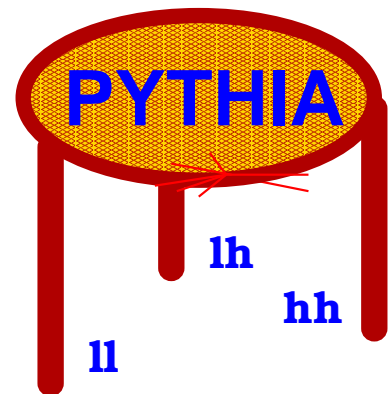




LUND UNIVERSITY

Workshop on  
Monte Carlo Generator Physics  
for Run II at the Tevatron,  
Fermilab, April 18-20, 2001

# Matching of ME with PS in PYTHIA



Torbjörn Sjöstrand

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Introduction

Merging

...in FSR

...in ISR

NLO Matching I (S. Mrenna)

NLO Matching II (L. Lönnblad)

(N)LO Matching III

3 persons  $\Rightarrow \geq 4$  strategies

# Introduction

## ME : Matrix Elements

- + systematic expansion in  $\alpha_S$  ('exact')
- + powerful for multiparton Born level
- + flexible phase space cuts
- loop calculations very tough
- negative cross section in collinear regions
  - ⇒ unproductive jet structure
- no easy match to hadronization

## PS : Parton Showers

- approximate, to LL (or NLL)
- main topology not predetermined
  - ⇒ inefficient for exclusive states
- + process-generic ⇒ simple multiparton
- + Sudakov form factors/resummation
  - ⇒ sensible jet structure
- + easy to match to hadronization

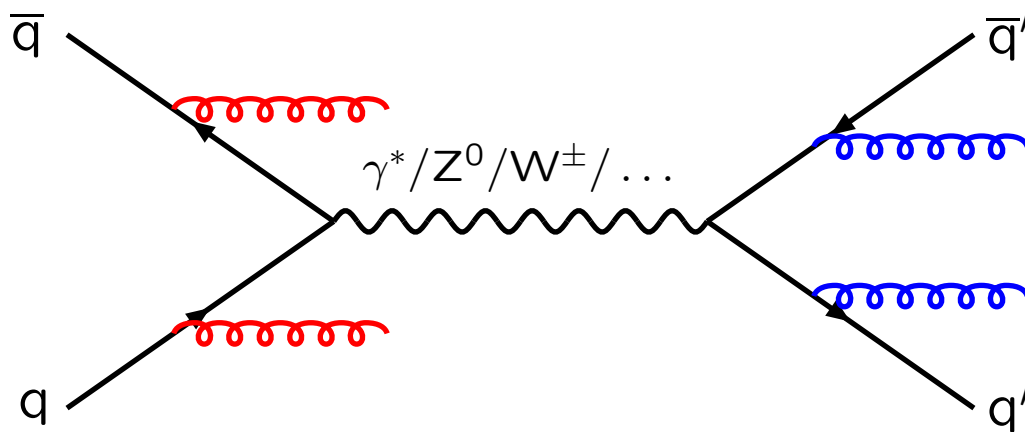
## Marriage desirable! But how?

- Problems:
- gaps in coverage?
  - doublecounting of radiation?
  - Sudakov?
  - NLO consistency?

# Merging

= smooth transition ME/PS, no sharp edge.

Default strategy in PYTHIA



FSR = final-state radiation: timelike shower  
forwards evolution in  $Q^2 = m^2$

ISR = initial-state radiation: spacelike shower  
backwards evolution in  $Q^2 = -m^2$

+ emissions can cover full phase space  
– coherence not straightforward

Want shower to reproduce

$$W^{\text{ME}} = \frac{1}{\sigma(\text{LO})} \frac{\sigma(\text{LO} + g)}{d(\text{phasespace})}$$

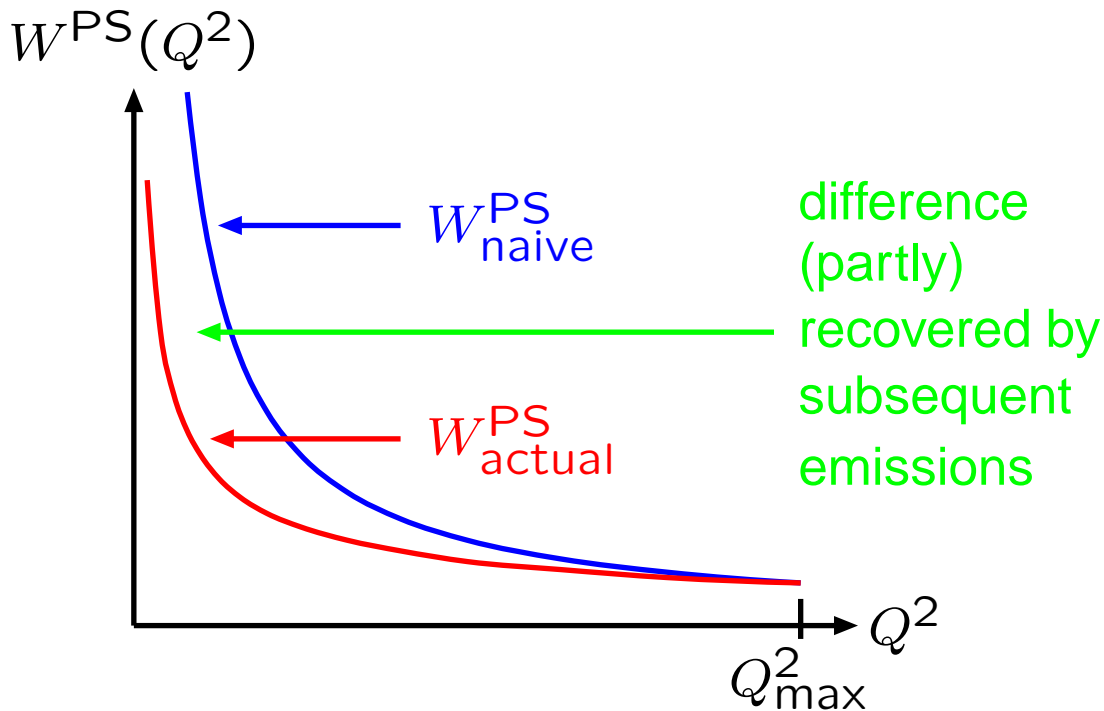
by shower generation + correction procedure

$$\underbrace{W^{\text{ME}}}_{\text{wanted}} = \underbrace{W^{\text{PS}}}_{\text{generated}} \cdot \frac{\overbrace{W^{\text{ME}}}_{\text{correction}}}{W^{\text{PS}}}$$

