Introduction to Event Generators 3

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CTEQ/MCnet School, DESY, 12 July 2016
An event consists of many different physics steps, which have to be modelled by event generators:
Event topologies

Expect and observe high multiplicities at the LHC. What are production mechanisms behind this?
What is minimum bias (MB)?

MB \approx \text{“all events, with no bias from restricted trigger conditions”}

\[ \sigma_{\text{tot}} = \sigma_{\text{elastic}} + \sigma_{\text{single-diffractive}} + \sigma_{\text{double-diffractive}} + \cdots + \sigma_{\text{non-diffractive}} \]

Schematically:

Reality: can only observe events with particles in central detector: no universally accepted, detector-independent definition

\[ \sigma_{\text{min-bias}} \approx \sigma_{\text{non-diffractive}} + \sigma_{\text{double-diffractive}} \approx \frac{2}{3} \times \sigma_{\text{tot}} \]
What is underlying event (UE)?

In an event containing a jet pair or another hard process, how much further activity is there, that does not have its origin in the hard process itself, but in other physics processes?

Pedestal effect: the UE contains more activity than a normal MB event does (even discarding diffractive events).

Trigger bias: a jet ”trigger” criterion $E_{\perp \text{jet}} > E_{\perp \text{min}}$ is more easily fulfilled in events with upwards-fluctuating UE activity, since the UE $E_{\perp}$ in the jet cone counts towards the $E_{\perp \text{jet}}$. Not enough!
What is pileup?

\[ \langle n \rangle = \overline{\mathcal{L}} \sigma \]

where \( \overline{\mathcal{L}} \) is machine luminosity per bunch crossing, \( \overline{\mathcal{L}} \sim n_1 n_2 / A \) and \( \sigma \sim \sigma_{\text{tot}} \approx 100 \text{ mb} \).

Current LHC machine conditions \( \Rightarrow \langle n \rangle \sim 10 - 20 \).

Pileup introduces no new physics, and is thus not further considered here, but can be a nuisance.

However, keep in mind concept of bunches of hadrons leading to multiple collisions.
The divergence of the QCD cross section

Cross section for $2 \rightarrow 2$ interactions is dominated by $t$-channel gluon exchange, so diverges like $\frac{d\hat{\sigma}}{dp_\perp^2} \approx \frac{1}{p_\perp^4}$ for $p_\perp \rightarrow 0$.

Integrate QCD $2 \rightarrow 2$

- $qq' \rightarrow qq'$
- $q\bar{q} \rightarrow q'\bar{q}'$
- $q\bar{q} \rightarrow gg$
- $qg \rightarrow qg$
- $gg \rightarrow gg$
- $gg \rightarrow q\bar{q}$

(with CTEQ 5L PDF's)
What is multiple partonic interactions (MPI)?

Note that $\sigma_{\text{int}}(p_{\perp\text{min}})$, the number of $(2 \to 2 \text{ QCD})$ interactions above $p_{\perp\text{min}}$, involves integral over PDFs,

$$\sigma_{\text{int}}(p_{\perp\text{min}}) = \int \int \int_{p_{\perp\text{min}}} dx_1 \, dx_2 \, dp_2^2 \, f_1(x_1, p_2^2) \, f_2(x_2, p_2^2) \, \frac{d\hat{\sigma}}{dp_2^2}$$

with $\int dx \, f(x, p_2^2) = \infty$, i.e. infinitely many partons.

So half a solution to $\sigma_{\text{int}}(p_{\perp\text{min}}) > \sigma_{\text{tot}}$ is

**many interactions per event: MPI**

$$\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_n$$

$$\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1$$
If interactions occur independently then **Poissonian statistics**

\[
\mathcal{P}_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}
\]

but \( n = 0 \Rightarrow \) no event (in many models) and energy–momentum conservation \( \Rightarrow \) large \( n \) suppressed so narrower than Poissonian

