The Parton-Shower Approach

\[ 2 \rightarrow n = (2 \rightarrow 2) \oplus \text{ISR} \oplus \text{FSR} \]

**FSR** = Final-State Radiation = timelike shower

\[ Q_i^2 \sim m^2 > 0 \text{ decreasing} \]

**ISR** = Initial-State Radiation = spacelike showers

\[ Q_i^2 \sim -m^2 > 0 \text{ increasing} \]
Why “time” like and “space” like?

Consider four-momentum conservation in a branching $a \rightarrow b c$

\[ p_{\perp a} = 0 \Rightarrow p_{\perp c} = -p_{\perp b} \]

\[ p_+ = E + p_L \Rightarrow p_{+a} = p_{+b} + p_{+c} \]

\[ p_- = E - p_L \Rightarrow p_{-a} = p_{-b} + p_{-c} \]

Define $p_{+b} = z p_{+a}$, $p_{+c} = (1 - z) p_{+a}$

Use $p_+ p_- = E^2 - p_L^2 = m^2 + p_{\perp}^2$

\[
\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{z p_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1 - z) p_{+a}}
\]

\[ \Rightarrow m_a^2 = \frac{m_b^2 + p_{\perp}^2}{z} + \frac{m_c^2 + p_{\perp}^2}{1 - z} = \frac{m_b^2}{z} + \frac{m_c^2}{1 - z} + \frac{p_{\perp}^2}{z(1 - z)} \]

Final-state shower: $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_{\perp}^2}{z(1 - z)} > 0 \Rightarrow$ timelike

Initial-state shower: $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_{\perp}^2}{1 - z} < 0 \Rightarrow$ spacelike
Showers and cross sections

Shower evolution is viewed as a probabilistic process, which occurs with unit total probability: the cross section is not directly affected.
Shower evolution is viewed as a probabilistic process, which occurs with unit total probability: *the cross section is not directly affected*

However, more complicated than that

- PDF evolution $\approx$ showers $\Rightarrow$ enters in convoluted cross section, e.g. for $2 \rightarrow 2$ processes

$$\sigma = \iiint dx_1 \, dx_2 \, d\hat{t} \, f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{d\hat{\sigma}_{ij}}{d\hat{t}}$$

- Shower affects event shape
  
  E.g. start from 2-jet event with $p_{\perp 1} = p_{\perp 2} = 100$ GeV. ISR gives third jet, plus recoil to existing two, so $p_{\perp 1} = 110$ GeV, $p_{\perp 2} = 90$ GeV, $p_{\perp 1} = 20$ GeV:
  
  - inclusive $p_{\perp \text{jet}}$ spectrum goes up
  - hardest $p_{\perp \text{jet}}$ spectrum goes up
  - two-jets with both jets above some $p_{\perp \text{min}}$ comes down
  - three-jet rate goes up
Doublecounting

A $2 \rightarrow n$ graph can be “simplified” to $2 \rightarrow 2$ in different ways:

- $g \rightarrow q\bar{q} \oplus qg \rightarrow qg$
- $g \rightarrow gg \oplus gg \rightarrow q\bar{q}$

or deform

FSR

or

ISR

Do not doublecount: $2 \rightarrow 2 = $ most virtual $= $ shortest distance

(detailed handling of borders $\Rightarrow$ match & merge)
Final-state radiation

Standard process $e^+e^- \rightarrow q\bar{q}g$ by two Feynman diagrams:

\[
x_i = \frac{2E_i}{E_{cm}}
\]

\[
x_1 + x_2 + x_3 = 2
\]

\[
\frac{d\sigma_{ME}}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \, dx_1 \, dx_2
\]
Final-state radiation

Standard process \(\text{e}^+\text{e}^- \rightarrow q\bar{q}g\) by two Feynman diagrams:

\[
\begin{align*}
\frac{d\sigma_{\text{ME}}}{\sigma_0} &= \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \, dx_1 \, dx_2 \\
\end{align*}
\]

