Theory of Hadronic Collisions
Part II: Phenomenology

Torbjörn Sjöstrand
Lund University

1. (yesterday) Introduction and Overview; Parton Showers
2. (today) Matching Issues; Multiple Interactions I
3. (on Monday) Hadronization; MI II/LHC; Generators & Conclusions
Matrix Elements vs. Parton Showers

**ME : Matrix Elements**
- systematic expansion in $\alpha_s$ (*exact*)
- powerful for multiparton Born level
- flexible phase space cuts
  - loop calculations very tough
  - negative cross section in collinear regions
    $\Rightarrow$ unpredictable jet/event structure
  - *no easy match to hadronization*

**PS : Parton Showers**
- approximate, to LL (or NLL)
- main topology not predetermined
  $\Rightarrow$ inefficient for exclusive states
+ process-generic $\Rightarrow$ simple multiparton
+ Sudakov form factors/resummation
  $\Rightarrow$ sensible jet/event structure
+ *easy to match to hadronization*
\( p_\perp \) (1 jet) \hfill \( p_\perp^{\text{max}} \) (2 jets) \hfill \( p_\perp^{\text{min}} \) (2 jets)

\[
\begin{align*}
\frac{d\sigma}{dp_T} &\text{(pb/GeV)} \\
|\eta_j| < 5, \Delta R_{jj} > 0.4 \\
K_{\text{Pythia}} = 1.8 \\

\text{LHC: sps1a} \\
&\text{Susy-MadGraph} \\
&\text{Pythia: } p_T^2 \text{ (power)} \\
&\text{ } p_T^2 \text{ (wimpy)} \\
&\text{ } Q^2 \text{ (power)} \\
&\text{ } Q^2 \text{ (wimpy)} \\
&\text{ } Q^2 \text{ (tune A)}
\end{align*}
\]

\[
\begin{align*}
\frac{d\sigma}{dp_T} &\text{(pb/GeV)} \\
|\eta_j| < 5, \Delta R_{jj} > 0.4 \\
K_{\text{Pythia}} = 1.75 \\

\text{LHC: } \text{sps1amod} \\
&\text{Susy-MadGraph} \\
&\text{Pythia: } p_T^2 \text{ (power)} \\
&\text{ } p_T^2 \text{ (wimpy)} \\
&\text{ } Q^2 \text{ (power)} \\
&\text{ } Q^2 \text{ (wimpy)} \\
&\text{ } Q^2 \text{ (tune A)}
\end{align*}
\]

power: \( Q^2_{\text{max}} = s \); \ hpace{5mm} \text{wimpy: } Q^2_{\text{max}} = m^2_\perp \); \ hpace{5mm} \text{tune A: } Q^2_{\text{max}} = 4m^2_\perp \)

\( m_t = 175 \text{ GeV}, \ m\bar{g} = 608 \text{ GeV}, \ m\bar{u}_L = 567 \text{ GeV} \)

(T. Plehn, D. Rainwater, P. Skands)
Matrix Elements and Parton Showers

Recall complementary strengths:

- ME’s good for well separated jets
- PS’s good for structure inside jets

Marriage desirable! But how?

Problems:
- gaps in coverage?
- doublecounting of radiation?
- Sudakov?
- NLO consistency?

Much work ongoing \( \implies \) no established orthodoxy

Three main areas, in ascending order of complication:

1) Match to lowest-order nontrivial process — merging
2) Combine leading-order multiparton process — vetoed parton showers
3) Match to next-to-leading order process — MC@NLO
Merging

= cover full phase space with smooth transition ME/PS

Want to reproduce

\[ W^{\text{ME}} = \frac{1}{\sigma(\text{LO})} \frac{d\sigma(\text{LO} + g)}{d(\text{phasespace})} \]

by shower generation + correction procedure

\[
\begin{align*}
\text{wanted} \quad W^{\text{ME}} & = \quad \text{generated} \quad W^{\text{PS}} \\
\text{correction} \quad \frac{W^{\text{ME}}}{W^{\text{PS}}} & =
\end{align*}
\]

- Exponentiate ME correction by shower Sudakov form factor:

\[
W^{\text{PS}}_{\text{actual}}(Q^2) = W^{\text{ME}}(Q^2) \exp \left( - \int_{Q^2}^{Q^2_{\text{max}}} W^{\text{ME}}(Q'^2) dQ'^2 \right)
\]

