 1} - \mathbf{p}_{\perp 2}$ and $\mathbf{p}_{\perp 3} - \mathbf{p}_{\perp 4}$

Double Parton Scattering

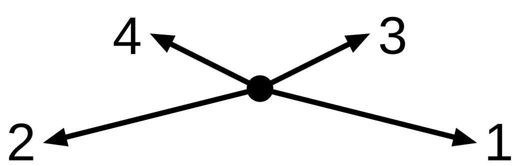


$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \approx 0$

$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \approx 0$

$\mathrm{d}\sigma/\mathrm{d}\varphi$ flat

Double BremsStrahlung



$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \gg 0$

$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \gg 0$

$\mathrm{d}\sigma/\mathrm{d}\varphi$ peaked at $\varphi \approx 0$

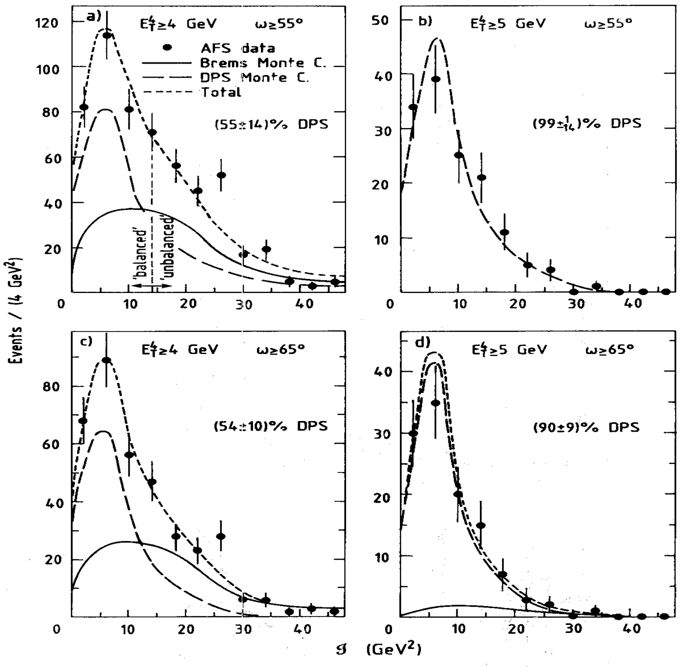


Fig. 3

AFS 4-jet analysis (pp at 63 GeV);
double bremsstrahlung subtracted:

observed	6	in arbitrary units
no MI	0	
simple MI	1	
double Gaussian	3.7	

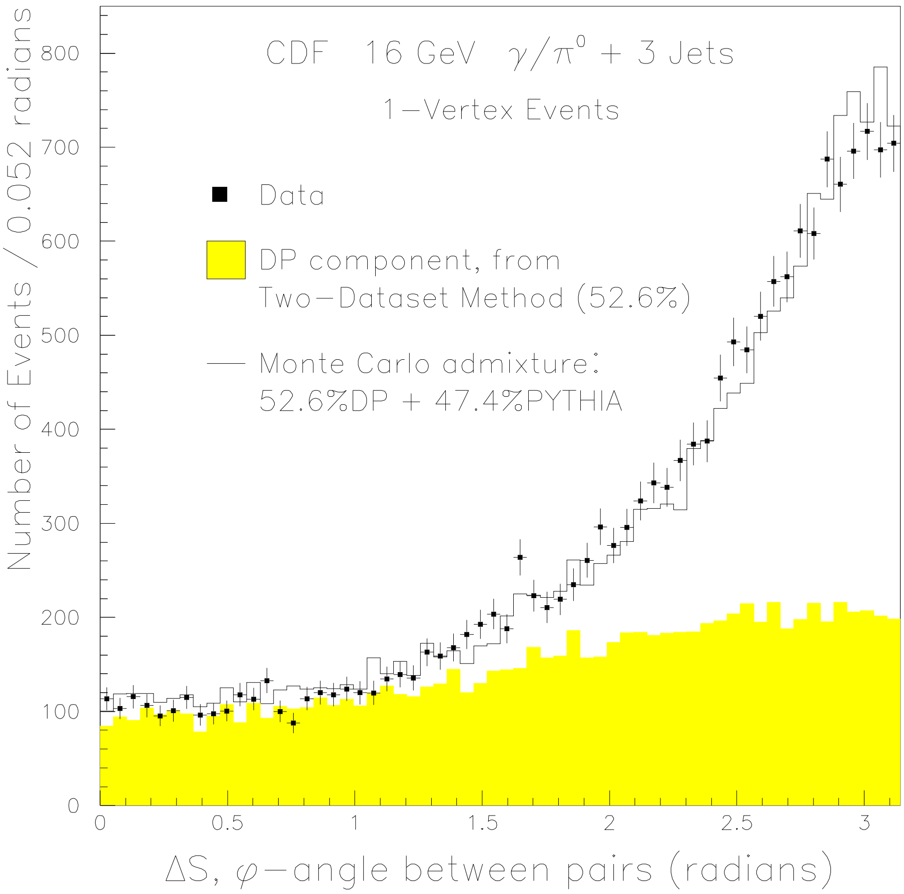
UA2 4-jet analysis (at 630 GeV):