Convenient (but arbitrary) subdivision to “split” radiation:

\[
\frac{1}{(1-x_1)(1-x_2)} \frac{(1-x_1) + (1-x_2)}{x_3} = \frac{1}{(1-x_2)x_3} + \frac{1}{(1-x_1)x_3}
\]

\[
x_i = \frac{2E_i}{E_{cm}}
\]

\(x_1 + x_2 + x_3 = 2\)
Rewrite for $x_2 \rightarrow 1$, i.e. $q-g$ collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\text{cm}}^2} = \frac{Q^2}{E_{\text{cm}}^2} \Rightarrow dx_2 = \frac{dQ^2}{E_{\text{cm}}^2}$$

define $z$ as fraction $q$ retains in branching $q \rightarrow qg$

$$x_1 \approx z \Rightarrow dx_1 \approx dz$$

$$x_3 \approx 1 - z$$

$$\Rightarrow d\mathcal{P} = \frac{d\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{dx_2}{(1 - x_2)} \frac{4}{3} \frac{x_2^2 + x_1^2}{(1 - x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \frac{4}{3} \frac{1 + z^2}{1 - z} dz$$

In limit $x_1 \rightarrow 1$ same result, but for $\bar{q} \rightarrow \bar{q}g$.

$$dQ^2/Q^2 = dm^2/m^2: \text{“mass (or collinear) singularity”}$$

$$dz/(1 - z) = d\omega/\omega \text{ “soft singularity”}$$
The DGLAP equations

Generalizes to

\[
\begin{align*}
\frac{d \mathcal{P}_{a \rightarrow bc}}{2\pi} &= \frac{\alpha_s}{Q^2} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) \, dz \\

P_{q \rightarrow qg} &= \frac{4}{3} \left( \frac{1 + z^2}{1 - z} \right) \\

P_{g \rightarrow gg} &= \frac{3}{2} \left( \frac{(1 - z(1 - z))^2}{z(1 - z)} \right) \\

P_{g \rightarrow q\bar{q}} &= \frac{n_f}{2} \left( z^2 + (1 - z)^2 \right) \quad (n_f = \text{no. of quark flavours})
\end{align*}
\]
The DGLAP equations

Generalizes to

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

\[
\begin{align*}
\frac{dP_{a\to bc}}{dQ^2} &= \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) \, dz \\
P_{q\to qg} &= 4 \frac{1 + z^2}{3} \\
P_{g\to gg} &= 3 \frac{(1 - z(1 - z))^2}{z(1 - z)} \\
P_{g\to q\bar{q}} &= \frac{n_f}{2} \left( z^2 + (1 - z)^2 \right) \quad (n_f = \text{no. of quark flavours})
\end{align*}
\]

Universality: any matrix element reduces to DGLAP in collinear limit.

\[
\begin{align*}
e.g. \quad \frac{d\sigma(H^0 \to q\bar{q}g)}{d\sigma(H^0 \to q\bar{q})} &= \frac{d\sigma(Z^0 \to q\bar{q}g)}{d\sigma(Z^0 \to q\bar{q})} \quad \text{in collinear limit}
\end{align*}
\]
The iterative structure

Generalizes to many consecutive emissions if strongly ordered, \( Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots \) (\( \approx \) time-ordered).

To cover “all” of phase space use DGLAP in whole region \( Q_1^2 > Q_2^2 > Q_3^2 \ldots \).

Iteration gives final-state parton showers:

Need soft/collinear cuts to stay away from nonperturbative physics. Details model-dependent, but around 1 GeV scale.
The Sudakov form factor – 1

Time evolution, conservation of total probability:
\[ P(\text{no emission}) = 1 - P(\text{emission}). \]