- Do not normalize \( W^{\text{ME}} \) to \( \sigma(\text{NLO}) \) (error \( O(\alpha_s^2) \) either way)

\[
\begin{align*}
1 + O(\alpha_s) & \quad \int = 1 \\
d\sigma & = K \sigma_0 \ dW^{\text{PS}}
\end{align*}
\]

- Normally several shower histories \( \Rightarrow \sim \) equivalent approaches
Merging with \( \gamma^*/Z^0 \rightarrow q\bar{q}g \) for \( m_q = 0 \) since long


For \( m_q > 0 \) pick \( Q_i^2 = m_i^2 - m_i^{2,\text{onshell}} \) as evolution variable since

\[
W^{\text{ME}} = \frac{(\ldots)}{Q_1^2} - \frac{(\ldots)}{Q_2^4} - \frac{(\ldots)}{Q_2^4}
\]

Coloured decaying particle also radiates:

\[
\Rightarrow \text{can merge PS with generic } a \rightarrow bcg \text{ ME}
\]

(E. Norrbin & TS, NPB603 (2001) 297)

Subsequent branchings \( q \rightarrow qg \): also matched to ME, with reduced energy of system
PYTHIA performs merging with generic FSR \( a \to b c g \) ME,
in SM: \( \gamma^*/Z^0/W^\pm \to q\bar{q}, t \to bW^+, H^0 \to q\bar{q}, \)
and MSSM: \( t \to bH^+, Z^0 \to q\bar{q}, \tilde{q} \to q'h^+ \), \( H^0 \to q\bar{q}, \tilde{q} \to q'h^+, \chi \to q\bar{q}, \chi \to q\bar{q}, \tilde{q} \to q\chi, t \to \tilde{t}\chi, \tilde{g} \to q\bar{q}, \tilde{q} \to q\tilde{g}, t \to \tilde{t}\tilde{g} \)
g emission for different colour, spin and parity:

\[ R^b_{3\ell}(y_c) : \text{mass effects in Higgs decay:} \]
Initial-State Shower Merging

\[ \frac{d\sigma}{dp_{\perp Z}} \]

- Z + 1 jet 'exact'
- physical
- shower: ditto
  + accompanying jets (exclusive)

Merged with matrix elements for
\[ q\bar{q} \rightarrow (\gamma^*/Z^0/W^\pm)g \]
\[ qg \rightarrow (\gamma^*/Z^0/W^\pm)q' \]

\[ \left( \frac{W^{ME}}{W^{PS}} \right)_{q\bar{q}' \rightarrow gW} = \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{s}^2 + m_W^4} \leq 1 \]

with \( Q^2 = -m^2 \)
and \( z = m_W^2/\hat{s} \)

\[ \left( \frac{W^{ME}}{W^{PS}} \right)_{qg \rightarrow q'W} = \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{(\hat{s} - m_W^2)^2 + m_W^4} < 3 \]
Merging in HERWIG

HERWIG also contains merging, for
- $Z^0 \rightarrow q\bar{q}$
- $t \rightarrow bW^+$
- $q\bar{q} \rightarrow Z^0$
and some more

Special problem:
angular ordering does not cover full phase space; so
(1) fill in “dead zone” with ME
(2) apply ME correction in allowed region

Important for agreement with data:
Vetoed Parton Showers


Generic method to combine ME’s of several different orders to NLL accuracy; will be a ‘standard tool’ in the future

Basic idea:
• consider (differential) cross sections \( \sigma_0, \sigma_1, \sigma_2, \sigma_3, \ldots \), corresponding to a lowest-order process (e.g. W or H production), with more jets added to describe more complicated topologies, in each case to the respective leading order
• \( \sigma_i, i \geq 1 \), are divergent in soft/collinear limits
• absent virtual corrections would have ensured “detailed balance”, i.e. an emission that adds to \( \sigma_{i+1} \) subtracts from \( \sigma_i \)
• such virtual corrections correspond (approximately) to the Sudakov form factors of parton showers
• so use shower routines to provide missing virtual corrections \( \Rightarrow \) rejection of events (especially) in soft/collinear regions
Veto scheme:
1) Pick hard process, mixing according to \( \sigma_0 : \sigma_1 : \sigma_2 : \ldots \), above some ME cutoff (e.g. all \( p_{\perp i} > p_{\perp 0} \), all \( R_{ij} > R_0 \)), with large fixed \( \alpha_s s_0 \)
2) Reconstruct imagined shower history (in different ways)
3) Weight \( W_\alpha = \prod \text{branchings} (\alpha_s (k_{\perp i}^2) / \alpha_s s_0) \Rightarrow \text{accept/reject} 

