LUND UNIVERSITY

CTEQ-MCnet School 2010
Lauterbad, Germany
26 July - 4 August 2010

Introduction to Monte Carlo Event Generators

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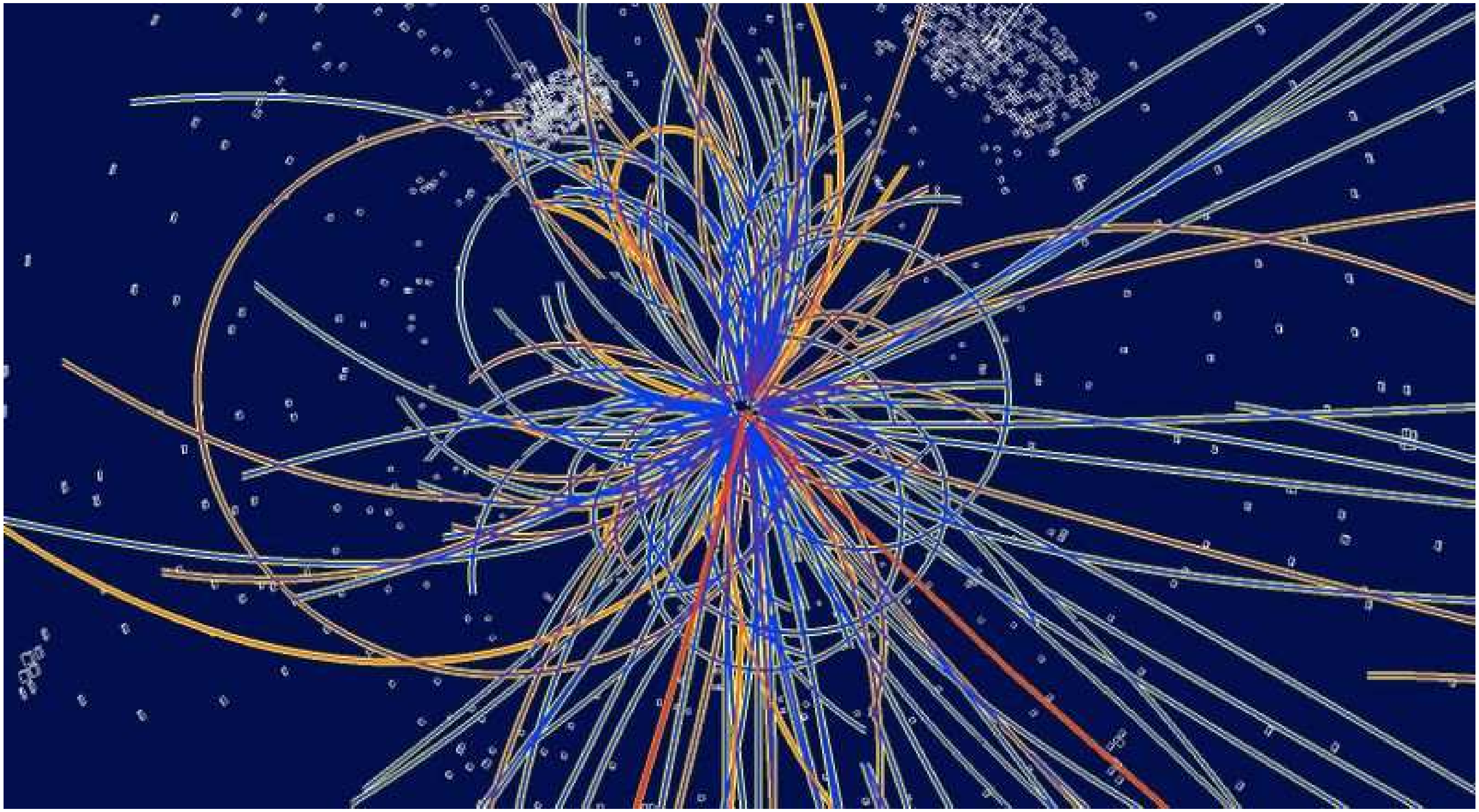
1. (today) Introduction and Overview; Monte Carlo Techniques
2. **(today) Matrix Elements; Parton Showers I**
3. (tomorrow) Parton Showers II; Matching Issues
4. (tomorrow) Multiple Parton–Parton Interactions
5. (Wednesday) Hadronization and Decays; Generator Status

Matrix Elements and Their Usage

$\mathcal{L} \Rightarrow$ Feynman rules $\Rightarrow$ Matrix Elements $\Rightarrow$ Cross Sections

$+$ Kinematics $\Rightarrow$ Processes $\Rightarrow \ldots \Rightarrow$



(Higgs simulation in CMS)

QCD at Fixed Order

Distribution of observable: O

In production of X + anything

Fixed Order (all orders)

$$\left.\frac{d\sigma}{d\mathcal{O}}\right|_{ME} = \sum_{k=0} \int d\Phi_{X+k} \left|\sum_{\ell=0} M^{(\ell)}_{X+k}\right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

Sum over identical amplitudes, then square

Momentum configuration

Phase Space

Cross Section differentially in O

Sum over "anything" ≈ legs

Matrix Elements for X+k at (l) loops

Evaluate observable → differential in O

Truncate at k=n, l=0

→ **Leading Order** for X + n

Lowest order at which X + n happens

(borrowed from Peter Skands)

Loops and Legs

Another representation

Loops \ Legs					
	$X^{(2)}$	$X{+}1^{(2)}$	...		
	$X^{(1)}$	$X{+}1^{(1)}$	$X{+}2^{(1)}$	$X{+}3^{(1)}$	...
	Born	$X{+}1^{(0)}$	$X{+}2^{(0)}$	$X{+}3^{(0)}$	...

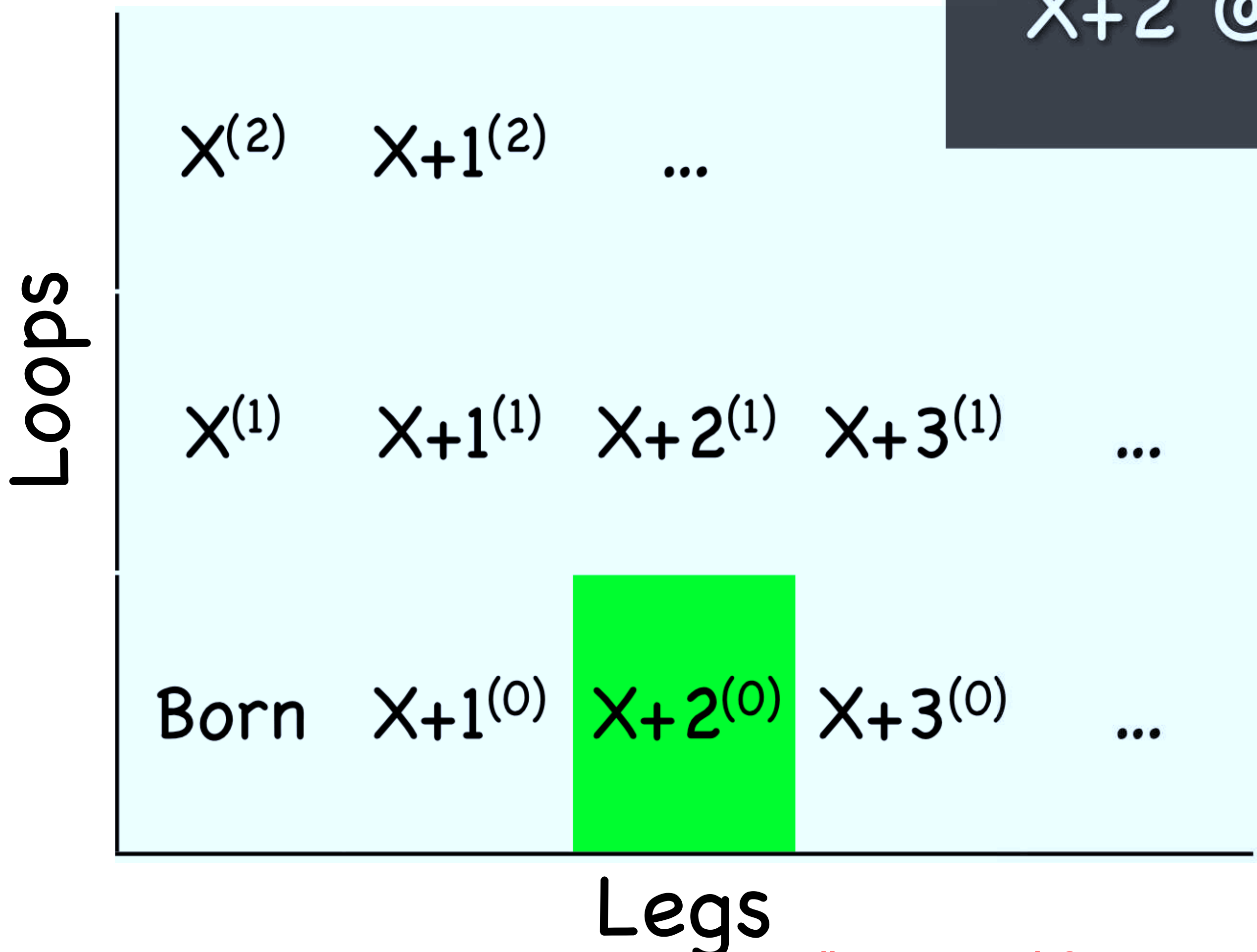
Legs

(borrowed from Peter Skands)