Multiplicativeness, with \( T_i = (i/n)T, \) \( 0 \leq i \leq n: \)

\[
P_{\text{no}}(0 \leq t < T) = \lim_{n \to \infty} \prod_{i=0}^{n-1} P_{\text{no}}(T_i \leq t < T_{i+1})
\]

\[
= \lim_{n \to \infty} \prod_{i=0}^{n-1} (1 - P_{\text{em}}(T_i \leq t < T_{i+1}))
\]

\[
= \exp \left( - \lim_{n \to \infty} \sum_{i=0}^{n-1} P_{\text{em}}(T_i \leq t < T_{i+1}) \right)
\]

\[
= \exp \left( - \int_0^T \frac{dP_{\text{em}}(t)}{dt} dt \right)
\]

\[ \Rightarrow \quad dP_{\text{first}}(T) = dP_{\text{em}}(T) \exp \left( - \int_0^T \frac{dP_{\text{em}}(t)}{dt} dt \right) \]

cf. radioactive decay in lecture 1.
The Sudakov form factor – 2

Expanded, with $Q \sim 1/t$ (Heisenberg)

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) dz$$

$$\times \exp \left( - \sum_{b,c} \int_{Q^2}^{Q_{\text{max}}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a\to bc}(z') dz' \right)$$

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c} \int \int d\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow \text{convenient for Monte Carlo}$

($\equiv 1$ if extended over whole phase space, else possibly nothing happens before you reach $Q_0 \approx 1$ GeV).
Sudakov regulates singularity for *first* emission ...  
... but in limit of *repeated soft* emissions $q \rightarrow qg$ (but no $g \rightarrow gg$) one obtains the same inclusive $Q$ emission spectrum as for ME, i.e. divergent ME spectrum  
$\iff$ infinite number of PS emissions  

More complicated in reality:  
- energy-momentum conservation effects big since $\alpha_s$ big, so hard emissions frequent  
- $g \rightarrow gg$ branchings leads to accelerated multiplication of partons
The ordering variable

In the evolution with

\[ d\mathcal{P}_{a\rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\rightarrow bc}(z) \, dz \]

\( Q^2 \) orders the emissions (memory).

If \( Q^2 = m^2 \) is one possible evolution variable then \( Q'^2 = f(z)Q^2 \) is also allowed, since

\[
\left| \frac{d(Q'^2, z)}{d(Q^2, z)} \right| = \begin{vmatrix}
\frac{\partial Q'^2}{\partial Q^2} & \frac{\partial Q'^2}{\partial z} \\
\frac{\partial Q^2}{\partial z} & \frac{\partial Q^2}{\partial z}
\end{vmatrix} = \begin{vmatrix}
f(z) & f'(z)Q^2 \\
0 & 1
\end{vmatrix} = f(z)
\]

\[ \Rightarrow d\mathcal{P}_{a\rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{f(z)dQ^2}{f(z)Q^2} P_{a\rightarrow bc}(z) \, dz = \frac{\alpha_s}{2\pi} \frac{dQ'^2}{Q'^2} P_{a\rightarrow bc}(z) \, dz \]

- \( Q'^2 = E_{a\rightarrow bc}^2 \theta_{a\rightarrow bc}^2 \approx m^2/(z(1 - z)); \text{ angular-ordered shower} \)
- \( Q'^2 = p_{\perp}^2 \approx m^2 z(1 - z); \text{ transverse-momentum-ordered} \)
Coherence

QED: Chudakov effect (mid-fifties)

- • requiring emission angles to be decreasing
- • requiring transverse momenta to be decreasing

Cosmic ray $\gamma$ atom

Emulsion plate  Reduced ionization  Normal ionization

Torbjörn Sjöstrand Event Generators 2 slide 14/33
QED: Chudakov effect (mid-fifties)

QCD: colour coherence for *soft* gluon emission

solved by
- requiring *emission angles* to be decreasing
- requiring *transverse momenta* to be decreasing
Ordering variables in the LEP/Tevatron era

**PYTHIA:** $Q^2 = m^2$

- Large mass first
- “Hardness” ordered
- Coherence brute force
- Covers phase space
- ME merging simple
- $g \rightarrow q\bar{q}$ simple
- Not Lorentz invariant
- No stop/restart
- ISR: $m^2 \rightarrow -m^2$