CKKW-L:
4) Sudakov factor for non-emission on all lines above ME cutoff
   \[ W_{\text{Sud}} = \prod \text{“propagators”} \]
   \[ \text{Sudakov}(k_{\perp \text{beg}}, k_{\perp \text{end}}) \]
4a) CKKW : use NLL Sudakovs
4b) L: use trial showers
5) \( W_{\text{Sud}} \Rightarrow \text{accept/reject} 
6) \text{do shower, vetoing emissions above cutoff}

MLM:
4) do parton showers
5) (cone-)cluster showered event
6) match partons and jets
7) if all partons are matched, and \( n_{\text{jet}} = n_{\text{parton}} \), keep the event, else discard it
CKKW mix of $W + (0, 1, 2, 3, 4)$ partons, hadronized and clustered to jets:

(S.Mrenna, P. Richardson)
MC@NLO

Objectives:
- Total rate should be accurate to NLO.
- NLO results are obtained for all observables when (formally) expanded in powers of $\alpha_s$.
- Hard emissions are treated as in the NLO computations.
- Soft/collinear emissions are treated as in shower MC.
- The matching between hard and soft emissions is smooth.
- The outcome is a set of “normal” events, that can be processed further.

Basic scheme (simplified!):
1) Calculate the NLO matrix element corrections to an $n$-body process (using the subtraction approach).
2) Calculate analytically (no Sudakov!) how the first shower emission off an $n$-body topology populates $(n + 1)$-body phase space.
3) Subtract the shower expression from the $(n + 1)$ ME to get the “true” $(n + 1)$ events, and consider the rest of $\sigma_{\text{NLO}}$ as $n$-body.
4) Add showers to both kinds of events.
\( \frac{d\sigma}{dp_{\perp Z}} \)

**simplified example**

- **Z + 1 jet ‘exact’**
  - \( Z + 1 \) jet according to shower (first emission, without Sudakov)
  - generate as \( Z + 1 \) jet + shower

Disadvantage: not perfect match everywhere, so can lead to events with negative weight, \( \sim 10\% \) when normalized to \( \pm 1 \).

**MC@NLO in comparison:**
- Superior with respect to “total” cross sections.
- Equivalent to merging for event shapes (differences higher order).
- Inferior to CKKW–L for multijet topologies.

\( \Rightarrow \) pick according to current task and availability.
Works identically to HERWIG: the very same analysis routines can be used.

- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- Features a self contained library of PDFs with old and new sets alike
- LHAPDF will also be implemented
These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly.

**Solid:** MC@NLO

**Dashed:** HERWIG $\times \frac{\sigma_{NLO}}{\sigma_{LO}}$

**Dotted:** NLO
Multiple Interactions
What is multiple interactions?

Cross section for $2 \rightarrow 2$ interactions is dominated by $t$-channel gluon exchange, so diverges like $d\sigma/dp_T^2 \approx 1/p_T^4$ for $p_T \rightarrow 0$.

Integrate QCD $2 \rightarrow 2$

- $qq' \rightarrow qq'$
- $q\bar{q} \rightarrow q'\bar{q}'$
- $q\bar{q} \rightarrow gg$
- $qq \rightarrow qg$
- $gg \rightarrow gg$
- $gg \rightarrow q\bar{q}$

with CTEQ 5L PDF's

[Graph showing integrated cross section above $p_T$ for pp at 14 TeV]
So $\sigma_{\text{int}}(p_{\perp \text{min}}) > \sigma_{\text{tot}}$ for $p_{\perp \text{min}} \lessapprox 5$ GeV

Half a solution: many interactions per event

\[
\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n
\]

\[
\sigma_{\text{int}} = \sum_{n=0}^{\infty} n \sigma_n
\]

\[
\sigma_{\text{int}} > \sigma_{\text{tot}} \iff \langle n \rangle > 1
\]