Comments:

- Do not normalize  $W^{\text{ME}}$  to  $\sigma(\text{NLO})$ , since extra work without clear gain (expect radiation also in events added by  $K\text{-factor} \geq 1$ )
- Shower contains Sudakov form factor

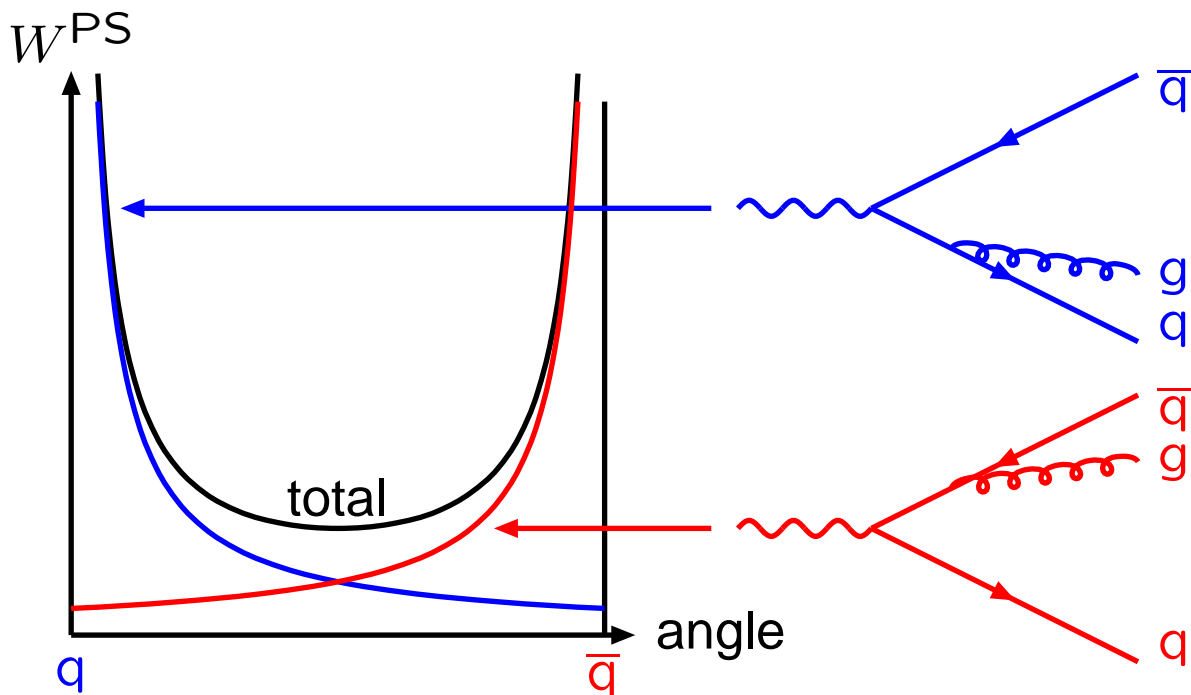
$$W_{\text{actual}}^{\text{PS}}(Q^2) = W_{\text{naive}}^{\text{PS}}(Q^2) \exp\left(-\int_{Q^2}^{Q_{\text{max}}^2} W_{\text{naive}}^{\text{PS}}(Q'^2) dQ'^2\right)$$



merger: exponentiate ME correction

$$W_{\text{actual}}^{\text{PS}}(Q^2) = W^{\text{ME}}(Q^2) \exp\left(-\int_{Q^2}^{Q_{\text{max}}^2} W^{\text{ME}}(Q'^2) dQ'^2\right)$$

- Normally several shower histories



(often  $\sim$  Feynman graphs; cf. ME amplitudes)

$$\Rightarrow W^{\text{PS}} = \sum_i W_i^{\text{PS}}; \text{ also(?) } W^{\text{ME}} = \sum_i W_i^{\text{ME}}$$

Alternative 1 (assuming  $W^{\text{ME}} \leq W^{\text{PS}}$ ):

$$W^{\text{ME}} = W^{\text{PS}} \frac{W^{\text{ME}}}{W^{\text{PS}}} = \sum_i W_i^{\text{PS}} \frac{W_i^{\text{ME}}}{\sum_j W_j^{\text{PS}}}$$

Alternative 2 (assuming  $W_i^{\text{ME}} \leq W_i^{\text{PS}}$ ):

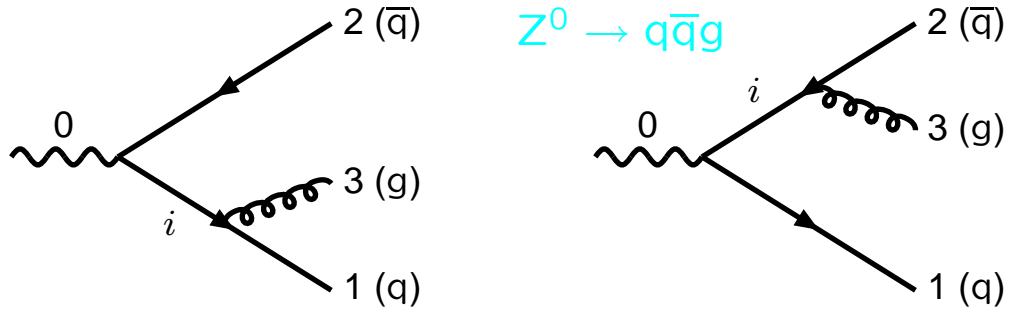
$$W^{\text{ME}} = \sum_i W_i^{\text{ME}} = \sum_i W_i^{\text{PS}} \frac{W_i^{\text{ME}}}{W_i^{\text{PS}}}$$

largely equivalent (some difference in Sudakov)