**HERWIG:** $Q^2 \sim E^2 \theta^2$

- Large angle first
- Hardness not ordered
- Coherence inherent
- Gaps in coverage
- ME merging messy
- $g \rightarrow q\bar{q}$ simple
- Not Lorentz invariant
- No stop/restart
- ISR: $\theta \rightarrow \theta$

**ARIADNE:** $Q^2 = p_{\perp}^2$

- Large $p_{\perp}$ first
- “Hardness” ordered
- Coherence inherent
- Covers phase space
- ME merging simple
- $g \rightarrow q\bar{q}$ messy
- Lorentz invariant
- Can stop/restart
- ISR: more messy
Quark vs. gluon jets

\[ \frac{P_{g \rightarrow gg}}{P_{q \rightarrow qg}} \approx \frac{N_c}{C_F} = \frac{3}{4/3} = \frac{9}{4} \approx 2 \]

⇒ gluon jets are softer and broader than quark ones (also helped by hadronization models, lecture 4).

Note transition g jets → q jets for increasing \( p_\perp \).
**Heavy flavours: the dead cone**

Matrix element for $e^+e^- \rightarrow q\bar{q}g$ for small $\theta_{13}$

\[
\frac{d\sigma_{q\bar{q}g}}{\sigma_{q\bar{q}}} \propto \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \approx \frac{d\omega}{\omega} \frac{d\theta_{13}^2}{\theta_{13}^2}
\]

is modified for heavy quark $Q$:

\[
\frac{d\sigma_{q\bar{q}g}}{\sigma_{q\bar{q}}} \propto \frac{d\omega}{\omega} \frac{d\theta_{13}^2}{\theta_{13}^2} \left(\frac{\theta_{13}^2}{\theta_{13}^2 + m_1^2/E_1^2}\right)^2
\]

\[
= \frac{d\omega}{\omega} \frac{\theta_{13}^2 d\theta_{13}^2}{(\theta_{13}^2 + m_1^2/E_1^2)^2}
\]

so “dead cone” for $\theta_{13} < m_1/E_1$

For charm and bottom largely filled in by their decay products.
Hadrons are composite, with time-dependent structure:

\[ f_i(x, Q^2) = \text{number density of partons } i \]

at momentum fraction \( x \) and probing scale \( Q^2 \).

Linguistics (example):

\[
F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2)
\]

structure function \hspace{1cm} \text{parton distributions}
PDF evolution

Initial conditions at small $Q_0^2$ unknown: nonperturbative.
Resolution dependence perturbative, by DGLAP:

\[
\frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(y, Q^2) \frac{\alpha_s}{2\pi} P_{a\rightarrow bc} \left( z = \frac{x}{y} \right)
\]

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)
PDF evolution

**Initial conditions at small $Q_0^2$ unknown: nonperturbative.**

**Resolution dependence perturbative, by DGLAP:**

### DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

\[
\frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(y, Q^2) \frac{\alpha_s}{2\pi} P_{a\rightarrow bc} \left( z = \frac{x}{y} \right)
\]

DGLAP already introduced for (final-state) showers:

\[
dP_{a\rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\rightarrow bc}(z) dz
\]

Same equation, but different context:

- $dP_{a\rightarrow bc}$ is probability for the individual parton to branch; while
- $df_b(x, Q^2)$ describes how the ensemble of partons evolve by the branchings of individual partons as above.
Initial-State Shower Basics

- Parton cascades in $p$ are continuously born and recombined.
- Structure at $Q$ is resolved at a time $t \sim 1/Q$ before collision.
- A hard scattering at $Q^2$ probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.
• Parton cascades in $p$ are continuously born and recombined.
• Structure at $Q$ is resolved at a time $t \sim 1/Q$ before collision.
• A hard scattering at $Q^2$ probes fluctuations up to that scale.
• A hard scattering inhibits full recombination of the cascade.