If interactions occur independently then Poissonian statistics

\[
P_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle}
\]

but energy–momentum conservation $\Rightarrow$ large $n$ suppressed
Other half of solution:
perturbative QCD not valid at small $p_\perp$ since q, g not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp \text{min}} \approx \frac{\hbar}{r_p} \approx \frac{0.2 \text{ GeV} \cdot \text{fm}}{0.7 \text{ fm}} \approx 0.3 \text{ GeV} \approx \Lambda_{\text{QCD}}$$

...but better replace $r_p$ by (unknown) colour screening length $d$ in hadron

\[ \lambda \sim \frac{1}{p_\perp} \]
so modify

\[
\frac{d\hat{\sigma}}{dp_\perp^2} \propto \frac{\alpha_S^2(p_\perp^2)}{p_\perp^4} \rightarrow \frac{\alpha_S^2(p_\perp^2)}{p_\perp^4} \theta(p_\perp - p_\perp\text{min}) \quad \text{(simpler)}
\]

or

\[
\rightarrow \frac{\alpha_S^2(p_{\perp0}^2 + p_\perp^2)}{(p_{\perp0}^2 + p_\perp^2)^2} \quad \text{(more physical)}
\]

where \(p_{\perp\text{min}}\) or \(p_{\perp0}\) are free parameters, empirically of order 2 GeV

Typically 2 – 3 interactions/event at the Tevatron, 4 – 5 at the LHC, but may be more in “interesting” high-\(p_\perp\) ones.
Modelling multiple interactions

T. Sjöstrand, M. van Zijl, PRD36 (1987) 2019: first model(s) for event properties based on perturbative multiple interactions

(1) Simple scenario:
- Sharp cut-off at $p_{\perp\text{min}}$ main free parameter
- Is only a model for nondiffractive events, i.e. for $\sigma_{\text{nd}} \simeq (2/3)\sigma_{\text{tot}}$
- Average number of interactions is $\langle n \rangle = \sigma_{\text{int}}(p_{\perp\text{min}})/\sigma_{\text{nd}}$
- Interactions occur almost independently, i.e.
  Poissonian statistics $P_n = \langle n \rangle^n e^{-\langle n \rangle}/n!$
  with fraction $P_0 = e^{-\langle n \rangle}$ pure low-$p_{\perp}$ events
- Interactions generated in ordered sequence $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > \cdots$
  by “Sudakov” trick (what happens “first”?)

\[
\frac{dP}{dp_{\perp i}} = \frac{1}{\sigma_{\text{nd}}dp_{\perp}} \exp \left[ - \int_{p_{\perp}}^{p_{\perp(i-1)}} \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp_{\perp}dp_{\perp}} \right]
\]

- Momentum conservation in PDF’s $\Rightarrow P_n$ narrower than Poissonian
- Simplify after first interaction: only $gg$ or $q\bar{q}$ outgoing, no showers, ...
(2) More sophisticated scenario:

- Smooth turn-off at $p_{\perp 0}$ scale
- Require $\geq 1$ interaction in an event
- Hadrons are extended, e.g. double Gaussian ("hot spots"):

$$\rho_{\text{matter}}(r) = N_1 \exp \left( -\frac{r_2^2}{r_1^2} \right) + N_2 \exp \left( -\frac{r_2^2}{r_2^2} \right)$$

where $r_2 \neq r_1$ represents "hot spots"

- Events are distributed in impact parameter $b$
- Overlap of hadrons during collision

$$\mathcal{O}(b) = \int d^3x \, dt \, \rho_{1,\text{matter}}^{\text{boosted}}(x, t) \rho_{2,\text{matter}}^{\text{boosted}}(x, t)$$

- Average activity at $b$ proportional to $\mathcal{O}(b)$
  $\Rightarrow$ central collisions normally more active
  $\Rightarrow P_n$ broader than Poissonian
- More time-consuming ($b, p_{\perp}$) generation
- Need for simplifications remains
(3) HERWIG
Soft Underlying Event (SUE), based on UA5 Monte Carlo

- Distribute a (∼ negative binomial) number of clusters independently in rapidity and transverse momentum according to parametrization/extrapolation of data
- modify for overall energy/momentum/flavour conservation
- no minijets; correlations only by cluster decays

(4) Jimmy (HERWIG add-on)
- similar to PYTHIA (2) above; but details different
- matter profile by electromagnetic form factor
- no $p_\perp$-ordering of emissions, no rescaling of PDF: abrupt stop when (if) run out of energy