$\Rightarrow$  pick by convenience

# Merging in FSR: massless case

(M. Bengtsson & TS, PLB185 (1987) 435, NPB289 (1987) 810)



$$\frac{1}{\sigma_0} \frac{d\sigma_{ME}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{\overbrace{x_1^2 + x_2^2}^{\approx 2}}{(1-x_1)(1-x_2)}$$

$$\frac{1}{\sigma_0} \frac{d\sigma_{PS}}{dQ^2 dz} = \frac{\alpha_s}{2\pi} C_F \frac{dQ^2}{Q^2} \frac{\overbrace{1+z^2}^{\approx 2}}{1-z} dz \cdot (\text{Sudakov})$$

$$Q_1^2 = m_i^2 = (p_0 - p_2)^2 = (1-x_2)E_{CM}^2$$

$$z_1 = \frac{p_0 p_1}{p_0 p_i} = \frac{E_1}{E_i} = \frac{x_1}{x_1 + x_3} = \frac{x_1}{2-x_2},$$

$$\Rightarrow \frac{dQ_1^2}{Q_1^2} \frac{dz_1}{1-z_1} = \frac{dx_2}{1-x_2} \frac{dx_1}{x_3}$$

$$\begin{aligned} \Rightarrow \frac{1}{\sigma_0} \frac{d\sigma_{PS}}{dx_1 dx_2} &\propto \frac{2}{(1-x_2)x_3} + \frac{2}{(1-x_1)x_3} \\ &= \frac{2}{(1-x_1)(1-x_2)} \end{aligned}$$

$W^{ME} < W^{PS}$ , i.e. good MC starting point

# Merging in FSR: massive case

(E. Norrbin & TS, LUTP 00-42 [hep-ph/0010012]  $\Rightarrow$  NPB)

$Q_i^2 = m_i^2$  gives wrong singularity structure,

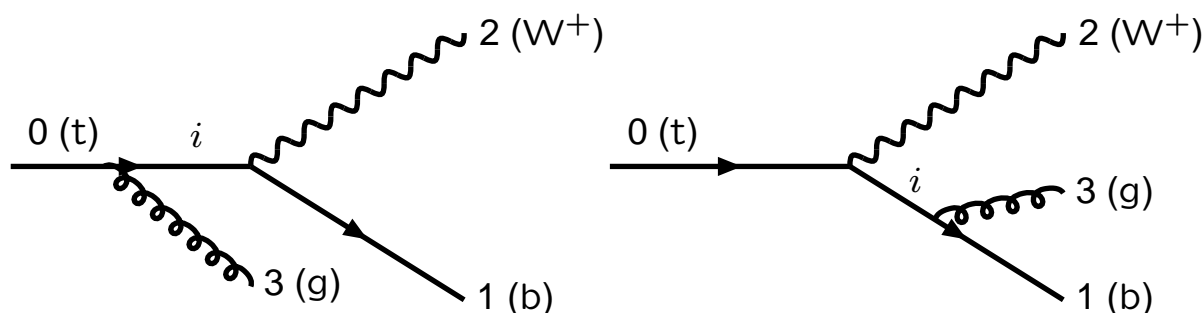
$Q_i^2 = m_i^2 - m_{i,\text{onshell}}^2$  is relevant propagator;

$$\frac{1}{\sigma_0} \frac{d\sigma_{\text{ME}}}{dx_1 dx_2} = \frac{(\dots)}{Q_1^2 Q_2^2} - \frac{(\dots)}{Q_1^4} - \frac{(\dots)}{Q_2^4}$$

$z$  now more messy but  $dz/(1-z) = dx_1/x_3$  still

$$W_i^{\text{ME}} = \frac{1/Q_i^2}{1/Q_1^2 + 1/Q_2^2} \frac{1}{\sigma_0} \frac{d\sigma_{\text{ME}}}{dx_1 dx_2}$$

Also radiation from decaying particle:



$$Q_0^2 = |m_i^2 - m_0^2| = |(p_0 - p_3)^2 - m_0^2| = x_3 E_{\text{CM}}^2$$

ME  $\frac{1}{Q_0^2 Q_1^2}$  matches PS  $b \rightarrow bg$

$\Rightarrow$  can match PS to generic  $a \rightarrow bcg$  ME

subsequent branchings  $q \rightarrow qg$ : also matched to ME, with reduced energy of system

Calculate for  $1 \rightarrow 2$  processes in SM + MSSM:

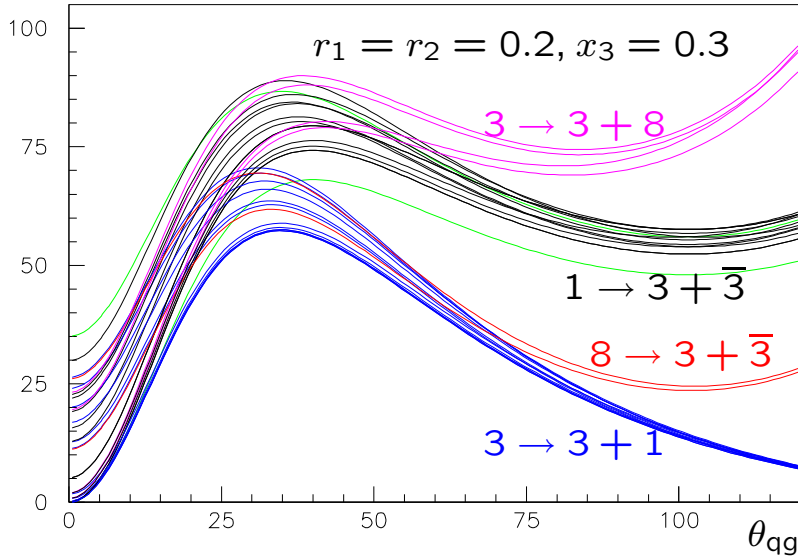
$$W^{ME}(x_1, x_2) = \frac{1}{\sigma(a \rightarrow bc)} \frac{d\sigma(a \rightarrow bcd)}{dx_1 dx_2}$$

Depends on

- mass ratios  $r_1 = m_b/m_a$  and  $r_2 = m_c/m_a$
- colour and spin structure
- vector vs. axial vector etc. ( $\gamma_5$ )