Loops and Legs

Another representation

X+2 @ LO

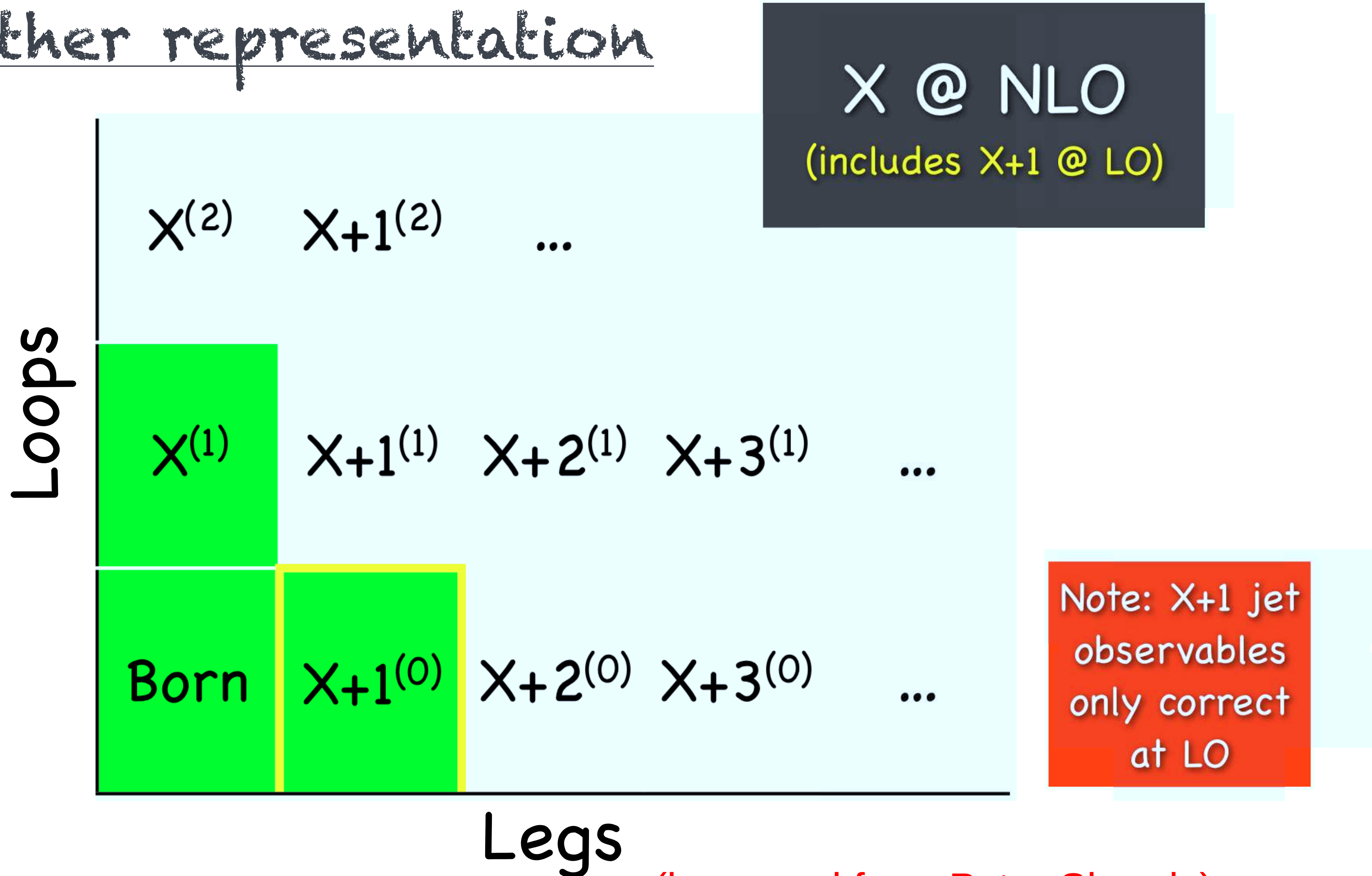


Note: $\sigma \to \infty$ if both jets not resolved

(borrowed from Peter Skands)

Loops and Legs

Another representation



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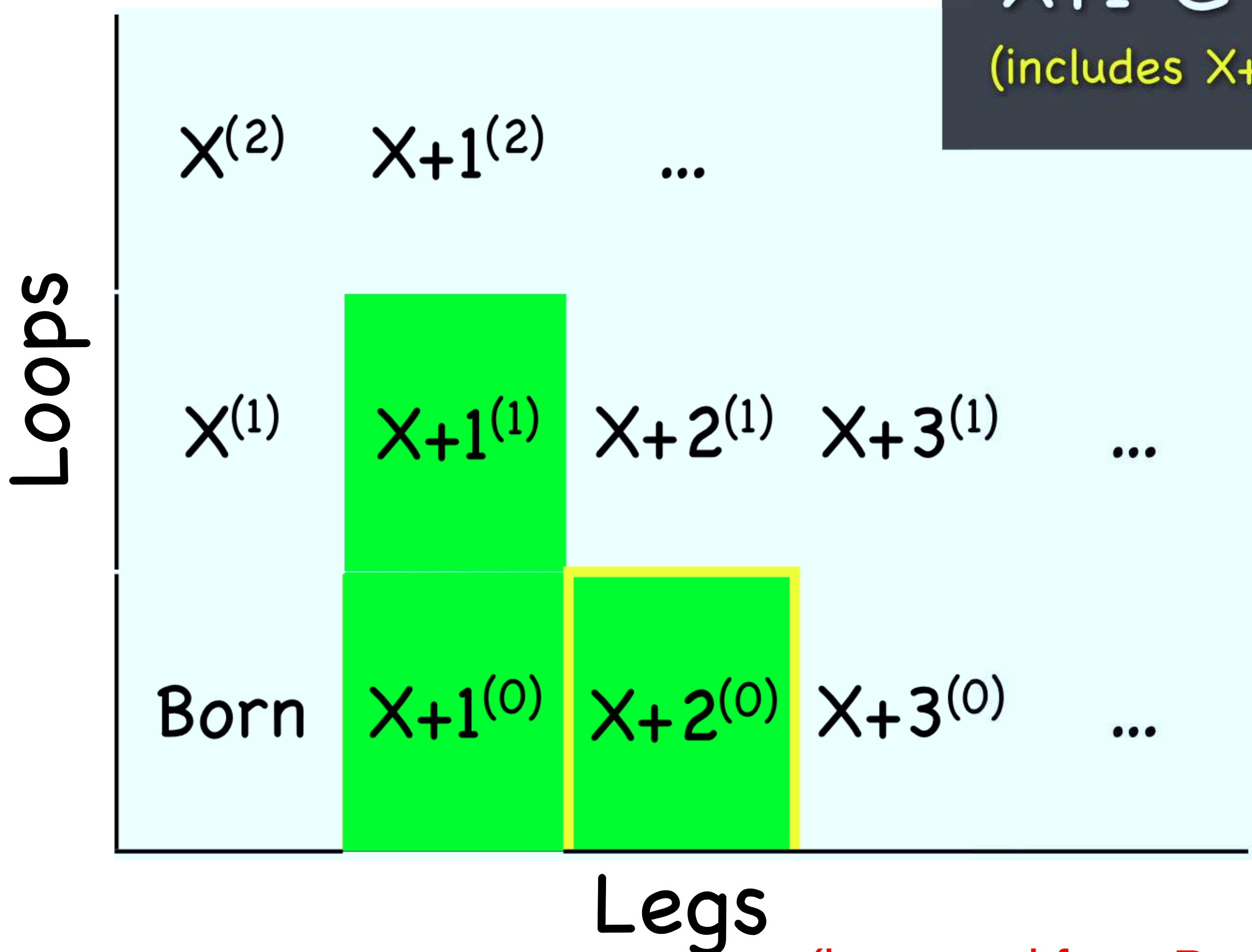
Loops and Legs

Another representation



X+1 @ NLO
(includes X+2 @ LO)

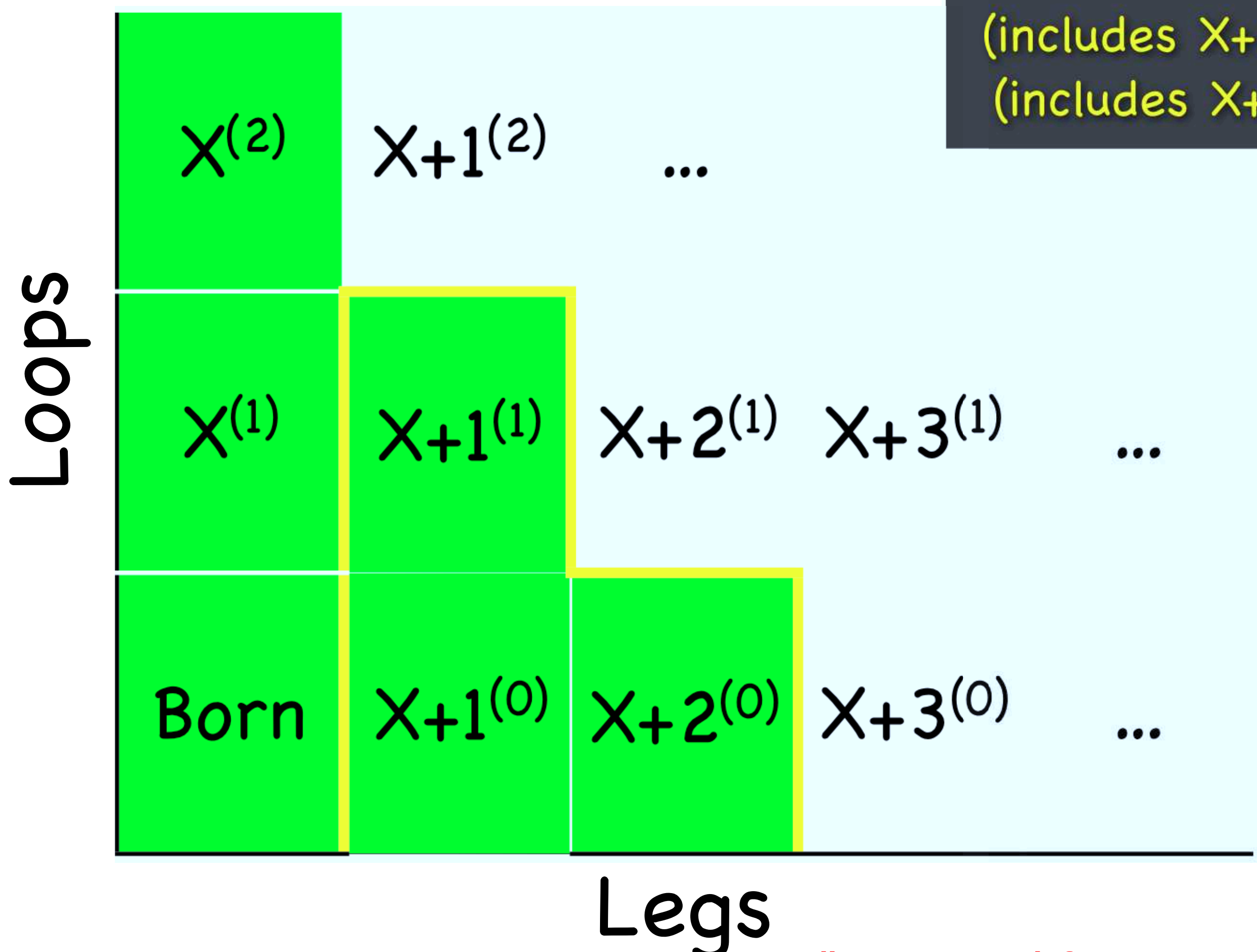
Note: $\sigma \to \infty$ if no jet resolved

Note: X+2 jet observables only correct at LO

(borrowed from Peter Skands)

Loops and Legs

Another representation



X @ NNLO

(includes X+1 @ NLO)

(includes X+2 @ LO)