with ansatz $\sigma_{\mathrm{DPS}} = \frac{1}{2}\frac{\sigma_{\mathrm{2jet}}^2}{\sigma_{\mathrm{eff}}}$
limit $\sigma_{\mathrm{eff}} > 8.3$ mb at 95% CL
i.e. $\sigma_{\mathrm{DPS}} < 4.5$ in 'AFS units'
. . . but best value 2.5 ± 1

CDF 4-jet analysis (at 1800 GeV):

$$\sigma_{\text{eff}} = 12.1^{+10.7}_{-5.4} \text{ mb}$$

CDF 3-jet + prompt photon analysis:

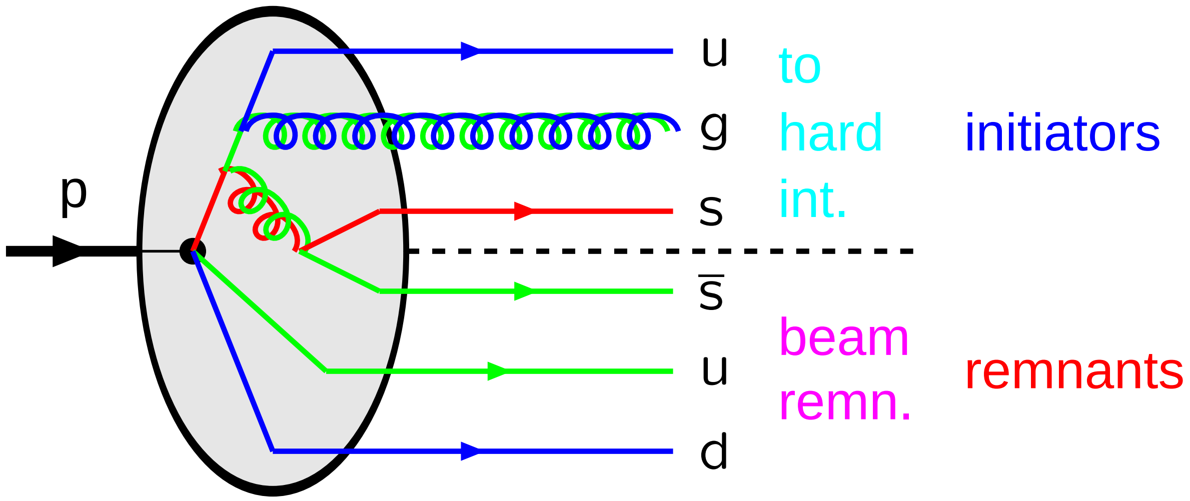


Warning!
PYTHIA
run with
multiple
interactions
off

$$\sigma_{\text{DPS}} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}} \qquad \text{for} A \neq B$$