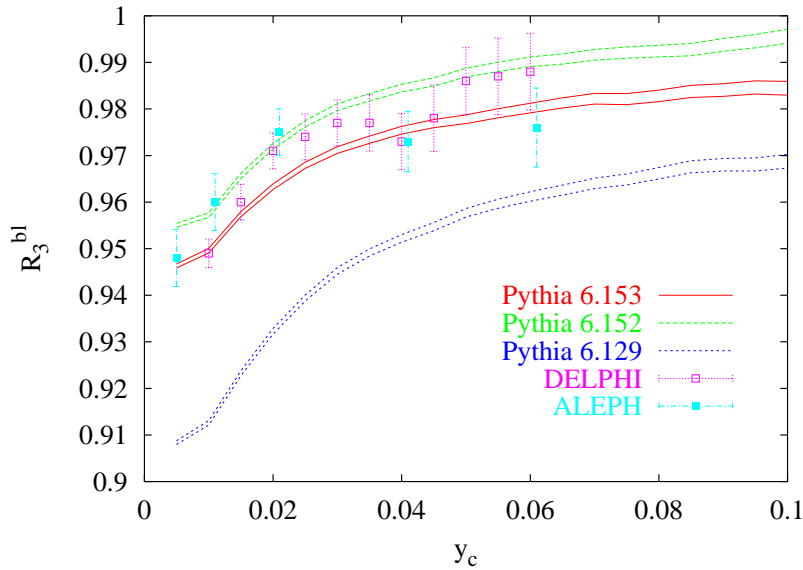
colour	spin	$\gamma_5$	example
$1 \rightarrow 3 + \bar{3}$	—	—	(eikonal)
$1 \rightarrow 3 + \bar{3}$	$1 \rightarrow \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$Z^0 \rightarrow q\bar{q}$
$3 \rightarrow 3 + 1$	$\frac{1}{2} \rightarrow \frac{1}{2} + 1$	$1, \gamma_5, 1 \pm \gamma_5$	$t \rightarrow bW^+$
$1 \rightarrow 3 + \bar{3}$	$0 \rightarrow \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$H^0 \rightarrow q\bar{q}$
$3 \rightarrow 3 + 1$	$\frac{1}{2} \rightarrow \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$t \rightarrow bH^+$
$1 \rightarrow 3 + \bar{3}$	$1 \rightarrow 0 + 0$	1	$Z^0 \rightarrow \tilde{q}\bar{\tilde{q}}$
$3 \rightarrow 3 + 1$	$0 \rightarrow 0 + 1$	1	$\tilde{q} \rightarrow \tilde{q}'W^+$
$1 \rightarrow 3 + \bar{3}$	$0 \rightarrow 0 + 0$	1	$H^0 \rightarrow \tilde{q}\bar{\tilde{q}}$
$3 \rightarrow 3 + 1$	$0 \rightarrow 0 + 0$	1	$\tilde{q} \rightarrow \tilde{q}'H^+$
$1 \rightarrow 3 + \bar{3}$	$\frac{1}{2} \rightarrow \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$\chi \rightarrow q\bar{\tilde{q}}$
$3 \rightarrow 3 + 1$	$0 \rightarrow \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{q} \rightarrow q\chi$
$3 \rightarrow 3 + 1$	$\frac{1}{2} \rightarrow 0 + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$t \rightarrow \tilde{t}\chi$
$8 \rightarrow 3 + \bar{3}$	$\frac{1}{2} \rightarrow \frac{1}{2} + 0$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{g} \rightarrow q\bar{\tilde{q}}$
$3 \rightarrow 3 + 8$	$0 \rightarrow \frac{1}{2} + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$\tilde{q} \rightarrow q\tilde{g}$
$3 \rightarrow 3 + 8$	$\frac{1}{2} \rightarrow 0 + \frac{1}{2}$	$1, \gamma_5, 1 \pm \gamma_5$	$t \rightarrow \tilde{t}\tilde{g}$





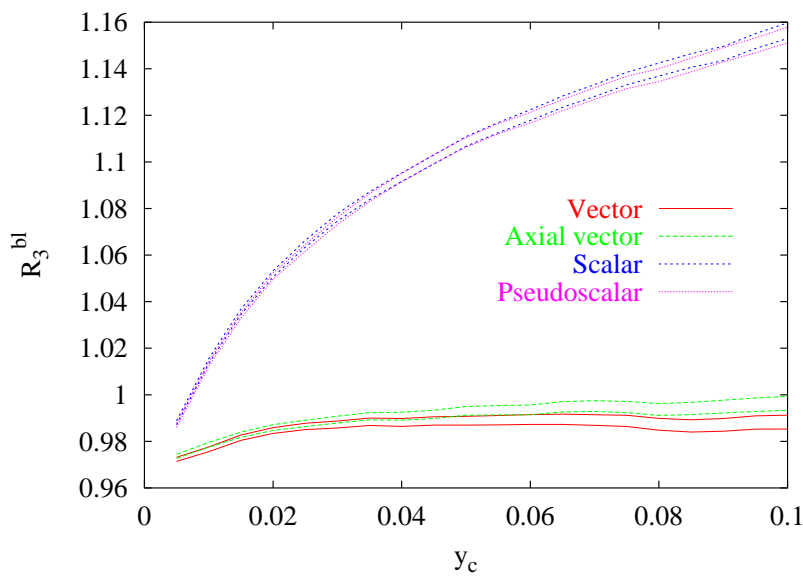
$$W^{\text{ME}}(x_1, x_2)$$

g emission rate  
 for different  
 colour, spin and  
 parity structures



$$R_3^{\text{bl}}(y_c)$$

$E_{\text{CM}} = 91 \text{ GeV}$   
 $m_b = 4.8 \text{ GeV}$   
 ratio of 3-jets  
 in b and uds (=l)  
 events