(5) Phojet/DTUjet
- comes from “historical” tradition of soft physics of “cut Pomerons” $\approx p_\perp \to 0$ limit of multiple interactions
- extended also to “hard” interactions similarly to PYTHIA
without multiple interactions

FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low $p_T$ only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.
FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\text{min}}=2.0$ GeV; solid line, $p_{T\text{min}}=1.6$ GeV; dashed-dotted line, $p_{T\text{min}}=1.2$ GeV.

with multiple interactions

FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.
Evidence for multiple interactions

- Width of multiplicity distribution: UA5, E735
  (previous slides)

- Forward–backward correlations: UA5
  (previous slides)

- Minijet rates: UA1

<table>
<thead>
<tr>
<th>No. jets</th>
<th>UA1 (%)</th>
<th>no MI</th>
<th>simple</th>
<th>double Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.96</td>
<td>14.30</td>
<td>11.51</td>
<td>8.88</td>
</tr>
<tr>
<td>2</td>
<td>3.45</td>
<td>2.45</td>
<td>2.45</td>
<td>2.67</td>
</tr>
<tr>
<td>3</td>
<td>1.12</td>
<td>0.22</td>
<td>0.32</td>
<td>0.74</td>
</tr>
<tr>
<td>4</td>
<td>0.22</td>
<td>0.01</td>
<td>0.04</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>0.00</td>
<td>0.00</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Direct observation: AFS, (UA2,) CDF

Order 4 jets $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > p_{\perp 4}$ and define $\varphi$ as angle between $p_{\perp 1} - p_{\perp 2}$ and $p_{\perp 3} - p_{\perp 4}$

Double Parton Scattering

\[ |p_{\perp 1} + p_{\perp 2}| \approx 0 \]
\[ |p_{\perp 3} + p_{\perp 4}| \approx 0 \] 
\[ \frac{d\sigma}{d\varphi} \text{ flat} \]

Double BremsStrahlung

\[ |p_{\perp 1} + p_{\perp 2}| \gg 0 \]
\[ |p_{\perp 3} + p_{\perp 4}| \gg 0 \] 
\[ \frac{d\sigma}{d\varphi} \text{ peaked at } \varphi \approx 0 \]

AFS 4-jet analysis (pp at 63 GeV);
double bremsstrahlung subtracted:

<table>
<thead>
<tr>
<th>Observed</th>
<th>No MI</th>
<th>Simple MI</th>
<th>Double Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0</td>
<td>1</td>
<td>3.7</td>
</tr>
</tbody>
</table>

in arbitrary units
CDF 3-jet + prompt photon analysis

Yellow region = double parton scattering (DPS)

The rest = PYTHIA showers

\[ \sigma_{\text{DPS}} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}} \] for \( A \neq B \) \[ \Rightarrow \sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb} \]

Strong enhancement relative to naive expectations!
● Jet pedestal effect: UA1, H1, CDF
Events with hard scale (jet, W/Z, ...) have more underlying activity!
Events with $n$ interactions have $n$ chances that one of them is hard,
so “trigger bias”: hard scale $\Rightarrow$ central collision
$\Rightarrow$ more interactions $\Rightarrow$ larger underlying activity.
Centrality effect saturates at $p_{\perp\text{hard}} \sim 10$ GeV.

Studied in detail by Rick Field, comparing with CDF data:

“MAX/MIN Transverse” Densities

- Define the MAX and MIN “transverse” regions on an event-by-event basis with
  MAX (MIN) having the largest (smallest) density.
Plot shows the “Transverse” charged particle density versus $P_T(chgjet#1)$ compared to the QCD hard scattering predictions of two tuned versions of PYTHIA 6.206 (CTEQ5L, Set B (PARP(67)=1) and Tune A CDF Run 2 Default!)}

New PYTHIA default (less initial-state radiation)

Old PYTHIA default (more initial-state radiation)
Tuned PYTHIA 6.206

“Transverse” $P_T$ Distribution

Compares the average “transverse” charge particle density ($|\eta|<1$, $P_T>0.5$ GeV) versus $P_T$(charged jet#1) and the $P_T$ distribution of the “transverse” density, $dN_{chg}/d\eta d\phi dP_T$ with the QCD Monte-Carlo predictions of two tuned versions of PYTHIA 6.206 ($P_T$(hard) > 0, CTEQ5L, Set B (PARP(67)=1) and Set A (PARP(67)=4)).
Leading Jet: “MAX & MIN Transverse” Densities