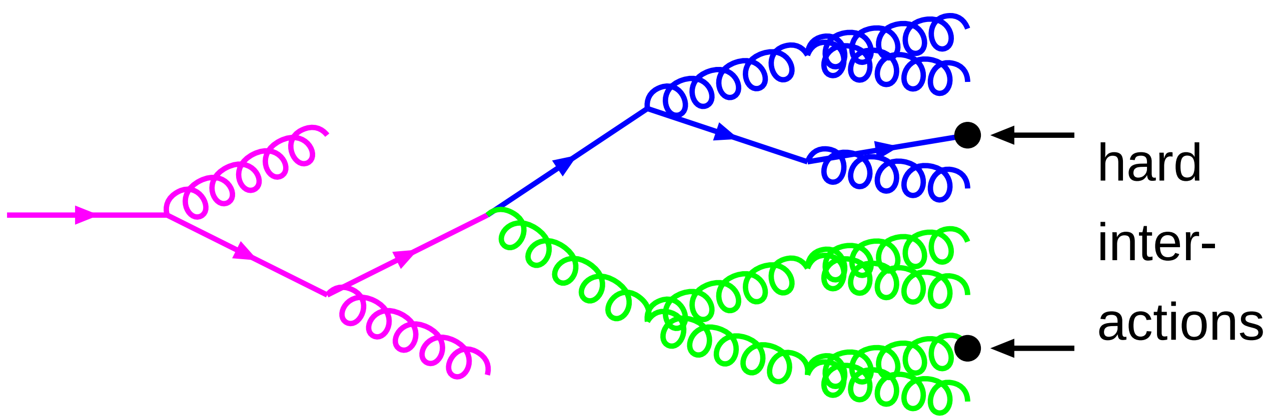
$$\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}$$

Initiators and Remnants

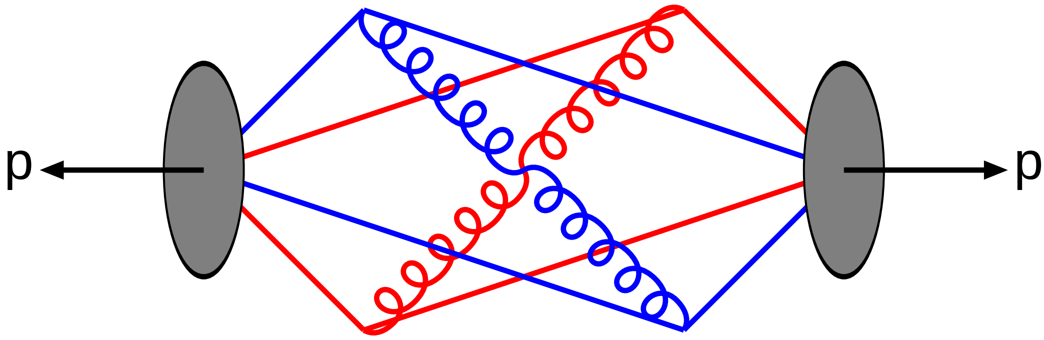


Need to assign:

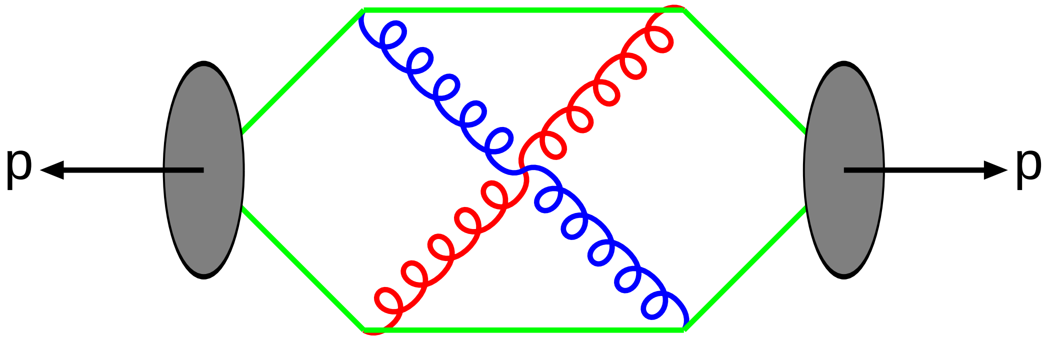
- correlated flavours
  (Where does the baryon number go?)
- correlated $x_i = p_{zi}/p_{z\mathrm{tot}}$
- correlated primordial $k_{\perp i}$
- correlated colours
- correlated showers (?)