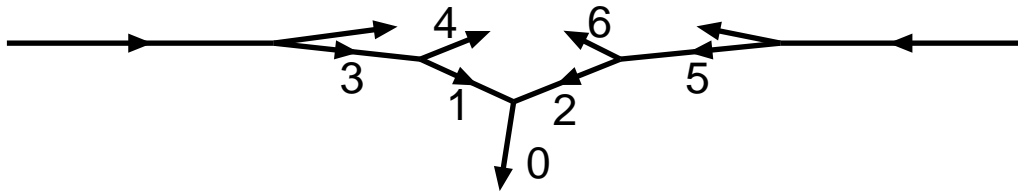


$$R_3^{\text{bl}}(y_c)$$

$E_{\text{CM}} = m_{h/H/A}$   
 $= 120 \text{ GeV}$   
 $m_b = 4.8 \text{ GeV}$   
 reference light q  
 from  $\gamma^*/Z^*$

# Merging in ISR: massless case

(G. Miu & TS, PLB449 (1999) 313)



2  $\rightarrow$  2 process  $q(3) + \bar{q}'(2) \rightarrow g(4) + W(0)$ :

$$\hat{s} = (p_3 + p_2)^2 = \frac{(p_1 + p_2)^2}{z} = \frac{m_W^2}{z}$$

$$\hat{t} = (p_3 - p_4)^2 = p_1^2 = -Q^2$$

$$\hat{u} = m_W^2 - \hat{s} - \hat{t} = Q^2 - \frac{1-z}{z} m_W^2$$

Relate **ME** and **PS** rates:

$$\left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{\text{ME}} = \frac{\sigma_0}{\hat{s}} \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2 \hat{s}}{\hat{t}\hat{u}}$$

$$\xrightarrow{Q^2 \rightarrow 0} \sigma_0 \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z} \frac{1}{Q^2} = \left. \frac{d\hat{\sigma}}{dQ^2} \right|_{\text{PS1}}$$

$$\left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{\text{PS1}} = \frac{\sigma_0}{\hat{s}} \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{\hat{s}^2 + m_W^4}{\hat{t}(\hat{t} + \hat{u})}$$

Add mirror  $q(1) + \bar{q}'(5) \rightarrow g(6) + W(0)$ :

$$\left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{\text{PS}} = \left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{\text{PS1}} + \left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{\text{PS2}} = \frac{\sigma_0}{\hat{s}} \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{\hat{s}^2 + m_W^4}{\hat{t}\hat{u}}$$

$$R_{q\bar{q}' \rightarrow gW}(\hat{s}, \hat{t}) = \frac{(d\hat{\sigma}/d\hat{t})_{ME}}{(d\hat{\sigma}/d\hat{t})_{PS}} = \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{s}^2 + m_W^4}$$

$$\frac{1}{2} < R_{q\bar{q}' \rightarrow gW}(\hat{s}, \hat{t}) \leq 1$$

Similarly for  $q(1) + g(5) \rightarrow q'(6) + W(0)$ :

$$\left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{ME} = \frac{\sigma_0}{\hat{s}} \frac{\alpha_s}{2\pi} \frac{1}{2} \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{-\hat{s}\hat{u}}$$

$$\xrightarrow{Q^2 \rightarrow 0} \sigma_0 \frac{\alpha_s}{2\pi} \frac{1}{2} \frac{z^2 + (1-z)^2}{Q^2} = \left. \frac{d\hat{\sigma}}{dQ^2} \right|_{PS}$$

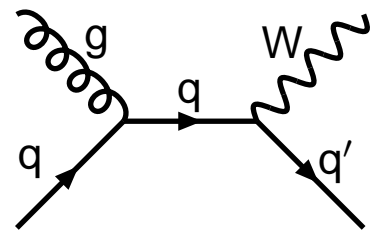
$$\left. \frac{d\hat{\sigma}}{d\hat{t}} \right|_{PS} = \frac{\sigma_0}{\hat{s}} \frac{\alpha_s}{2\pi} \frac{1}{2} \frac{\hat{s}^2 + 2m_W^2(\hat{t} + \hat{u})}{-\hat{s}\hat{u}}$$

$$R_{qg \rightarrow q'W}(\hat{s}, \hat{t}) = \frac{(d\hat{\sigma}/d\hat{t})_{ME}}{(d\hat{\sigma}/d\hat{t})_{PS}} = \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{(\hat{s} - m_W^2)^2 + m_W^4}$$