MPI is a logical consequence of the composite nature of protons,

\[
n_{\text{parton}} \sim \sum_{q, \bar{q}, g} \int f(x) \, dx > 3, \text{ which allows } \sigma_{\text{int}}(p_{\perp \text{min}}) > \sigma_{\text{tot}},
\]

but what about the limit \( p_{\perp \text{min}} \to 0? \)
Other half of solution is that perturbative QCD is not valid at small $p_\perp$ since $q, g$ are not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp \text{min}} \simeq \frac{\hbar}{r_p} \approx \frac{0.2 \text{ GeV} \cdot \text{fm}}{0.7 \text{ fm}} \approx 0.3 \text{ GeV} \simeq \Lambda_{\text{QCD}}$$

...but better replace $r_p$ by (unknown) colour screening length $d$ in hadron:
Regularization of low-$p_\perp$ divergence

so need **nonperturbative regularization for** $p_\perp \to 0$, e.g.

$$\frac{d\hat{\sigma}}{d p_\perp^2} \propto \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4} \to \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4} \theta(p_\perp - p_{\perp\text{min}}) \quad \text{(simpler)}$$

or $$\to \frac{\alpha_s^2(p_{\perp0}^2 + p_\perp^2)}{(p_{\perp0}^2 + p_\perp^2)^2} \quad \text{(more physical)}$$

where $p_{\perp\text{min}}$ or $p_{\perp0}$ are free parameters, empirically of order 2–3 GeV.

Typical number of interactions/event is 3 at 2 TeV, 4 – 5 at 13 TeV, but may be twice that in “interesting” high-$p_\perp$ ones.
So far assumed that all collisions have equivalent initial conditions, but hadrons are extended, so dependence on impact parameter $b$.

Overlap of protons during encounter is

$$\mathcal{O}(b) = \int d^3x \, dt \, \rho_1(x, t) \rho_2(x, t)$$

where $\rho$ is (boosted) matter distribution in $p$, e.g. Gaussian or electromagnetic form factor.

Average activity at $b$ proportional to $\mathcal{O}(b)$:

- central collisions more active
  $\Rightarrow \mathcal{P}_n$ broader than Poissonian;
- peripheral passages normally give no collisions $\Rightarrow$ finite $\sigma_{\text{tot}}$. 

\[ \langle n \rangle \text{ for } n \geq 1 \quad \text{all} \]
Double parton scattering (DPS): two hard processes in same event.

\[
\sigma_{\text{DPS}} = \begin{cases} 
\frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}} & \text{for } A \neq B \\
\frac{\sigma_A \sigma_B}{2 \sigma_{\text{eff}}} & \text{for } A = B 
\end{cases}
\]

(Poissonian $\Rightarrow 1/2$; $AB + BA \Rightarrow 2$)

Note inverse relationship on $\sigma_{\text{eff}}$.

Natural scale is $\sigma_{\text{ND}} \approx 50$ mb, but “reduced” by $b$ dependence.

Studied by
- 4 jets
- $\gamma$ + 3 jets
- 4 jets, whereof two b- or c-tagged
- $J/\psi$ or $\Upsilon + 2$ jets (including $\nu \overline{\nu}$)
- $W/Z + 2$ jets
- $W^- W^-$
Always non-DPS backgrounds, so kinematics cuts required.

Example: order 4 jets $\mathbf{p}_1 > \mathbf{p}_2 > \mathbf{p}_3 > \mathbf{p}_4$ and define $\varphi$ as angle between $\mathbf{p}_1 \mp \mathbf{p}_2$ and $\mathbf{p}_3 \mp \mathbf{p}_4$ for AFS/CDF.
Experimental summary on DPS rate

Note:
big error bars,
uncertain
methodology,
but consistent:
\[ \sigma_{\text{eff}} \approx \sigma_{\text{ND}} / 3 \]
\[ \Rightarrow \text{factor } \sim 3 \]
enhancement
relative to naive expectations
Multiplicity and MPI effects

DPS only probes high-$p_{\perp}$ tail of effects. More dramatic are effects on multiplicity distributions:

- New ATLAS analysis uses:
  - $p_T > 100$ MeV/c
  - $N_{ch} \geq 2$
  - Single diffraction is inhibited, and $p_T$ cut allows direct comparison with other experiments.