$\sigma \rightarrow \sigma_{NNLO}$ if no jet resolved

Note: X+2 jet observables only correct at LO

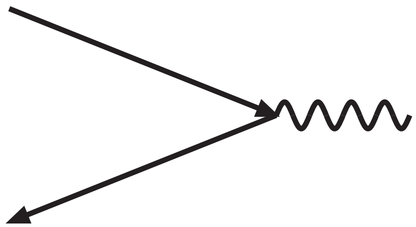
(borrowed from Peter Skands)

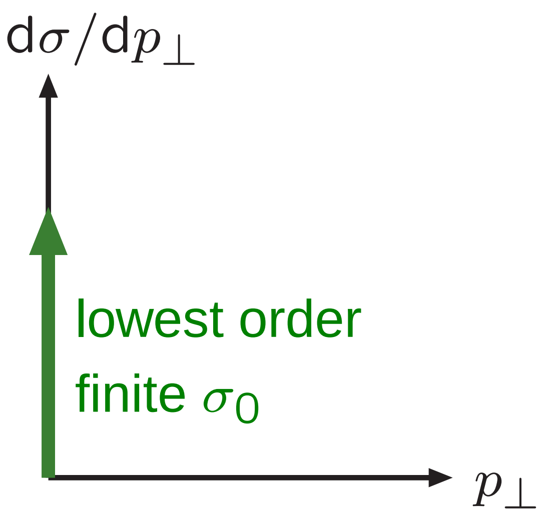
Next-to-leading order (NLO) calculations

I. Lowest order,

$\mathcal{O}(\alpha_{\mathrm{em}})$:

$q\overline{q} \to Z^0$





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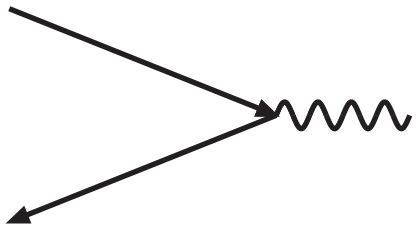
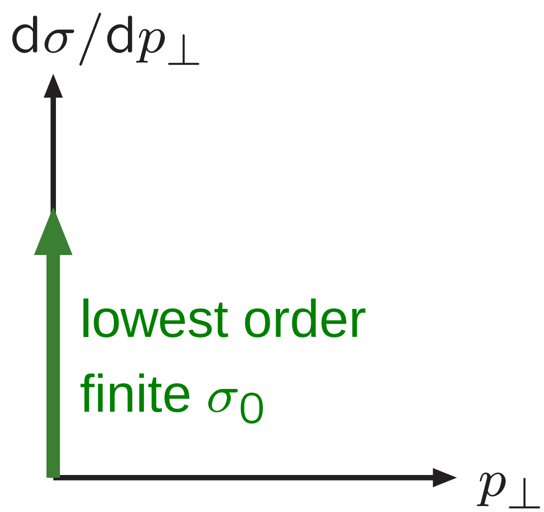
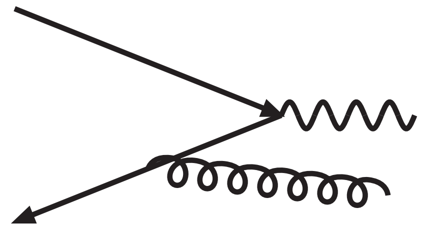
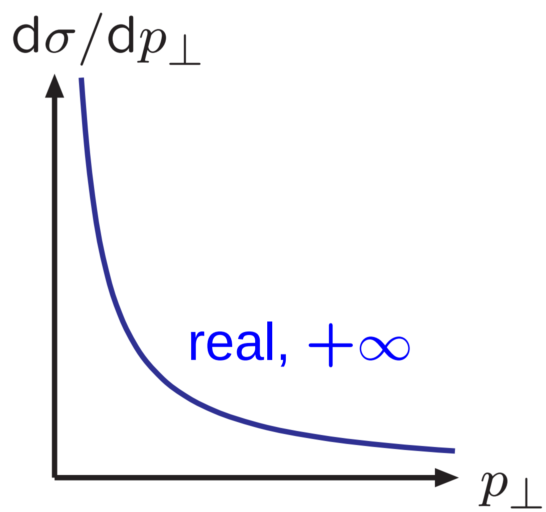




II. First-order real,
$\mathcal{O}(\alpha_{\mathrm{em}}\alpha_{\mathrm{s}})$:
$q\bar{q} \rightarrow Z^0 g$ etc.





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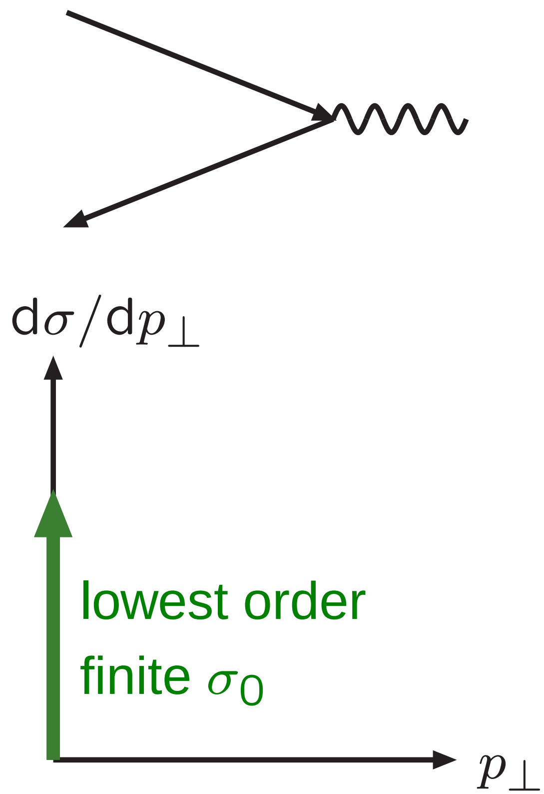

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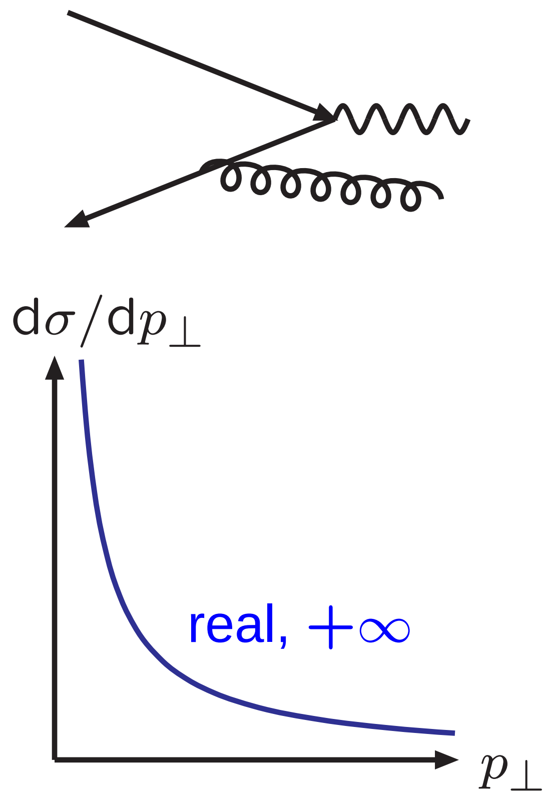
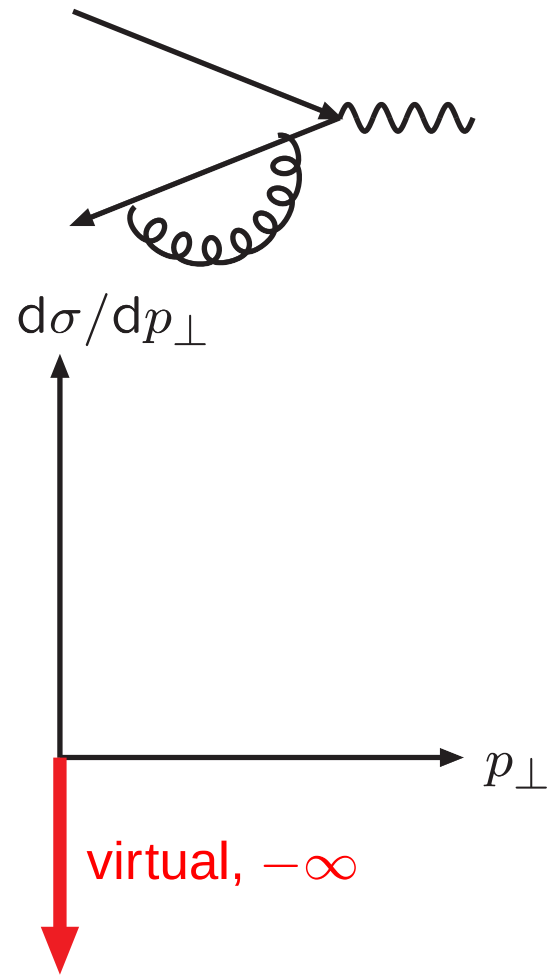


III. First-order virtual,
$\mathcal{O}(\alpha_{\mathrm{em}}\alpha_{\mathrm{s}})$:
$\mathrm{q}\overline{\mathrm{q}} \to \mathrm{Z}^0$ with loops



$$\sigma_{\mathsf{NLO}} = \int_n \mathrm{d}\sigma_{\mathsf{LO}} + \int_{n+1} \mathrm{d}\sigma_{\mathsf{Real}} + \int_n \mathrm{d}\sigma_{\mathsf{Virt}}$$

Simple one-dimensional example: $x \sim p_\perp / p_{\perp\mathsf{max}}$, $0 \leq x \leq 1$

Divergences regularized by $d = 4 - 2\epsilon$ dimensions, $\epsilon < 0$

$$\sigma_{\mathsf{R+V}} = \int_0^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} M(x) + \frac{1}{\epsilon} M_0$$

KLN cancellation theorem: $M(0) = M_0$

Phase Space Slicing:

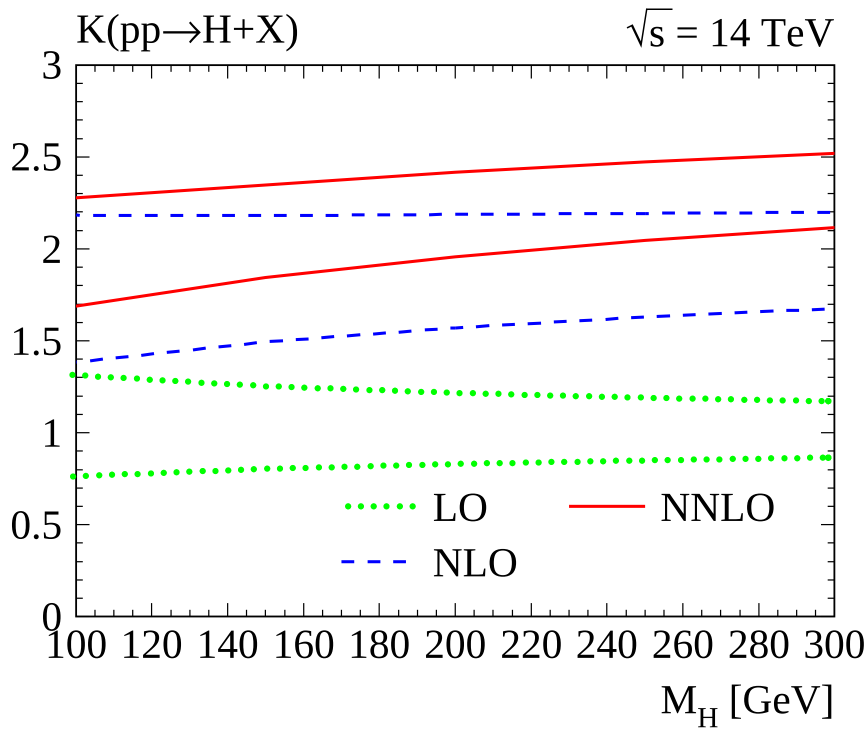
Introduce arbitrary *finite* cutoff $\delta << 1$ (so $\delta \gg |\epsilon|$)

$$\begin{aligned}
\sigma_{\mathsf{R+V}} &= \int_\delta^1 \frac{\mathrm{d}x}{x^{1+\epsilon}} M(x) + \int_0^\delta \frac{\mathrm{d}x}{x^{1+\epsilon}} M(x) + \frac{1}{\epsilon} M_0 \\
&\approx \int_\delta^1 \frac{\mathrm{d}x}{x} M(x) + \int_0^\delta \frac{\mathrm{d}x}{x^{1+\epsilon}} M_0 + \frac{1}{\epsilon} M_0 \\
&= \int_\delta^1 \frac{\mathrm{d}x}{x} M(x) + \frac{1}{\epsilon}\left(1 - \delta^{-\epsilon}\right) M_0 \\
&\approx \int_\delta^1 \frac{\mathrm{d}x}{x} M(x) + \ln\delta \, M_0
\end{aligned}$$

Alternatively **Subtraction**:

$$\begin{aligned}\sigma_{R+V} &= \int_0^1 \frac{dx}{x^{1+\epsilon}} M(x) - \int_0^1 \frac{dx}{x^{1+\epsilon}} M_0 + \int_0^1 \frac{dx}{x^{1+\epsilon}} M_0 + \frac{1}{\epsilon} M_0 \\ &= \int_0^1 \frac{M(x) - M_0}{x^{1+\epsilon}} dx + \left(-\frac{1}{\epsilon} + \frac{1}{\epsilon}\right) M_0 \\ &\approx \int_0^1 \frac{M(x) - M_0}{x} dx + \mathcal{O}(1) M_0\end{aligned}$$

NLO provides a more accurate answer for an integrated cross section:



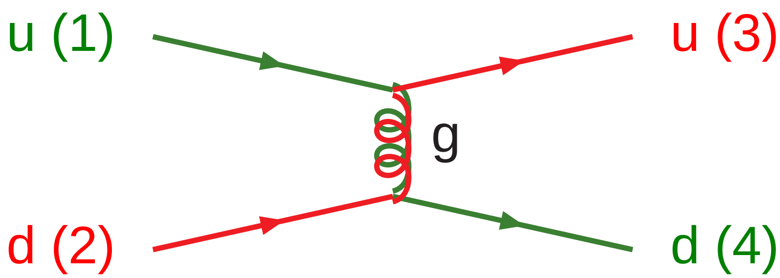
Warning!

Neither approach operates with positive definite quantities

No obvious event-generator implementation

No trivial connection to physical events

Cross sections and kinematics

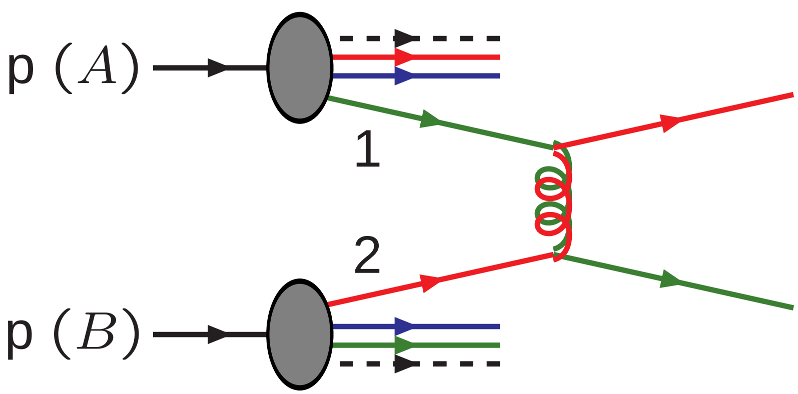


$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2 = -\hat{s}(1 - \cos\hat{\theta})/2$$

$$\hat{u} = (p_1 - p_4)^2 = -\hat{s}(1 + \cos\hat{\theta})/2$$

$$\mathrm{qq}' \to \mathrm{qq}' \quad : \quad \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\hat{t}} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \qquad (\sim \text{Rutherford})$$



$$s = (p_A + p_B)^2$$

$$x_1 \approx E_1/E_A$$

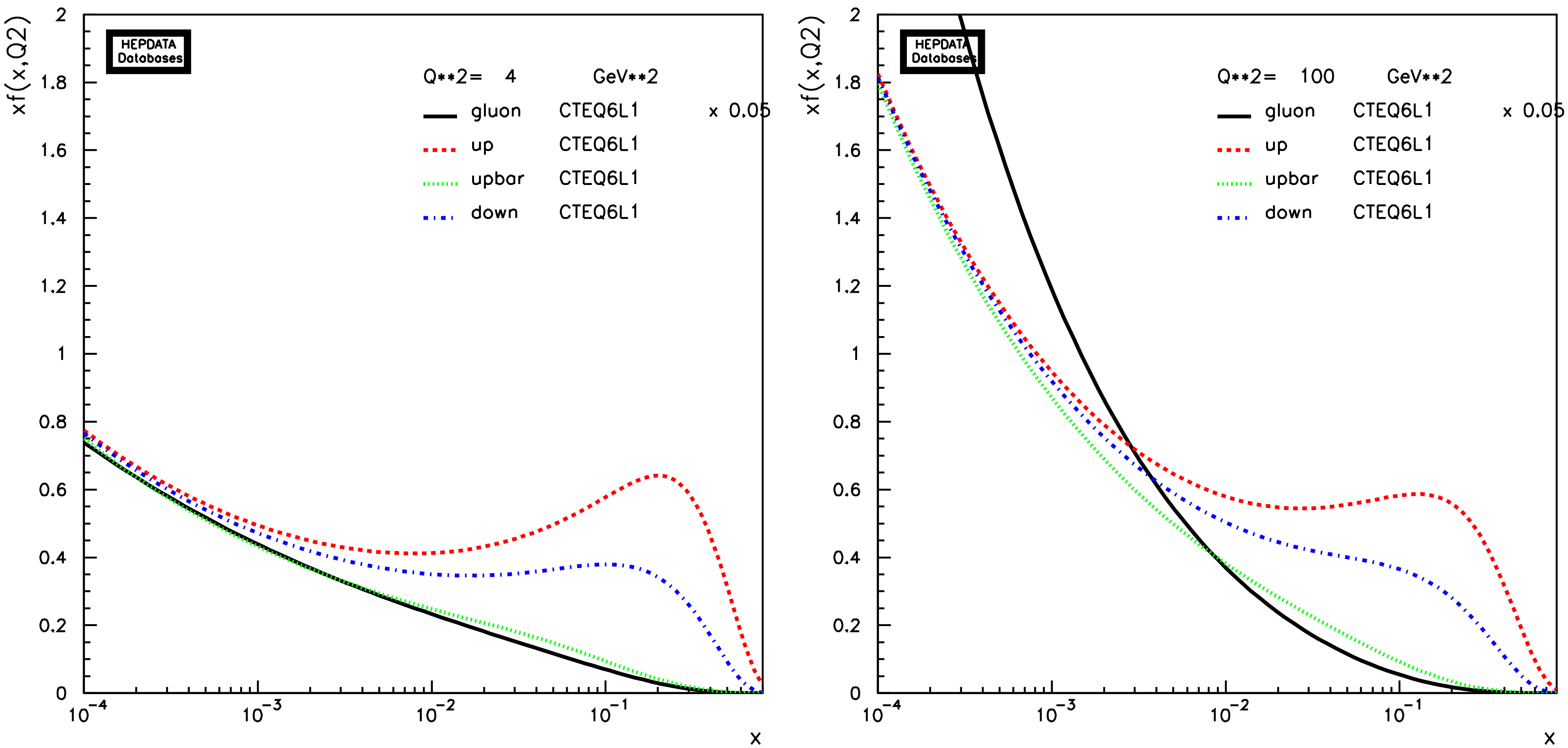
$$x_2 \approx E_2/E_B$$

$$\hat{s} = x_1 x_2 s$$

$$\sigma = \sum_{i,j} \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\hat{t} \, f_i^{(A)}(x_1, Q^2) \, f_j^{(B)}(x_2, Q^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

Factorization: proven for a few processes, assumed for more!