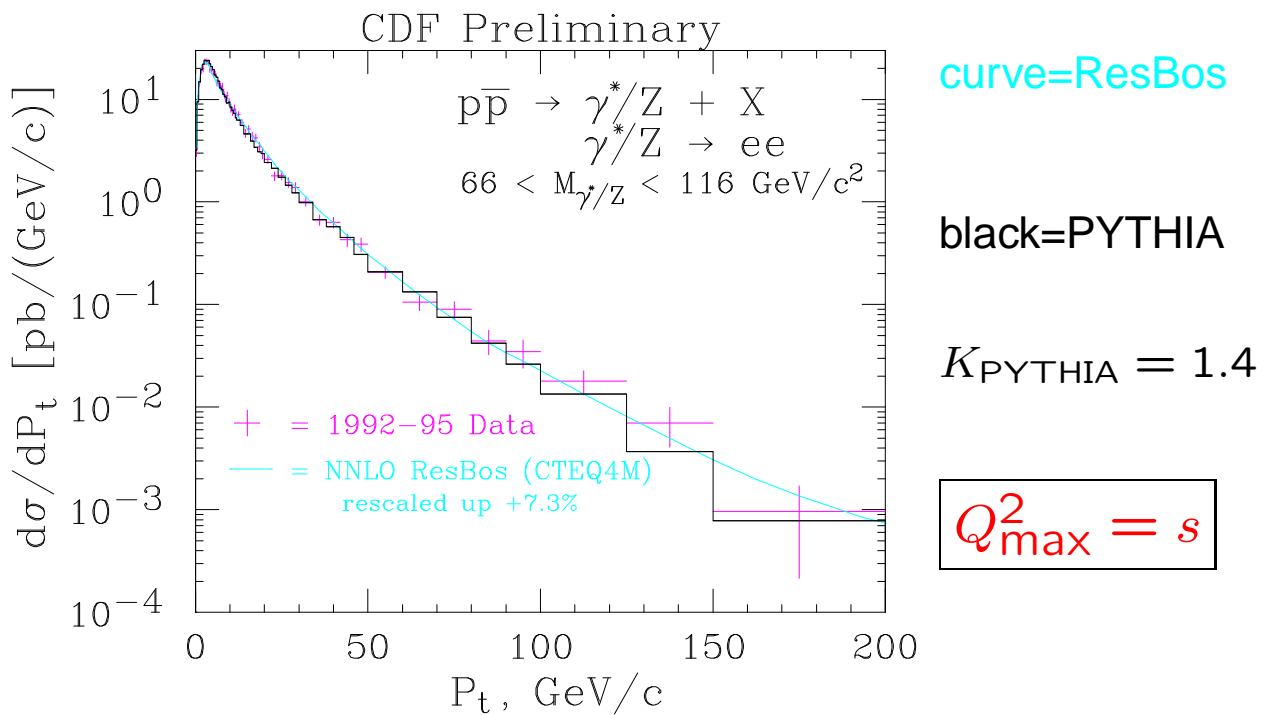
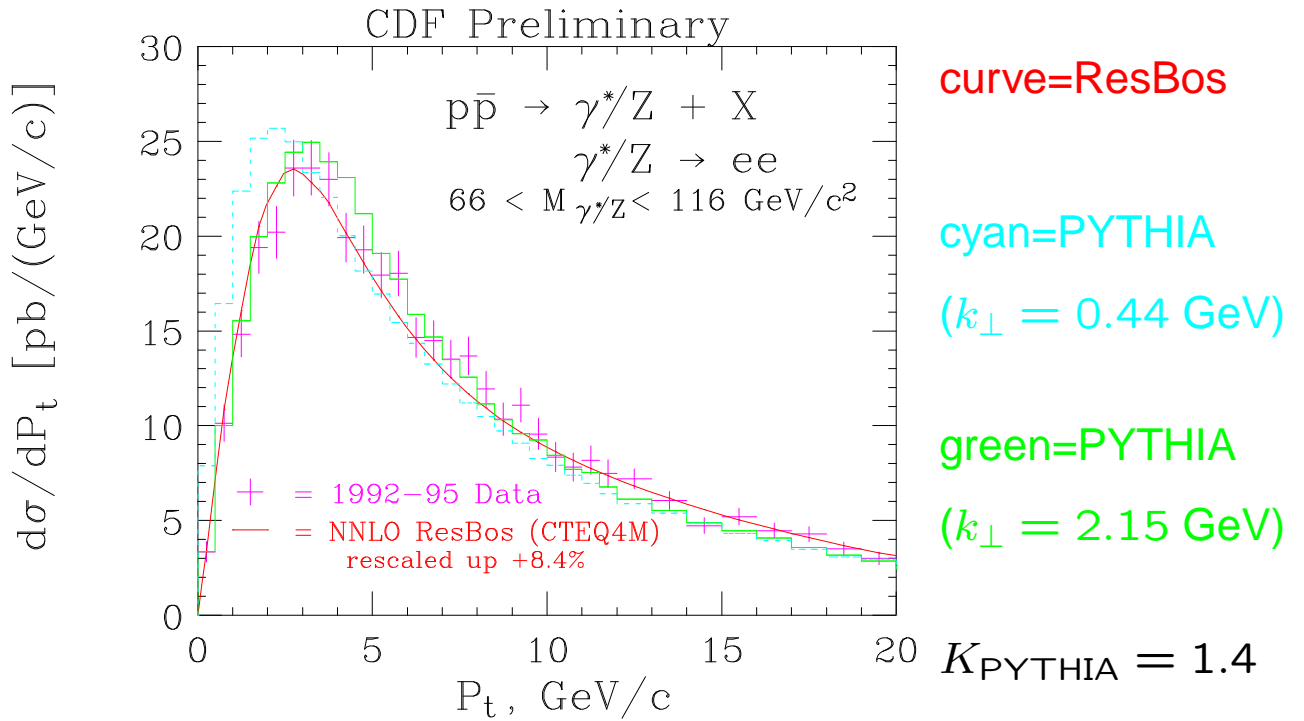
$$1 \leq R_{qg \rightarrow q'W}(\hat{s}, \hat{t}) \leq \frac{\sqrt{5} - 1}{2(\sqrt{5} - 2)} < 3$$

(?) Larger  $R_{qg}$  than  $R_{q\bar{q}'}$   
since PS misses  $s$ -channel  
graph of ME:

“resonance decay”,  
“final-state radiation”



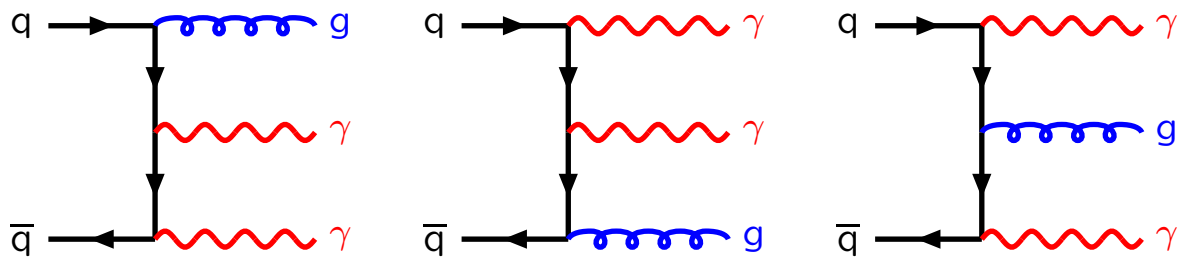
C. Balázs, J. Huston and I. Puljak,  
PRD63 (2001) 014021



# Merging: much more to do!

## ISR for gauge boson pairs

(S. Burby & TS, studies begun)



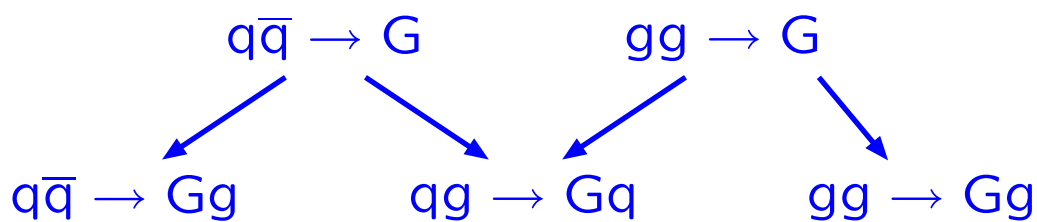
$q\bar{q} \rightarrow \gamma\gamma g$ :

not only  $(q\bar{q} \rightarrow \gamma\gamma) \oplus (q \rightarrow qg)$

but also  $(q\bar{q} \rightarrow g\gamma) \oplus (q \rightarrow q\gamma)$

$\Rightarrow$  more shower histories  $\approx$  divergences,  
more difficult to ensure  $W^{ME} \leq W^{PS}$