$\langle p_\perp \rangle(n_{\text{ch}})$ and colour correlations

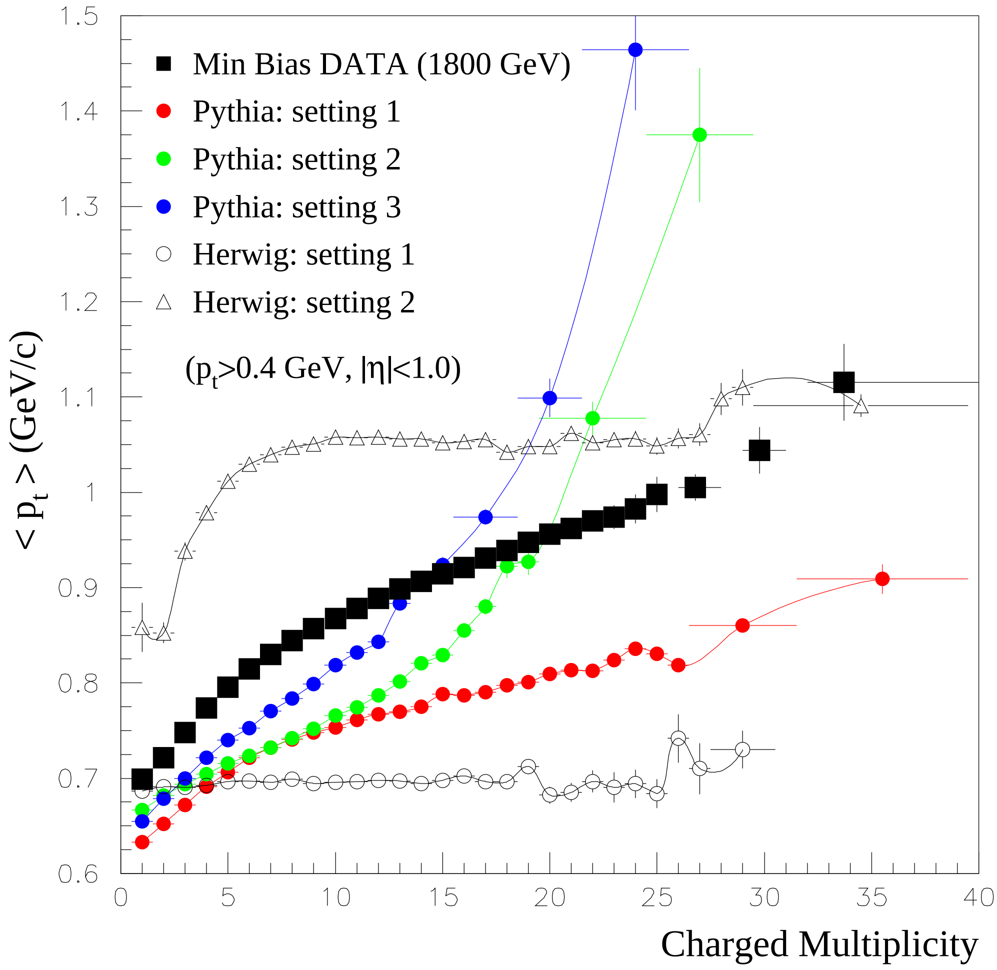


$l_{\text{string}} \nearrow \;\Rightarrow\; n_{\text{ch}}/n_{\text{int}} \nearrow \;\Rightarrow\; p_{\perp\text{min}} \nearrow \;\Rightarrow$
$n_{\text{int}} \searrow \;\Rightarrow\; p_{\perp\text{pert}} \searrow \;\Rightarrow\; \langle p_\perp \rangle(n_{\text{ch}}) \sim \text{flat}$