![Diagram](image_url)

• Convenient reinterpretation:

\[
\begin{align*}
m^2 &= 0 \\
m^2 &< 0 \\
m^2 &> 0 \\
Q^2 &= -m^2 > 0 \\
\end{align*}
\]
Forwards vs. backwards evolution

Event generation could be addressed by **forwards evolution**: pick a complete partonic set at low $Q_0$ and evolve, consider collisions at different $Q^2$ and pick by $\sigma$ of those.

**Inefficient:**

1. have to evolve and check for *all* potential collisions, but 99.9...% inert
2. impossible (or at least very complicated) to steer the production, e.g. of a narrow resonance (Higgs)

**Backwards evolution** is viable and $\sim$-equivalent alternative: start at hard interaction and trace what happened “before”
Monte Carlo approach, based on *conditional probability*: recast
\[
\frac{d f_b(x, Q^2)}{dt} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc}(z)
\]
with \( t = \ln(Q^2/\Lambda^2) \) and \( z = x/x' \) to
\[
d\mathcal{P}_b = \frac{df_b}{f_b} = |dt| \sum_a \int dz \frac{x' f_a(x', t)}{x f_b(x, t)} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)
\]
then solve for *decreasing* \( t \), i.e. backwards in time, starting at high \( Q^2 \) and moving towards lower, with Sudakov form factor \( \exp(-\int d\mathcal{P}_b) \).

Extra factor \( x' f_a/xf_b \) relative to final-state equations.
Coherence in spacelike showers

with $Q^2 = -m^2 = \text{spacelike virtuality}$

- **Kinematics only:**
  $Q_3^2 > z_1 Q_1^2, \ Q_5^2 > z_3 Q_3^2, \ldots$
  i.e. $Q_i^2$ need not even be ordered

- **Coherence of leading collinear singularities:**
  $Q_5^2 > Q_3^2 > Q_1^2$, i.e. $Q^2$ ordered

- **Coherence of leading soft singularities (more messy):**
  $E_3 \theta_4 > E_1 \theta_2$, i.e. $z_1 \theta_4 > \theta_2$
  $z \ll 1$: $E_1 \theta_2 \approx p_{\perp 2}^2 \approx Q_3^2, \ E_3 \theta_4 \approx p_{\perp 4}^2 \approx Q_5^2$
  i.e. reduces to $Q^2$ ordering as above
  $z \approx 1$: $\theta_4 > \theta_2$, i.e. angular ordering of soft gluons
  $\implies$ reduced phase space

$z_1 = E_3/E_1$
$z_3 = E_5/E_3$
$\theta_2 = \theta_{12}$
$\theta_4 = \theta_{14}!!$
Evolution procedures

DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi
evolution towards larger $Q^2$ and (implicitly) towards smaller $x$
BFKL: Balitsky–Fadin–Kuraev–Lipatov
evolution towards smaller $x$ (with small, unordered $Q^2$)
CCFM: Ciafaloni–Catani–Fiorani–Marchesini
interpolation of DGLAP and BFKL
GLR: Gribov–Levin–Ryskin
nonlinear equation in dense-packing (saturation) region,
where partons recombine, not only branch
Did we reach BFKL regime?

Study events with $\geq 2$ jets as a function of their $y$ separation; $\cos(\pi - \Delta \phi) = 1$ is back-to-back jets, i.e. little extra radiation.

Analytic BFKL calculations describe data for $\Delta y > 4$, but HEJ BFKL-inspired generator overshoots effect, and standard DGLAP Herwig++ almost spot on. No strong indications for BFKL/CCFM behaviour onset so far!
Initial- vs. final-state showers

Both controlled by same evolution equations

\[ d\mathcal{P}_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) \, dz \cdot (\text{Sudakov}) \]
Initial- vs. final-state showers

Both controlled by same evolution equations

\[ \frac{dP_{a\rightarrow bc}}{2\pi} = \frac{\alpha_s}{Q^2} \frac{dQ^2}{Q^2} P_{a\rightarrow bc}(z) \, dz \cdot \text{(Sudakov)} \]

but

Final-state showers:  
\( Q^2 \) timelike (\( \sim m^2 \))

decreasing \( E, m^2, \theta \)
both daughters \( m^2 \geq 0 \)
physics relatively simple
\( \Rightarrow \) "minor" variations:
\( Q^2 \), shower vs. dipole, . . .