## ISR in Graviton/Higgs/... production



also Higgs loop  $\Rightarrow$  much more messy

QCD  $2 \rightarrow 2$  processes, e.g.  $gg \rightarrow gg$ ,  
with consistent merging both to ISR and FSR

# NLO Showering I

(S. Mrenna, UCD-99-4 [hep-ph/0010012])

ISR PS is a  $Q_T$ -resummation calculation

$$= \mathbf{e}^{-S_0^1} \otimes \mathbf{e}^{-S_1^2} \dots \mathbf{e}^{-S_{N-1}^N} \sigma_0 \mathbf{e}^{-S_{M-1}^M} \dots \mathbf{e}^{-S_1^2} \otimes \mathbf{e}^{-S_0^1}$$

where  $\mathbf{e}^{-S_i^j} = r \in [0, 1]$  and  $\sigma_0 = \sigma(q\bar{q}' \rightarrow W)$

$$S_i^j = - \int_{t_i}^{t_j} dt'' \int_{\frac{x}{1-\epsilon}}^{\frac{x}{x+\epsilon}} dz \frac{\alpha_s(z, t'')}{2\pi} \hat{P}_{a \rightarrow bc}(z) \frac{x' f_a(x', t_i)}{x f_b(x, t_i)}$$

allow all branchings  $N + M = [0, \infty]$

$\Rightarrow$  generates NLL  $d\sigma/dQ_T^2$ , LO  $\sigma$

Can  $Q_T$ -resummation generate a PS?

$$\frac{d\sigma(WX)}{dQ^2 dQ_T^2 dy} = \frac{d}{dQ_T^2} \tilde{W}(Q_T, Q) + Y(Q_T, Q)$$

$$\tilde{W} = \mathbf{e}^{-S_{Q_T}^Q} (C \otimes f)[Q_T] (C \otimes f)[Q_T] H(Q)$$

$$S_{Q_T}^Q = \int_{Q_T^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[ \ln \left( \frac{Q^2}{\bar{\mu}^2} \right) A(\alpha_s(\bar{\mu})) + B(\alpha_s(\bar{\mu})) \right]$$

$$(C_{jl} \otimes f_{l/h_1})(x_1, \mu) = \int_{x_1}^1 \frac{d\xi_1}{\xi_1} C_{jl}\left(\frac{x_1}{\xi_1}, \mu\right) f_{l/h_1}(\xi_1, \mu)$$

with  $H, A, B, C$  calculable in pert. theory

Ignoring  $Y$  and other kinematical dependence,

$$\frac{d\sigma(WX)}{dQ_T^2} = \sigma_1 \frac{d}{dQ_T^2} \left[ e^{-S_{Q_T}^Q} \frac{(C \otimes f)[Q_T] (C \otimes f)[Q_T]}{(C \otimes f)[Q] (C \otimes f)[Q]} \right]$$

$$\sigma_1 = \kappa \int \frac{dx_1}{x_1} (C \otimes f)[Q] (C \otimes f)[Q]$$

$\sigma_1$  is the higher-order cross section ( $-Y$ )  
integral over  $Q_T^2$  is unity

$$\sigma_0 = \kappa \int \frac{dx_1}{x_1} f(x_1, Q) f(x_2, Q) \quad (C \sim \delta(1-x))$$

Are the Sudakov's the same?

$$\int_{z_m}^{1-z_m} dz C_F \left( \frac{1+z^2}{1-z} \right) \simeq A^{(1)} \ln(Q^2/Q_T^2) + B^{(1)}$$

new PS algorithm  $\Rightarrow$  use  $C \otimes f$  instead of  $f$  generates higher-order rate  
slightly modifies showering

What about  $Y$ ?

$Y$  corrects for soft gluon approximation

At NLO,  $Y \propto$

$$|\mathcal{M}(q\bar{q}' \rightarrow Wg)|^2 + |\mathcal{M}(qg \rightarrow W\bar{q}')|^2 - \widetilde{W}|_{1/Q_T^2}$$

for  $Q \geq Q_T$ , the total rate is given by the hard emission processes

But  $Y$  has  $+$  and  $-$  weights!

(Also  $\widetilde{W} < 0$  for  $Q_T \sim 50$  GeV)

How to make hard correction with explicit emissions?

keep track of signs and cancel?

modify  $\widetilde{W}$ ? (Promising)

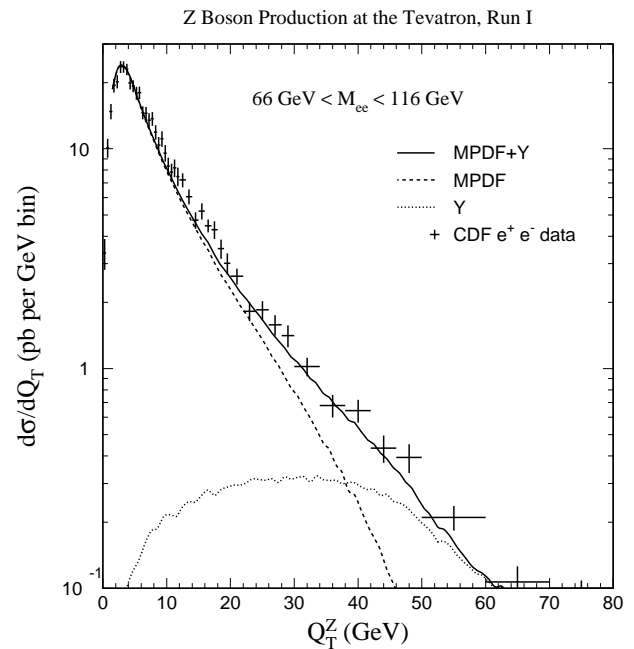
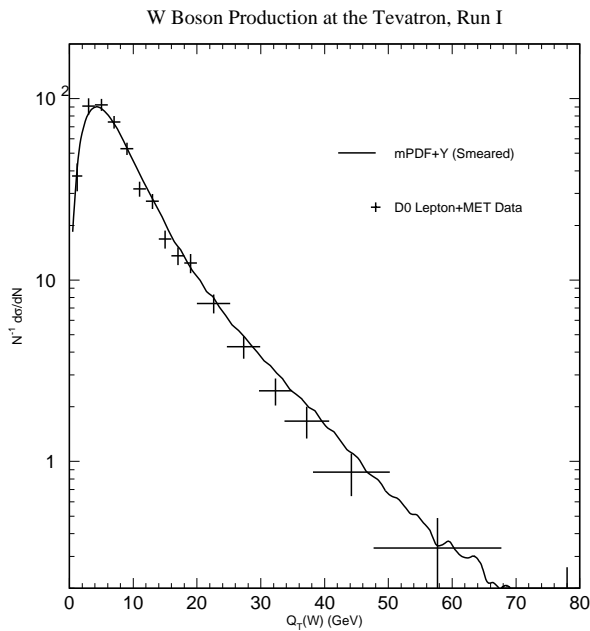
for now, guarantee average behavior