- ATLAS $\sqrt{s} = 13$ TeV
Forward-backward correlations

Global number, such as \#MPI, affects activity everywhere:

\[
\eta \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \\
\]

\[
\text{FB multiplicity correlation} \\
0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \quad 0.9 \\
\]

Data 2010
Pythia 6 MC09
Pythia 6 DW
Pythia 6 Perugia2011
Pythia 6 AMBT2B
Pythia 8 4C
Herwig++

\[
\text{ATLAS} \\
\sqrt{s} = 7 \text{ TeV} \\
\]

\[
\text{MC / data} \\
0.8 \quad 0.9 \quad 1 \quad 1.1 \\
\]

\[
\eta \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \\
\]

\[
\text{(note suppressed zero on vertical axis ⇒ big effects!)} \\
\]
Colour (re)connections and $\langle p_\perp \rangle(n_{ch})$

$\langle p_\perp \rangle(n_{ch})$ is very sensitive to colour flow

long strings to remnants $\Rightarrow$ much $n_{ch}$/interaction $\Rightarrow$ $\langle p_\perp \rangle(n_{ch})$ $\sim$ flat

short strings (more central) $\Rightarrow$ less $n_{ch}$/interaction $\Rightarrow$ $\langle p_\perp \rangle(n_{ch})$ rising

$n_{ch} \geq 1$, $p_T > 500$ MeV, $|\eta| < 0.8$
$\tau > 300$ ps

**ATLAS** $\sqrt{s} = 13$ TeV
Events with hard scale (jet, W/Z) have more underlying activity!
Events with $n$ interactions have $n$ chances that one of them is hard, so “trigger bias”: hard scale $\Rightarrow$ central collision $\Rightarrow$ more interactions $\Rightarrow$ larger underlying activity.

Studied in particular by Rick Field, with CDF/CMS data:

“MAX/MIN Transverse” Densities

- Define the **MAX** and **MIN** “transverse” regions on an event-by-event basis with **MAX** (MIN) having the largest (smallest) density.
Jet pedestal effect – 2

281 nb$^{-1}$ (13 TeV)

CMS

Preliminary

$\langle N_{\text{chg}} \rangle / \Delta\eta(\Delta\phi)$

13 TeV
7 TeV
2.76 TeV
0.9 TeV

transAVE

Leading Jet $p_T$ (GeV)
MPI in PYTHIA

- MPIs are generated in a falling sequence of $p_{\perp}$ values; recall Sudakov factor approach to parton showers.
- Energy, momentum and flavour conserved step by step: subtracted from proton by all “previous” collisions.
- Protons modelled as extended objects, allowing both central and peripheral collisions, with more or less activity.
- (Partons at small $x$ more broadly spread than at large $x$.)
- Colour screening increases with energy, i.e. $p_{\perp 0} = p_{\perp 0}(E_{\text{cm}})$, as more and more partons can interact.
- (Rescattering: one parton can scatter several times.)
- Colour connections: each interaction hooks up with colours from beam remnants, but also correlations inside remnants.
- Colour reconnections: many interaction “on top of” each other $\Rightarrow$ tightly packed partons $\Rightarrow$ colour memory loss?
Interleaved evolution in PYTHIA

- Transverse-momentum-ordered parton showers for ISR and FSR
- MPI also ordered in $p_\perp$

$\Rightarrow$ Allows interleaved evolution for ISR, FSR and MPI:

$$\frac{d\mathcal{P}}{dp_\perp} = \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp_\perp} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp_\perp} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp_\perp} \right)$$

$$\times \exp \left( - \int_{p_\perp}^{p_{\perp \text{max}}} \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp'_\perp} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp'_\perp} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp'_\perp} \right) dp'_\perp \right)$$

Ordered in decreasing $p_\perp$ using “Sudakov” trick.
Corresponds to increasing “resolution”: smaller $p_\perp$ fill in details of basic picture set at larger $p_\perp$.