**PYTHIA Tune A**

**HERWIG**

Charged particle density and PTsum density for “leading jet” events versus $E_T(jet\#1)$ for PYTHIA Tune A and HERWIG.
"Back-to-Back "Associated" Charged Particle Densities"

Shows the $\Delta \phi$ dependence of the "associated" charged particle density, $dN_{\text{ch}}/d\eta d\phi$, $p_T > 0.5$ GeV/c, $|\eta| < 1$, $\text{PTmaxT} > 2.0$ GeV/c (not including $\text{PTmaxT}$) relative to $\text{PTmaxT}$ (rotated to 180°) and the charged particle density, $dN_{\text{ch}}/d\eta d\phi$, $p_T > 0.5$ GeV/c, $|\eta| < 1$, relative to jet#1 (rotated to 270°) for "back-to-back events" with $30 < E_T(\text{jet#1}) < 70$ GeV.
For $PT_{maxT} > 2.0$ GeV both PYTHIA and HERWIG produce slightly too many “associated” particles in the direction of $PT_{maxT}$!

But HERWIG (without multiple parton interactions) produces too few particles in the direction opposite of $PT_{maxT}$!
**Average multiplicity** of charged particles in the underlying event associated to a leading jet with $P_t^{\text{jet}}$ (GeV).

**Average $p_T^{\text{sum}}$ (GeV) of charged particles in the underlying event associated to a leading jet with $P_t^{\text{jet}}$ (GeV).**

**UE tunings: Jimmy validation using CDF data**
Colour correlations

$\langle p_\perp \rangle (n_{\text{ch}})$ is very sensitive to colour flow

long strings to remnants $\Rightarrow$ much $n_{\text{ch}}$/interaction $\Rightarrow$ $\langle p_\perp \rangle (n_{\text{ch}}) \sim$ flat

short strings (more central) $\Rightarrow$ less $n_{\text{ch}}$/interaction $\Rightarrow$ $\langle p_\perp \rangle (n_{\text{ch}})$ rising

FIG. 27. Average transverse momentum of charged particles in $|\eta| < 2.5$ as a function of the multiplicity. UA1 data points (Ref. 49) at 900 GeV compared with the model for different assumptions about the nature of the subsequent (nonhardest) interactions. Dashed line, assuming $q\bar{q}$ scatterings only; dotted line, $gg$ scatterings with “maximal” string length; solid line $gg$ scatterings with “minimal” string length.
Look at the $<p_T>$ of particles in the “transverse” region ($p_T > 0.5$ GeV/c, $|\eta| < 1$) versus the number of particles in the “transverse” region: $<p_T>$ vs Nchg.

Shows $<p_T>$ versus Nchg in the “transverse” region ($p_T > 0.5$ GeV/c, $|\eta| < 1$) for “Leading Jet” and “Back-to-Back” events with $30 < E_T(jet#1) < 70$ GeV compared with “min-bias” collisions.
Initiators and Remnants

Initiators and Remnants

Need to assign:
- correlated flavours
- correlated $x_i = \frac{p_{zi}}{p_{ztot}}$
- correlated primordial $k_{\perp i}$
- correlated colours
- correlated showers

- PDF after preceding MI/ISR activity:
  0) Squeeze range $0 < x < 1$ into $0 < x < 1 - \sum x_i$ (ISR: $i \neq i_{\text{current}}$)
  1) Valence quarks: scale down by number already kicked out
  2) Introduce companion quark $q/\bar{q}$ to each kicked-out sea quark $\bar{q}/q$,
     with $x$ based on assumed $g \rightarrow q\bar{q}$ splitting
  3) Gluon and other sea: rescale for total momentum conservation
Interleaved Multiple Interactions

$p_\perp$

$p_\perp_{\text{max}}$

$p_\perp_1$

ISR

$p_\perp_2$

ISR

$p_\perp_3$

ISR

$p_\perp_1$ (ISR)

$p_\perp_2$ (ISR)

$p_\perp_3$ (ISR)

$p_\perp_4$

ISR

$p_\perp_{\text{min}}$

1 2 3 4

interaction number

hard int.

mult. int.

mult. int.

 ISR

 ISR

 ISR