Parton Distribution/Density Functions (PDFs)



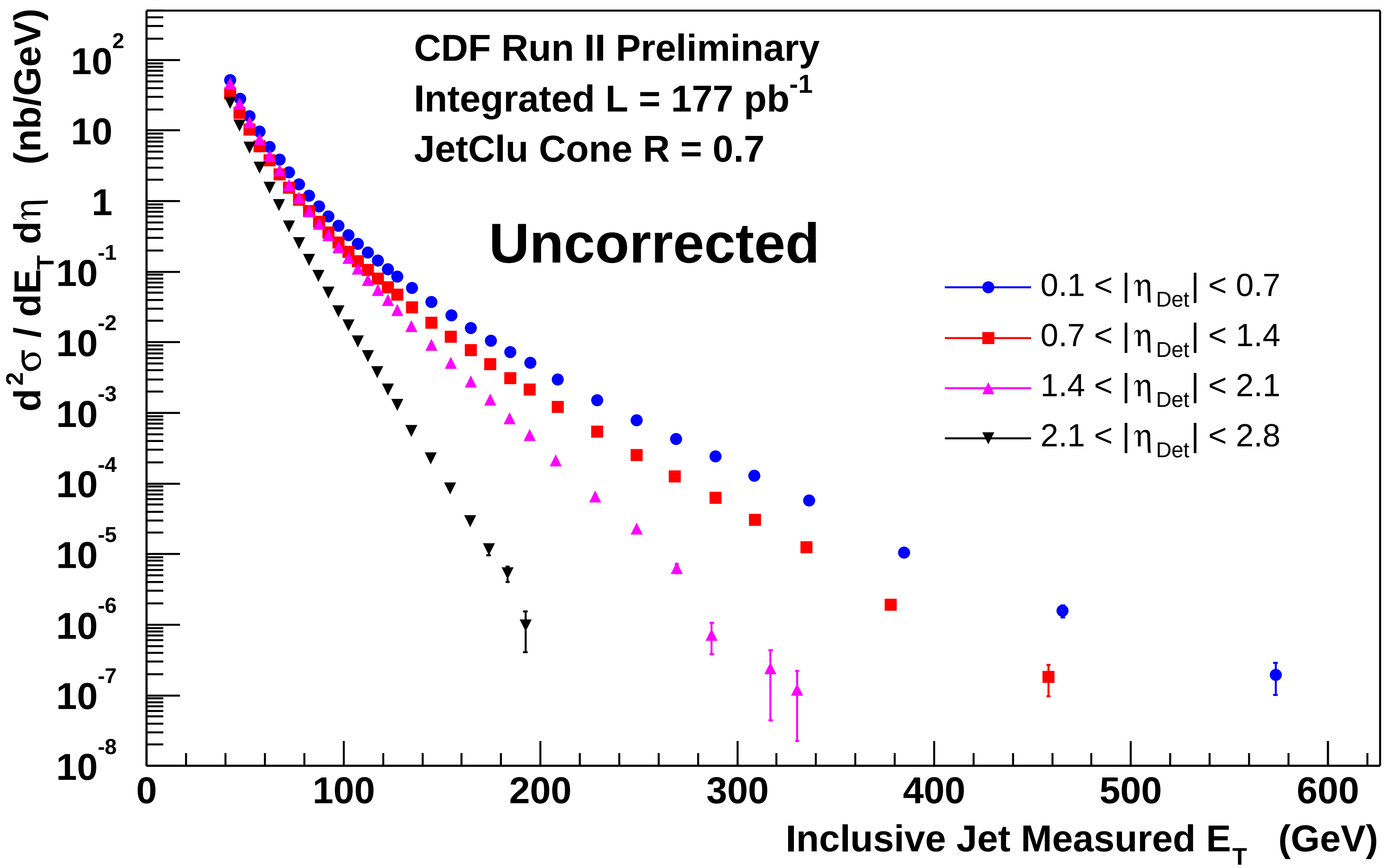
http://durpdg.dur.ac.uk/hepdata/pdf.html

Initial conditions nonperturbative; evolution perturbative (DGLAP):

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z}\, f_a(x',Q^2)\, \frac{\alpha_{\mathrm{s}}}{2\pi}\, P_{a\to bc}\left(z=\frac{x}{x'}\right)$$

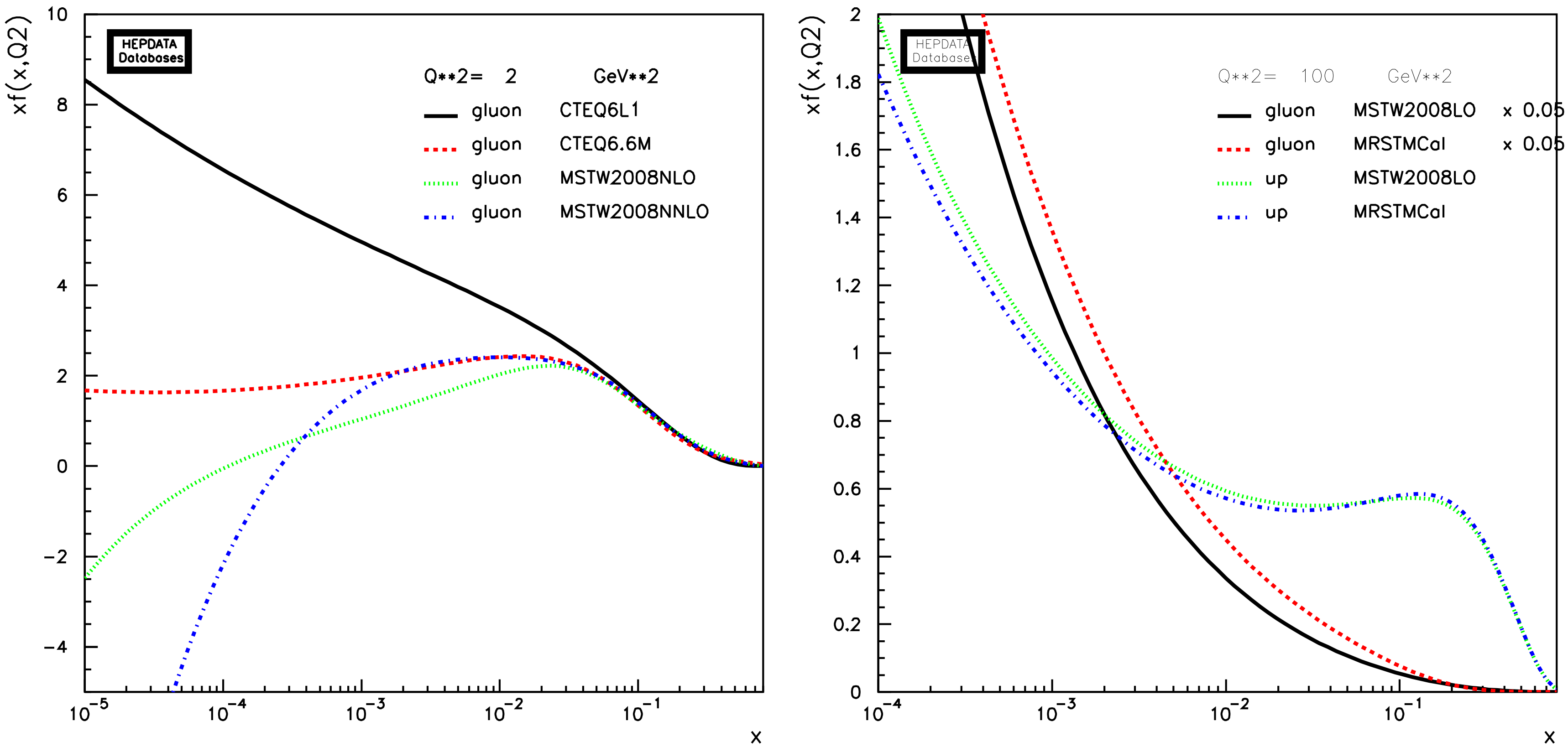
Peaking of PDF's at small x and of QCD ME's at low $p_\perp$

$\Longrightarrow$ most of the physics is at low transverse momenta . . .



. . . but New Physics likely to show up at large masses/$p_\perp$'s

At NLO PDFs are not physical objects and not required positive definite:
$\sigma = \hat{\sigma} \otimes$ PDF, and both can be negative.



Dangerous for LO MCs: recently introduce new MC-adapted PDFs

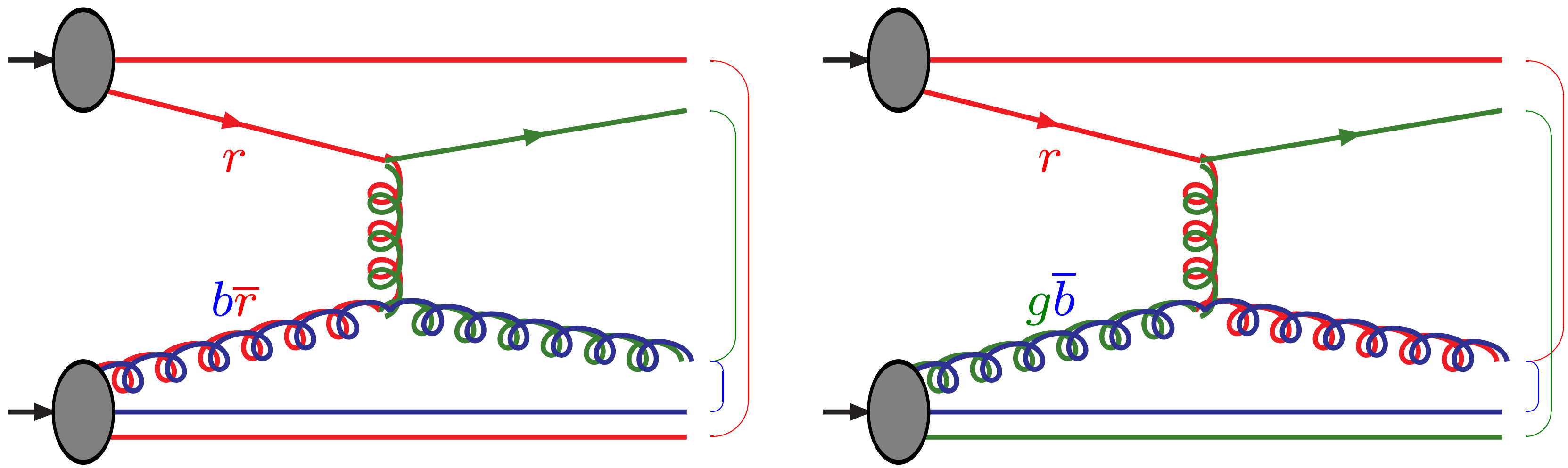
- allow $\sum_i \int_0^1 x f_i(x, Q^2) > 1$ as "built-in K factor"
- use NLO-calculated pseudodata as target for tunes

Current usage:

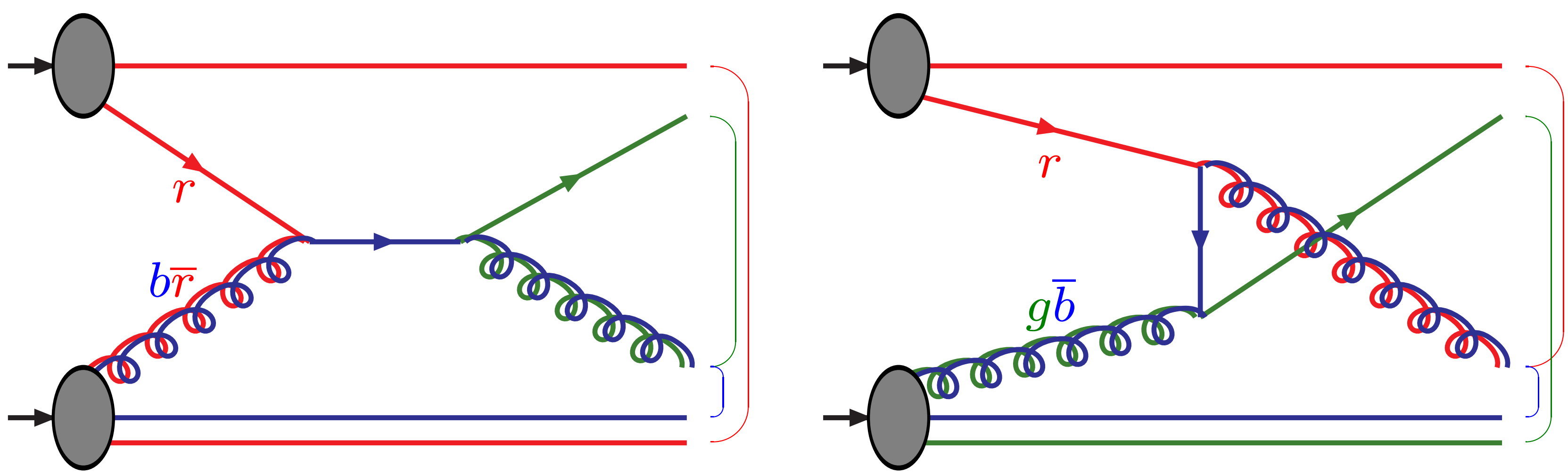
- conventional: CTEQ 5L, CTEQ 6L, CTEQ 6L1, MSTW 2008 LO
- MC-adapted: MRST LO* and LO**; CT09 MC1, MC2 and MCS

Colour flow in hard processes

One Feynman graph can correspond to several possible colour flows, e.g. for $qg \rightarrow qg$:



while other $qg \rightarrow qg$ graphs only admit one colour flow:



so nontrivial mix of kinematics variables $(\widehat{s}, \widehat{t})$
and colour flow topologies I, II:

$$\begin{aligned} |\mathcal{A}(\widehat{s},\widehat{t})|^2 &= |\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t}) + \mathcal{A}_{\mathrm{II}}(\widehat{s},\widehat{t})|^2 \\ &= |\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t})|^2 + |\mathcal{A}_{\mathrm{II}}(\widehat{s},\widehat{t})|^2 + 2\,\mathcal{R}e\left(\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t})\mathcal{A}^*_{\mathrm{II}}(\widehat{s},\widehat{t})\right) \end{aligned}$$

with $\mathcal{R}e\left(\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t})\mathcal{A}^*_{\mathrm{II}}(\widehat{s},\widehat{t})\right) \neq 0$
$\Rightarrow$ indeterminate colour flow, while

- showers *should* know it (coherence),
- hadronization *must* know it (hadrons singlets).

Normal solution:

$$\frac{\text{interference}}{\text{total}} \propto \frac{1}{N_C^2 - 1}$$

so split I : II according to proportions in the $N_C \to \infty$ limit, i.e.

$$\begin{aligned} |\mathcal{A}(\widehat{s},\widehat{t})|^2 &= |\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t})|^2_{\text{mod}} + |\mathcal{A}_{\mathrm{II}}(\widehat{s},\widehat{t})|^2_{\text{mod}} \\ |\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t})|^2_{\text{mod}} &= |\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t}) + \mathcal{A}_{\mathrm{II}}(\widehat{s},\widehat{t})|^2 \left(\frac{|\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t})|^2}{|\mathcal{A}_{\mathrm{I}}(\widehat{s},\widehat{t})|^2 + |\mathcal{A}_{\mathrm{II}}(\widehat{s},\widehat{t})|^2}\right)_{N_C\to\infty} \\ |\mathcal{A}_{\mathrm{II}}(\widehat{s},\widehat{t})|^2_{\text{mod}} &= \ldots \end{aligned}$$

Process Libraries

Traditionally generators come each with its own subprocess library, handcoded since before the days of automatic code generation. Subprocess lists with hundreds of entries *look* impressive, and are useful to rapidly get going, but:

$\star$ Processes usually only in lowest nontrivial order
$\Rightarrow$ need programs that include HO loop corrections to cross sections, alternatively do $(p_\perp, y)$-dependent rescaling by hand?

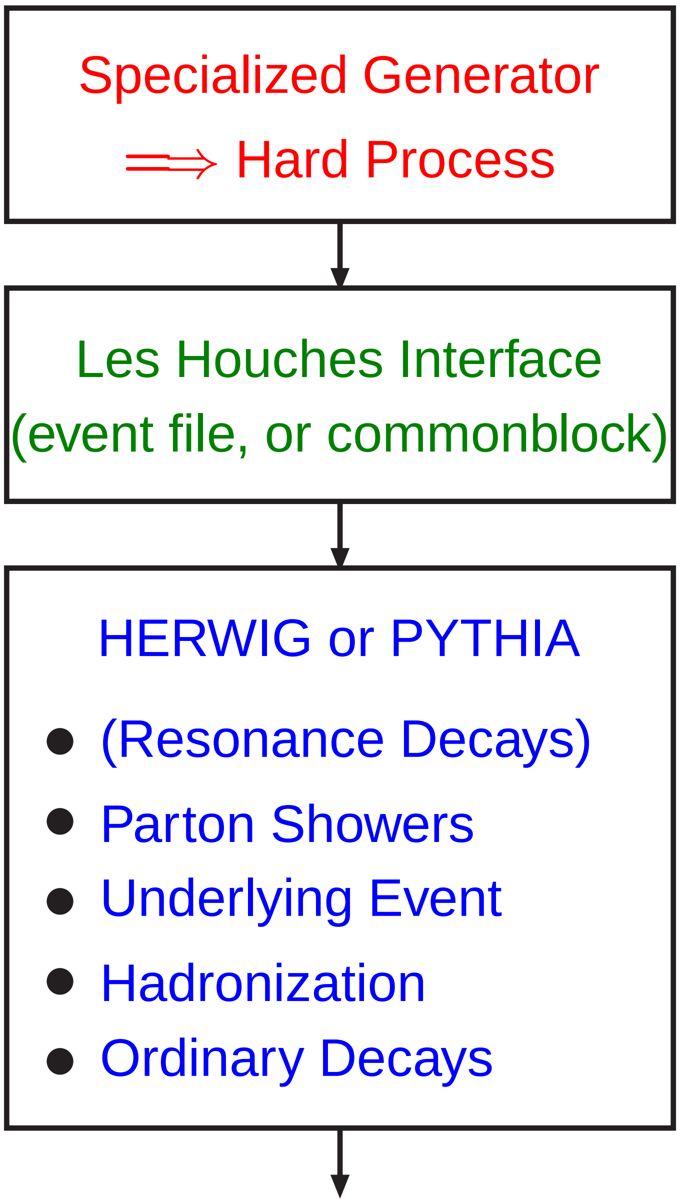
$\star$ No multijet topologies (except in SHERPA)
$\Rightarrow$ have to trust shower to get it right, alternatively match to HO (non-loop) ME generators

$\star$ Spin correlations often absent or incomplete (in PYTHIA)
e.g. top produced unpolarized, while $\mathrm{t} \to \mathrm{bW}^+ \to \mathrm{b}\ell^+\nu_\ell$ decay correct
$\Rightarrow$ have to use external programs when important

$\star$ New physics scenarios appear at rapid pace
$\Rightarrow$ need to have a bigger class of "one-issue experts" contributing code

$\Longrightarrow$ **The Les Houches Accord**

The Les Houches Accord



Some Specialized Generators:

- AcerMC: $\mathrm{t\bar{t}b\bar{b}}, \ldots$
- ALPGEN: $\mathrm{W/Z}+\leq 6\mathrm{j}$, $n\mathrm{W}+m\mathrm{Z}+k\mathrm{H}+\leq 3\mathrm{j}, \ldots$
- CalcHEP: generic LO
- Comix: generic LO
- CompHEP: generic LO
- GRACE+Bases/Spring: generic LO+ some NLO loops
- HELAC–PHEGAS: generic LO
- MadCUP: $\mathrm{W/Z}+\leq 3\mathrm{j}, \mathrm{t\bar{t}b\bar{b}}$
- MadGraph+HELAS: generic LO
- MCFM: NLO $\mathrm{W/Z}+\leq 2\mathrm{j}$, WZ, WH, $\mathrm{H}+\leq 1\mathrm{j}$
- O'Mega+WHIZARD: generic LO

Apologies for all unlisted programs

Do it yourself

MadGraph, CompHEP and CalcHEP can easily be run interactively:
- user specifies process, e.g. $\mathrm{gg} \to \mathrm{W}^+\overline{\mathrm{u}}\mathrm{d}$, and cuts
- program finds all contributing lowest-order Feynman graphs,
- the required amplitudes/cross sections are calculated,
- phase-space is sampled and unweighted to give parton-level events,
- parton-level properties can be histogrammed,
- Les Houches Accord $\Longrightarrow$ complete events.

CompHEP/CalcHEP (matrix-elements-based, good for $\sim\leq$ 4 outgoing):
`http://theory.sinp.msu.ru/comphep/`
`http://theory.sinp.msu.ru/~pukhov/calchep.html`

MadGraph (amplitude-based, can handle $\sim\leq$ 7 outgoing):
`http://madgraph.physics.uiuc.edu/`

Comix (in Sherpa): powerful new framework based on recursion relations

. . . but
- stiff price to pay for each additional parton $\Longrightarrow$ optimized LO libraries,
- confined to lowest-order processes $\Longrightarrow$ NLO libraries.

Ready-made libraries

Many leading-order (LO) ones, e.g.:

•ALPGEN: $\mathrm{W}/\mathrm{Z}+\leq 6\mathrm{j}$, $n\mathrm{W}+m\mathrm{Z}+k\mathrm{H}+\leq 3\mathrm{j}$, $\mathrm{Q}\overline{\mathrm{Q}}+\leq 6\mathrm{j}, \ldots$

http://mlm.home.cern.ch/mlm/alpgen/

•AcerMC: $\mathrm{t}\bar{\mathrm{t}}\mathrm{b}\bar{\mathrm{b}}$, $\mathrm{WWb}\bar{\mathrm{b}}, \ldots$

http://borut.home.cern.ch/borut/

• VECBOS: $\mathrm{W}/\mathrm{Z}+\leq 4\mathrm{j}$

• TopReX: $\mathrm{t}\bar{\mathrm{t}}, \ldots$

Not as many NLO, but still quite a few, e.g.