$$\widetilde{W}|_{1/Q_T^2} \rightarrow (1 - f_{\text{cor}})|\mathcal{M}|^2$$



DØ  $P_T^W$

CDF  $P_T^Z$



CTEQ4M and primordial  $k_T = 2 \text{ GeV}$

$Q_T < 50 \text{ GeV}$

$$f_{\text{COR}} = \frac{1}{2} (Q_T/50)^2 \times (1 + Q_T/25)$$

$Q_T \geq 50 \text{ GeV}$

hard  $W + g$  and  $W + \bar{q}(q)$  emissions

Cut off PS at  $Q_T = 50 \text{ GeV}$

(where analytic resum  $< 0$ ).

# NLO Matching II

(L. Lönnblad, work in progress)

Current study: resonance decay  
(e.g.  $e^+e^-$  annihilation)

Basic idea: add Sudakovs to tree-level  $\alpha_S^2$  ME  
by veto procedure.

- 1) Pick event according to ME tree-level ( $q\bar{q}$ ,  $qg\bar{q}$ ,  $qgg\bar{q}$  or  $q\bar{q}q'\bar{q}'$ ), fixed (overestimated)  $\alpha_{S0}$  and some cut-off  $Q_0$
- 2) Enumerate possible shower histories, find the corresponding emission scales  $k_{i\perp}^2$ , estimate probabilities and pick one history accordingly.
- 3) Throw with probability  $\prod_i \alpha_S(k_{i\perp}^2) / \alpha_{S0}$
- 4a) If a 2-jet event, make a trial emission. If it is larger than  $Q_0$  throw. Otherwise continue with emissions below  $Q_0$ .
- 4b) If a 3-jet event, make a trial emission from the original  $q\bar{q}$ . If it is larger than  $k_{1\perp}^2$  throw. Otherwise make trial emission from  $qg\bar{q}$ . If it is larger than  $Q_0$  throw. Otherwise continue with emissions below  $Q_0$ .

4c) If a 4-jet event, make a trial emission from the original  $q\bar{q}$ . If it is larger than  $k_{1\perp}^2$  throw. Otherwise make trial emission from reconstructed  $qg\bar{q}$ . If it is larger than  $k_{2\perp}^2$  throw. Otherwise continue with emissions below  $k_{2\perp}^2$ .

This will give emission diagrams correct to  $\alpha_S^2$  and leading log, and virtual diagrams correct to leading log accuracy.

Virtual diagrams could also be corrected by including the difference between the exact and leading log terms in the generation of the ME.

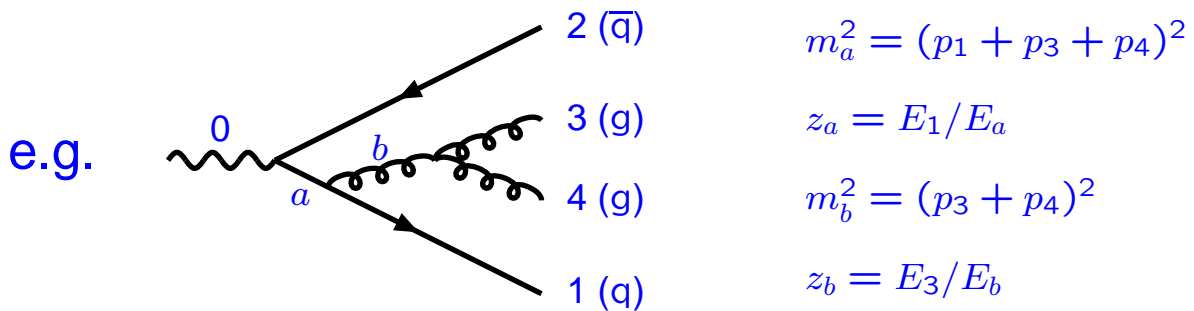
Is being implemented in Ariadne. Difficulties due to invariant mass cutoff in ME, but evolution in transverse momenta in dipole cascade.

# (N)LO Matching III

(J. André & TS, PRD57 (1998) 5767)

- 1) Pick event according to ME
- 2) Enumerate possible shower histories, finding equivalent shower variables  $(m^2, z, \varphi)$
- 3) Estimate probability for each PS history, and pick one accordingly
- 4) Develop shower from original parton pair, with some branchings now predetermined and the rest as normal

Currently only  $e^+e^- \rightarrow q\bar{q}gg, q\bar{q}q'\bar{q}'$



$$\mathcal{P} = \mathcal{P}_{a \rightarrow 1b} \mathcal{P}_{b \rightarrow 34} = \frac{1}{m_a^2} \frac{4}{3} \frac{1+z_a^2}{1-z_a} \frac{1}{m_b^2} 3 \frac{(1-z_b(1-z_b))^2}{z_b(1-z_b)}$$

Could be combined with NLO ME generator, by vetoing 'normal' shower emissions that doublecount emissions already in ME