- Start from fixed hard interaction $\Rightarrow$ underlying event
- No separate hard interaction $\Rightarrow$ minbias events
- Possible to choose two hard interactions, e.g. $W^-W^-$
Key point: two-component model

\[ p_\perp > p_\perp^{\text{min}}: \text{pure perturbation theory (no modification)} \]
\[ p_\perp < p_\perp^{\text{min}}: \text{pure nonperturbative ansatz} \]
Number of MPIs first picked; then generated unordered in $p_\perp$.

Interactions uncorrelated, up until energy used up.

Force ISR to reconstruct back to gluon after first interaction.

Impact parameter by em form factor shape, but tunable width.

$p_{\perp\text{min}}$ scale to be tuned energy-by-energy.

Colour reconnection essential to get $dn/d\eta$ correct.
• The only way we can create the QGP in the laboratory!
• By colliding heavy ions it is possible to create a large (»1fm$^3$) zone of hot and dense QCD matter
• Goal is to create and study the properties of the Quark Gluon Plasma
• Experimentally mainly the final state particles are observed, so the conclusions have to be inferred via models
The three systems — understanding before 2012

Pb-Pb

pp

p-Pb

Hot QCD matter:
This is where we expect the QGP to be created in central collisions.

QCD baseline:
This is the baseline for “standard” QCD phenomena.

Cold QCD matter:
This is to isolate nuclear effects, e.g. nuclear pdfs.
Strangeness enhancement

\[
\frac{\langle p_T \rangle (\text{GeV}/c) |y|<0.5}{\langle p_T \rangle (\text{GeV}/c) |y|<0.5} = 10^{-1}
\]

\[
\langle dN_{\text{ch}}/d\eta \rangle_{|\eta|<0.5} = 10^{-2}
\]

ALICE Preliminary - pp \( s=7 \text{ TeV} \)

Logarithmic fit

\[
\langle dN_{\text{ch}}/d\eta \rangle_{|\eta|<0.5} = 10^{-3}
\]

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Event Generators 3
Collective flow

ATLAS
\( \sqrt{s} = 13 \text{ TeV} \)

\( 0.5 < p_T^{a,b} < 5.0 \text{ GeV} \)

\( 0 < N_{\text{ch}}^{\text{rec}} < 30 \)

\( N_{\text{ch}}^{\text{rec}} > 120 \)

\( 50 \leq N_{\text{ch}}^{\text{rec}} < 60 \)

ATLAS Preliminary

Increasingly blurred line between pp, pA and AA!

QGP theory wrong?
Much smaller systems enough for QGP?

Standard pp generators wrong! Need mechanism for collectivity.

Increasingly blurred line between pp, pA and AA!

QGP theory wrong?
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Standard pp generators wrong! Need mechanism for collectivity.
The result for the total hadronic cross section presented here, $\sigma_{\text{tot}} = 9.35 \pm 1.36 \text{ mb}$, can be compared to the value measured by TOTEM in the same LHC fill using a luminosity-dependent analysis, $\sigma_{\text{tot}} = 9.66 \pm 2.2 \text{ mb}$\cite{11}. Assuming the uncertainties are uncorrelated, the difference between the ATLAS and TOTEM values corresponds to 1.3 $\sigma$. The uncertainty on the TOTEM result is dominated by the luminosity uncertainty of $\pm 4\%$, while the measurement reported here profits from a smaller luminosity uncertainty of only $\pm 2.3\%$.

In subsequent publications\cite{16,54}, TOTEM has used the same detector to perform a luminosity-independent measurement of the total cross section using a simultaneous determination of elastic and inelastic event yields. In addition, TOTEM made a $\rho$-independent measurement without using the optical theorem by summing directly the elastic and inelastic cross sections\cite{16}. The three TOTEM results are consistent with one another.