• MCFM: NLO $\mathrm{W}/\mathrm{Z}+\leq 2\mathrm{j}$, WZ, WH, $\mathrm{H}+\leq 1\mathrm{j}$

http://mcfm.fnal.gov/

• NLOJet++: 2j, 3j

http://nagyz.web.cern.ch/nagyz/Site/NLOJet++

• PHOX family: photons + jets

http://wwwlapp.in2p3.fr/lapth/PHOX_FAMILY/main.html

• MNR: $\mathrm{c}\bar{\mathrm{c}}$, $\mathrm{b}\bar{\mathrm{b}}$

• VBFNLO: WW, WZ, ZZ, ... (incl. Higgs contribution)

http://www-itp.particle.uni-karlsruhe.de/~vbfnloweb/

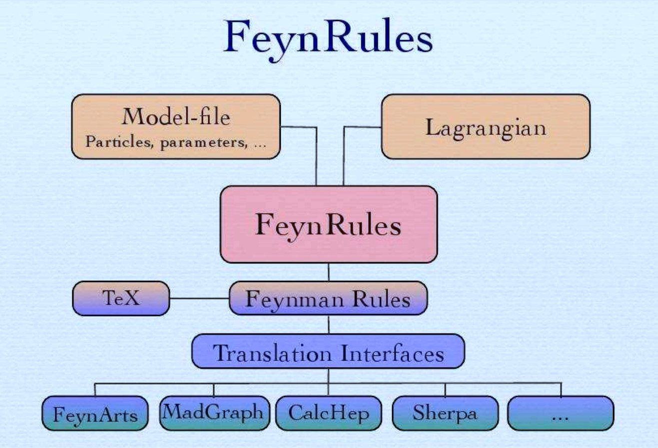
• HIGLU: $\mathrm{gg}\rightarrow\mathrm{H}$

• PROSPINO: $\tilde{\mathrm{q}}\tilde{\mathrm{q}}$, $\tilde{\mathrm{q}}\tilde{\mathrm{g}}$, $\tilde{\mathrm{g}}\tilde{\mathrm{g}}$

FeynRules: Implementing new models made easy

Aim

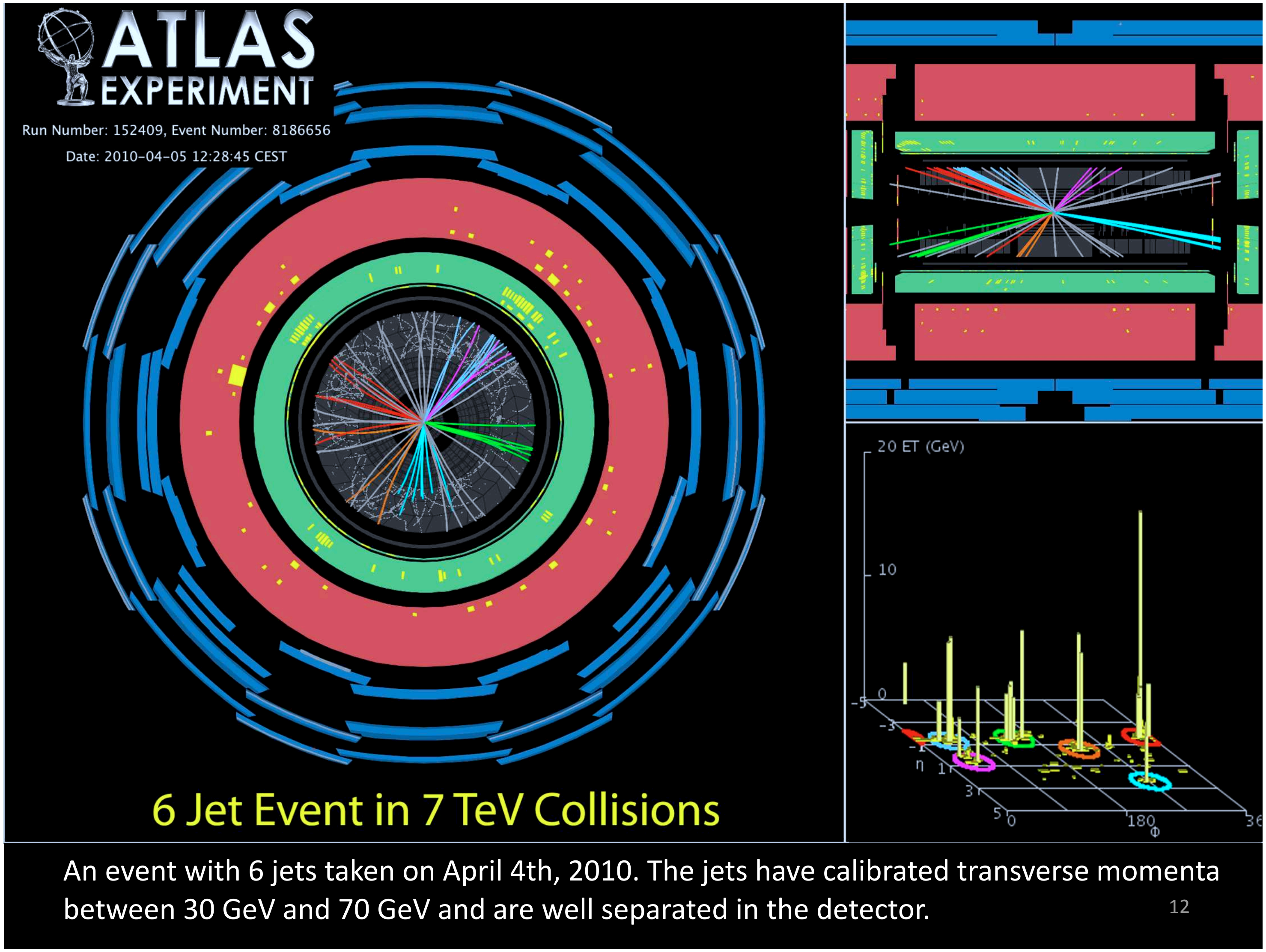
- Portable, transparent & reproducible implementation of (nearly arbitrary) new physics models.
- In most codes: New models given by new particles, their properties & interactions.
- Output to standard ME generators enabled (MadGraph, Sherpa, ...)



- Various models already implemented & validated for a list: http://feynrules.phys.ucl.ac.be

(borrowed from Frank Krauss)

Parton Showers



- Final-State (Timelike) Showers
- Initial-State (Spacelike) Showers
- Matching to Matrix Elements

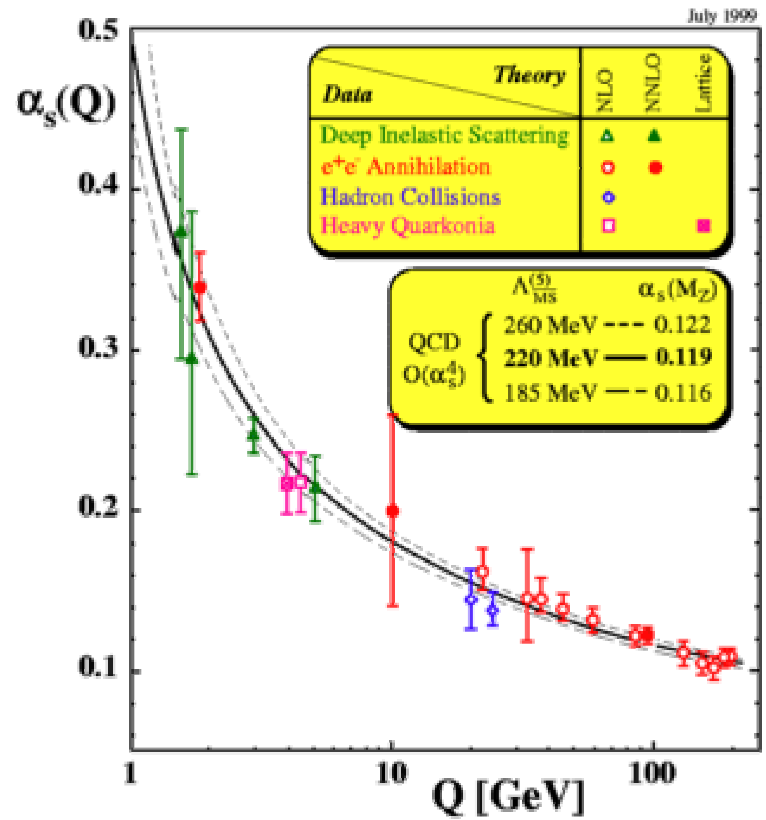
Divergences

Emission rate q $\longrightarrow$ qg diverges when

- collinear: opening angle $\theta_{\mathrm{qg}} \longrightarrow 0$
- soft: gluon energy $E_{\mathrm{g}} \longrightarrow 0$

Almost identical to e $\longrightarrow$ eγ (“bremsstrahlung”),
but QCD is non-Abelian so additionally

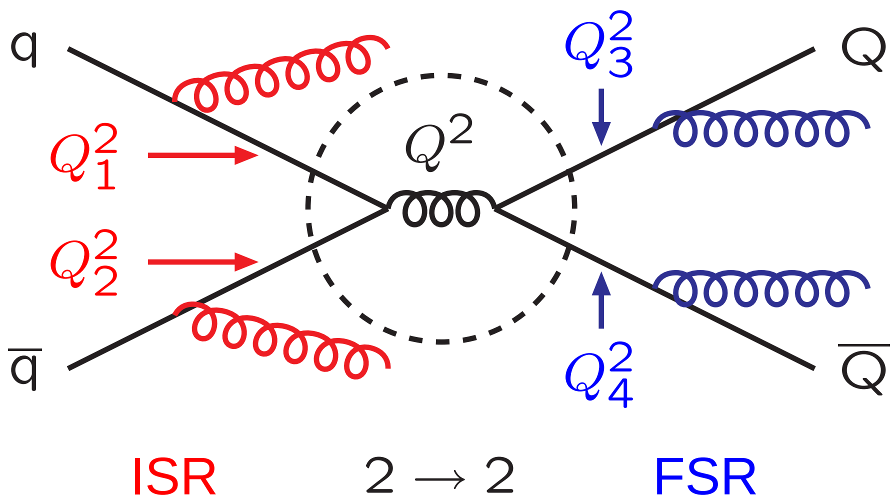
- g $\longrightarrow$ gg similarly divergent
- $\alpha_{\mathrm{s}}(Q^2)$ diverges for $Q^2 \longrightarrow 0$
  (actually for $Q^2 \longrightarrow \Lambda^2_{\mathrm{QCD}}$)



Big probability for one emission $\Longrightarrow$ also big for several
$\Longrightarrow$ with ME’s need to calculate to high order **and** with many loops
$\Longrightarrow$ extremely demanding technically (not solved!), and
involving big cancellations between positive and negative contributions.
Alternative approach: **parton showers**

The Parton-Shower Approach

$$2 \to n \;=\; (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$



FSR = Final-State Rad.;

timelike shower

$Q_i^2 \sim m^2 > 0$ decreasing

ISR = Initial-State Rad.;

spacelike shower

$Q_i^2 \sim -m^2 > 0$ increasing

$2 \to 2$ = hard scattering (on-shell):

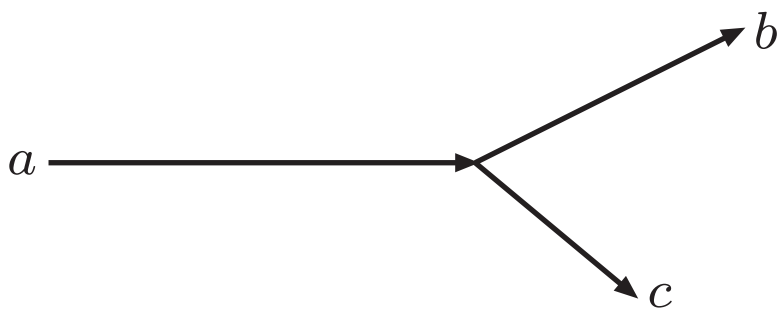
$$\sigma = \iiint \mathrm{d}x_1 \, \mathrm{d}x_2 \, \mathrm{d}\hat{t} \; f_i(x_1, Q^2) \, f_j(x_2, Q^2) \, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

Shower evolution is viewed as a probabilistic process, which occurs with unit total probability:

the cross section is not directly affected, but indirectly it is, via the changed event shape

Technical aside: why timelike/spacelike?