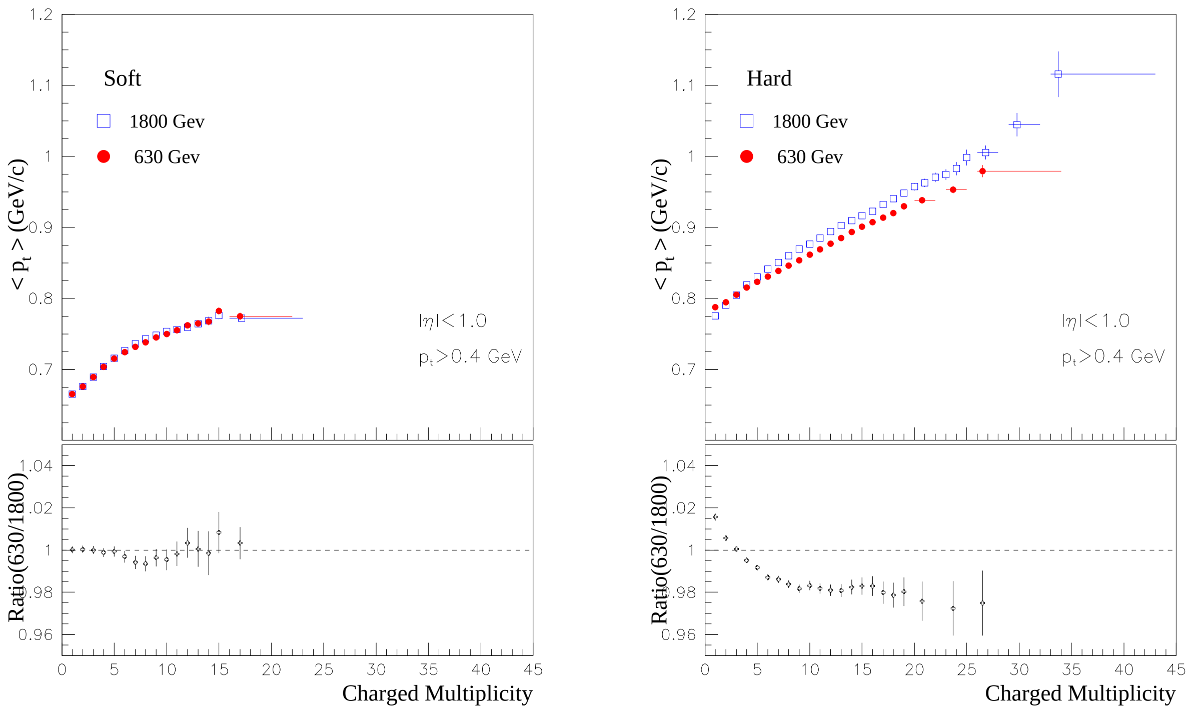


$l_{\text{string}} \searrow \;\Rightarrow\; n_{\text{ch}}/n_{\text{int}} \searrow \;\Rightarrow\; p_{\perp\text{min}} \searrow \;\Rightarrow$
$n_{\text{int}} \nearrow \;\Rightarrow\; p_{\perp\text{pert}} \nearrow \;\Rightarrow\; \langle p_\perp \rangle(n_{\text{ch}}) \nearrow$

. . . but need guiding principles
or other mechanism to accomplish the same


Min Bias DATA (1800 GeV)
Pythia: setting 1
Pythia: setting 2
Pythia: setting 3
Herwig: setting 1
Herwig: setting 2
(p_t>0.4 GeV, |η|<1.0)
< p_t > (GeV/c)
Charged Multiplicity