Initial-state showers:  
\( Q^2 \) spacelike (\( \approx -m^2 \))

decreasing \( E \), increasing \( Q^2, \theta \)
one daughter \( m^2 \geq 0 \), one \( m^2 < 0 \)
physics more complicated
\( \Rightarrow \) more formalisms:
DGLAP, BFKL, CCFM, GLR, . . .
Separate processing of ISR and FSR misses interference ($\sim$ colour dipoles)
Combining FSR with ISR

Separate processing of ISR and FSR misses interference (∼ colour dipoles)

ISR+FSR add coherently in regions of colour flow and destructively else in “normal” shower by azimuthal anisotropies automatic in dipole (by proper boosts)
Coherence tests

Current-day generators for pseudorapidity of third jet:

Pseudorapidity, $\eta$, of 3rd jet

and past incoherent:
1 → 2 branching = replace $m = 0$ parton by pair with $m > 0$. Breaks energy–momentum conservation.
Herwig angular-ordered shower: post-facto rescaling machinery.

Alternative: dipole picture (first Ariadne, now everybody else).
2 → 3 parton branching, or 1 → 2 colour dipole branching.
Can be viewed as radiator $a \rightarrow bc$ with recoiler $r$. 
Ariadne main splitting expressions for final-state radiation:

\[
\begin{align*}
    dP_{q\bar{q} \rightarrow q\bar{q}g} &= \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} \, dx_1 \, dx_2 \\
    dP_{qg \rightarrow qgg} &= \frac{\alpha_s}{2\pi} \frac{3}{2} \frac{x_1^2 + x_2^3}{(1 - x_1)(1 - x_2)} \, dx_1 \, dx_2 \\
    dP_{gg \rightarrow ggg} &= \frac{\alpha_s}{2\pi} \frac{3}{2} \frac{x_1^3 + x_2^3}{(1 - x_1)(1 - x_2)} \, dx_1 \, dx_2
\end{align*}
\]

does not define angular orientation.

The Catani–Seymour dipole is primarily a kinematics recipe how to map 2 partons \( ar \leftrightarrow 3 \) partons \( bcr' \) for both initial and final state:

\[
\begin{align*}
    p_a &= p_b + p_c - \frac{y}{1 - y} p_{r'} \\
    p_r &= \frac{1}{1 - y} p_{r'} \\
    y &= \frac{p_b p_c}{p_b p_c + p_b p_{r'} + p_c p_{r'}}
\end{align*}
\]
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<td>(High Energy Jets) BFKL-inspired description of well-separated multijets, with approximate matrix elements and virtual corrections</td>
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VINCIA: an Interleaved Antennae shower

Markovian process: no memory of path to reach current state.

Based on antenna factorization of amplitudes and phase space.

Smooth ordering fills whole phase space.

Step-by-step reweighting to new matrix elements: $Z \rightarrow Zj \rightarrow Zjj \rightarrow Zjjj$ (also Sudakov), e.g.

$$W = \frac{|M_{Zj}|^2}{\sum_i a_i |M_Z|^2_i}$$

Replaces PYTHIA normal showers; recent release.

![Graph showing CMS, $\Delta\phi(Z,J_1)$, $\sqrt{s} = 7$ TeV comparison between data and MC with different matrix elements.](image)
Joint Sherpa/PYTHIA development, but separate implementations, means technically well tested.

“Midpoint between dipole and parton shower”, dipole with emitter & spectator, but not quite CS ones: unified initial–initial, initial–final, final–initial, final–final.

Soft term of kernels in all dipole types is less singular

\[
\frac{1}{1-z} \to \frac{1-z}{(1-z)^2 + p_{\perp}^2/M^2}
\]