The results presented here are compared in Fig. 19 to the result of TOTEM and are also compared with results of experiments at lower energy\cite{29} and with cosmic ray experiments\cite{55–58}. The measured total cross section is furthermore compared to the best fit to the energy evolution of the total cross section from the COMPETE Collaboration\cite{26} assuming an energy dependence of $\ln s$. The elastic measurement is in turn compared to a second order polynomial fit in $\ln s$ of the elastic cross sections. The value of $\sigma_{\text{tot}}$ reported here is two standard deviations below the COMPETE parameterization. Some other models prefer a somewhat slower increase of the total cross section with energy, predicting values below 95 mb, and thus agree slightly better with the result reported here\cite{59–61}.

\[ \sigma_{\text{tot}} \]
Event-type breakdown

- Cross sections obtained for $\xi > 10^{-6}$, and full kinematics.
- Quite large error for extrapolation.
- Diffractive mass according to PYTHIA Donnachie-Landshoff parameterization.

\[
\sigma_{\xi>10^{-6}} = 68.2 \pm 0.08 \pm 1.3 \text{ mb}
\]
\[
\sigma_{\text{inel}} = 79.3 \pm 0.08 \pm 1.3 \pm 2.5 \text{ mb}
\]

- Phase space for diffractive masses and rapidity gaps roughly like $dM^2/M^2 = dy$, i.e. flat in rapidity.
- Rapidity integration means $\sigma_{\text{sd}}$ grows faster than $\sigma_{\text{tot}}$, $\sigma_{\text{dd}}$ even faster, etc.
  $\Rightarrow$ Need damping.
The Pomeron

Amplitude for (forward) elastic scattering from total cross section:

\[ p p \Rightarrow p p \Rightarrow p p = \Rightarrow p p \]

Introducing the Pomeron $IP$ as shorthand for the effective 2-gluon exchange.

Since $p \rightarrow p IP$ the Pomeron must have the quantum numbers of the vacuum: $0^+$ colour singlet.

Recall: elastic cross section requires squaring one more time:
Regge–Pomeranchuk theory of cross sections

\[ \frac{d\sigma_{AB}^{\text{tot}}}{dt} = \beta_A(0) \beta_B(0) \text{Im} G_{\text{IP}}(s/s_0, 0) \]

\[ \frac{d\sigma_{el}^{AB}}{dt} = \frac{1}{16\pi} \beta_A^2(t) \beta_B^2(t) |G_{\text{IP}}(s/s_0, t)|^2 \]

\[ \frac{d\sigma_{sd}^{AB \rightarrow AX}}{dt \, dM^2} = \frac{1}{16\pi M^2} g_{3\text{IP}} \beta_A^2(t) \beta_B(0) |G_{\text{IP}}(s/M^2, t)|^2 \text{Im} G(M^2/s_0, 0) \]

\[ \frac{d\sigma_{dd}^{AB \rightarrow X_1 X_2}}{dt \, dM_1^2 \, dM_2^2} = \frac{1}{16\pi M_1^2 M_2^2} g_{3\text{IP}}^2 \beta_A(0) \beta_B(0) |G_{\text{IP}}(s_0/(M_1^2 M_2^2), t)|^2 \times \text{Im} G(M_1^2/s_0, 0) \text{Im} G(M_2^2/s_0, 0) \]
Ingelman-Schlein: Pomeron as hadron with partonic content

Diffractive event = (Pomeron flux) \times (\text{IPp collision})

1) \sigma_{SD} and \sigma_{DD} set by Reggeon theory.
2) f_{IP/p}(x_{IP}, t) \Rightarrow diffractive mass spectrum, p_{\perp} of proton out.
3) Smooth transition from simple model at low masses to IPp with full pp machinery: multiple interactions, parton showers, etc.
4) Choice between different Pomeron PDFs.
5) Free parameter \sigma_{IPp} needed to fix \langle n_{interactions} \rangle = \sigma_{jet}/\sigma_{IPp}.
Non-diffractive fine, but wrong gap spectrum for diffraction.
Multiplicity in diffractive events

4 < Δ η^F < 6
\bar{s} = 7 TeV
p_T > 200 MeV

**ATLAS**

PYTHIA 6 lacks MPI, ISR, FSR in diffraction, so undershoots.