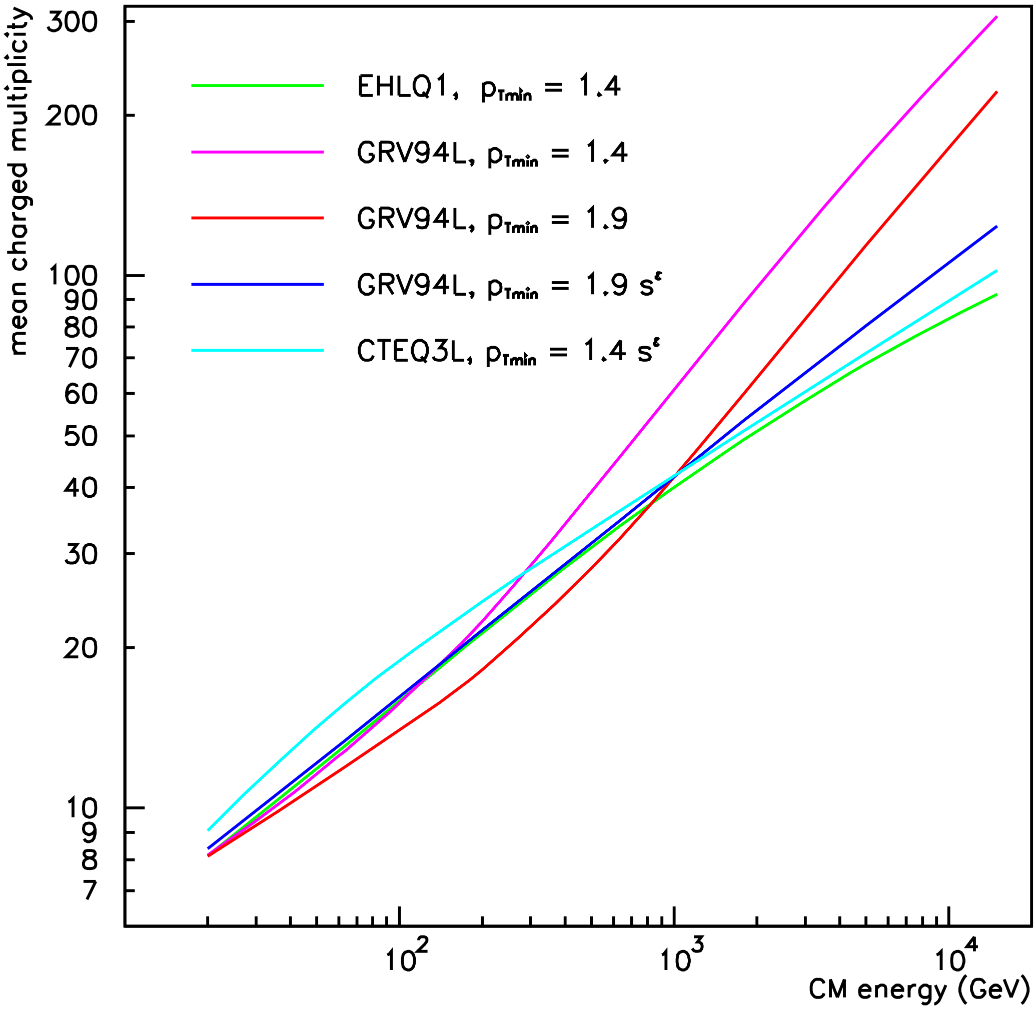
CDF


Soft
1800 Gev
630 Gev
|η|<1.0
p_t>0.4 GeV
< p_t > (GeV/c)
Ratio(630/1800)
Charged Multiplicity
Hard
1800 Gev
630 Gev
|η|<1.0
p_t>0.4 GeV
< p_t > (GeV/c)
Ratio(630/1800)
Charged Multiplicity

Energy dependence of $p_{\perp\mathrm{min}}$ and $p_{\perp 0}$

Larger collision energy
$\Rightarrow$ probe parton ($\approx$ gluon) density at smaller x
$\Rightarrow$ smaller colour screening length d
$\Rightarrow$ larger $p_{\perp\mathrm{min}}$ or $p_{\perp 0}$

Post-HERA PDF fits steeper at small x
$\Rightarrow$ stronger energy dependence



Current PYTHIA default, tied to CTEQ 5L, is

$$p_{\perp\mathrm{min}}(s) = 1.9 \text{ GeV} \left(\frac{s}{1 \text{ TeV}^2}\right)^{0.08}$$

How to make progress?

Need model building $\Leftrightarrow$ experimental tests

Reference samples over wide energy range:

- $\sim$ 20 GeV: fixed target
- $\sim$ 63 GeV: ISR
- $\sim$ 200 GeV: Sp$\overline{\text{p}}$S, RHIC
- $\sim$ 630 GeV: Sp$\overline{\text{p}}$S, Tevatron
- $\sim$ 2 TeV: Tevatron

Corrected and reliable distributions
(over all or well-defined phase space) of:

- n_{ch}
- y and $p_\perp$ single-particle spectra
- y correlations, $p_\perp$&φ correlations
- $\langle p_\perp \rangle (n_{\text{ch}})$
- n_{minijet} for different $E_{\perp\text{jet}}$
- jet profile and pedestal
- rapidity gap size and position
- other interesting properties?

in form usable to outsiders

$\Longrightarrow$ JetWeb?

Outlook

⋆ Multiple interactions concept compelling;
it *has to* exist at some level

⋆ By now
- strong direct evidence
- overwhelming indirect evidence

⋆ Understanding of multiple interactions
crucial for precision physics involving jets

⋆ Many details uncertain
- $p_{\perp \mathrm{min}}/p_{\perp 0}$ cut-off
- impact parameter picture
- energy dependence
- multiparton densities in incoming hadron
- colour correlations between scatterings
- interferences between showers
- ...

⋆ Above physics aspects must be present

If a model is simple, it is wrong!

...so let's keep on complicating matters!