Consider four-momentum conservation in a branching $a \to b\,c$



$$\mathbf{p}_{\perp a} = 0 \;\Rightarrow\; \mathbf{p}_{\perp c} = -\mathbf{p}_{\perp b}$$
$$p_+ = E + p_L \;\Rightarrow\; p_{+a} = p_{+b} + p_{+c}$$
$$p_- = E - p_L \;\Rightarrow\; p_{-a} = p_{-b} + p_{-c}$$

Define $p_{+b} = z\,p_{+a}, \quad p_{+c} = (1-z)\,p_{+a}$

Use $p_+ p_- = E^2 - p_L^2 = m^2 + p_\perp^2$

$$\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{z\,p_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1-z)\,p_{+a}}$$

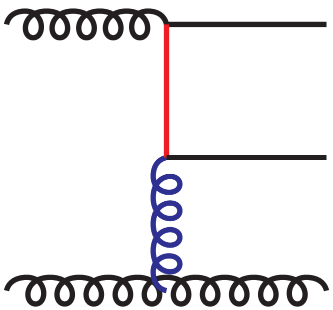
$$\Rightarrow \quad m_a^2 = \frac{m_b^2 + p_\perp^2}{z} + \frac{m_c^2 + p_\perp^2}{1-z} = \frac{m_b^2}{z} + \frac{m_c^2}{1-z} + \frac{p_\perp^2}{z(1-z)}$$

Final-state shower: $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_\perp^2}{z(1-z)} > 0 \Rightarrow$ timelike

Initial-state shower: $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_\perp^2}{1-z} < 0 \Rightarrow$ spacelike

Doublecounting

A $2 \rightarrow n$ graph can be "simplified" to $2 \rightarrow 2$ in different ways:



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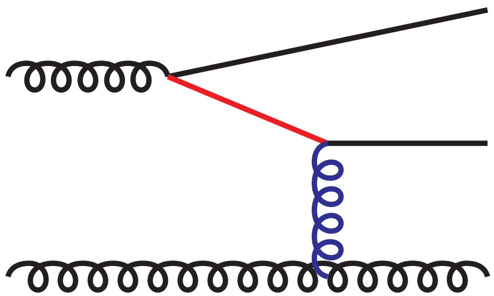
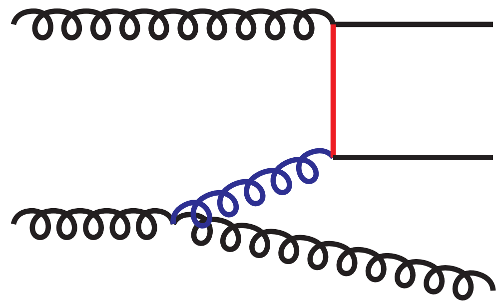


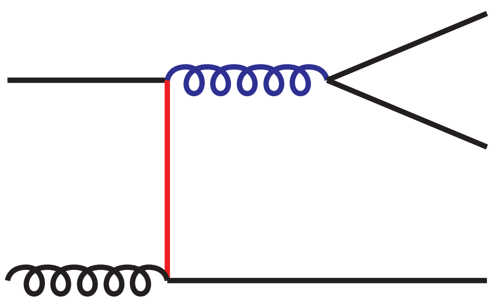
g $\longrightarrow$ q$\overline{\text{q}}$ $\oplus$ qg $\longrightarrow$ qg

or



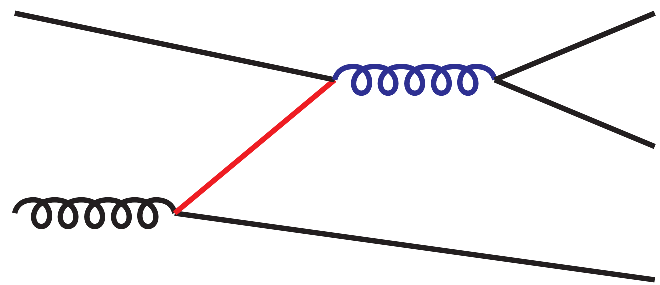
g $\longrightarrow$ gg $\oplus$ gg $\longrightarrow$ q$\overline{\text{q}}$

or deform



FSR

to

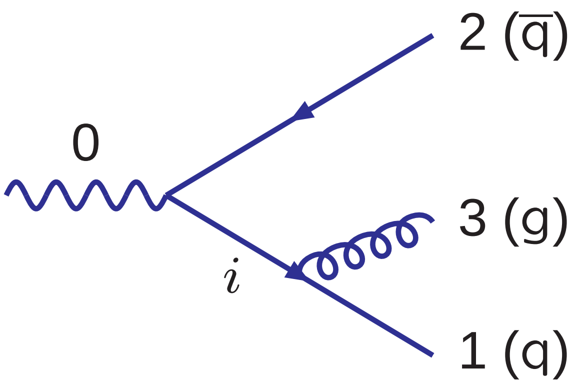


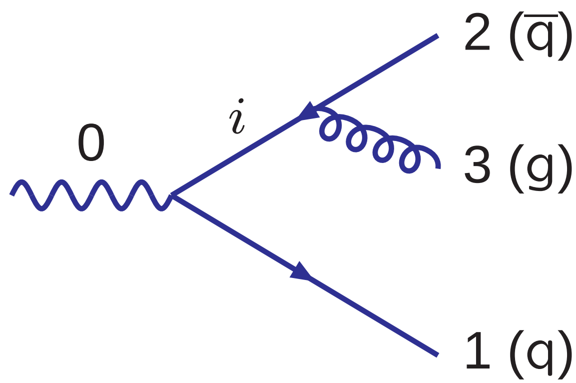
ISR

Do not doublecount: $2 \rightarrow 2$ *= most virtual = shortest distance*

Conflict: theory derivations often assume virtualities strongly ordered; interesting physics often in regions where this is not true!

From Matrix Elements to Parton Showers





$e^+e^- \to q\overline{q}g$

$x_j = 2E_j/E_{cm} \Rightarrow$

$x_1 + x_2 + x_3 = 2$

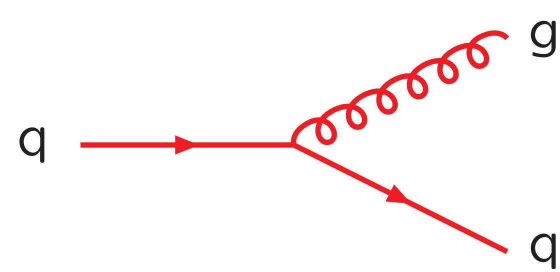
$$m_q = 0: \quad \frac{d\sigma_{ME}}{\sigma_0} = \frac{\alpha_s}{2\pi}\frac{4}{3}\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}\, dx_1\, dx_2$$

Rewrite for $x_2 \to 1$, i.e. q–g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{cm}^2} = \frac{Q^2}{E_{cm}^2} \quad \Rightarrow \quad dx_2 = \frac{dQ^2}{E_{cm}^2}$$

$$x_1 \approx z \quad \Rightarrow \quad dx_1 \approx dz$$

$$x_3 \approx 1 - z$$



$$\Rightarrow \; d\mathcal{P} = \frac{d\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi}\frac{dx_2}{(1-x_2)}\frac{4}{3}\frac{x_2^2 + x_1^2}{(1-x_1)}\, dx_1 \approx \frac{\alpha_s}{2\pi}\frac{dQ^2}{Q^2}\frac{4}{3}\frac{1+z^2}{1-z}\, dz$$

Generalizes to DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

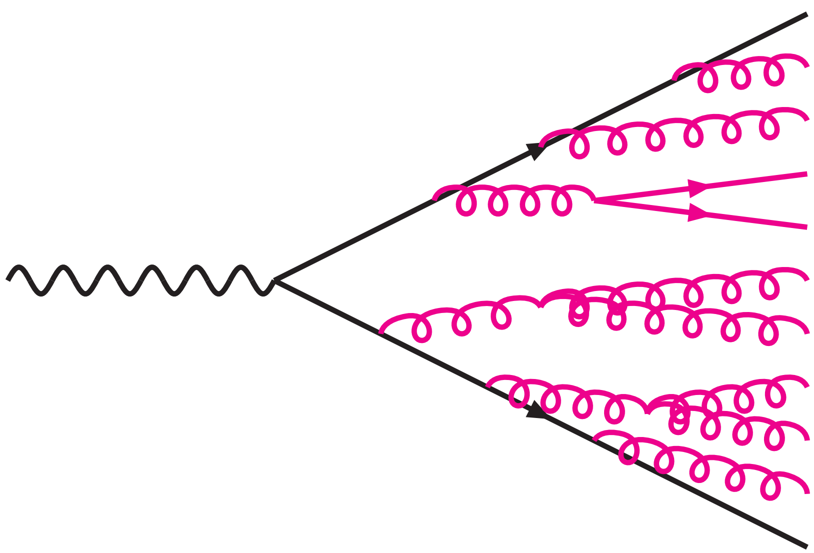
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\,\mathrm{d}z$$

$$P_{q\to qg} = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$P_{g\to gg} = 3\frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{g\to q\overline{q}} = \frac{n_f}{2}(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})$$

Iteration gives final-state parton showers



Need soft/collinear cut-offs to stay away from nonperturbative physics.

Details model-dependent, e.g.

$Q > m_0 = \min(m_{ij}) \approx 1$ GeV,

$z_{\min}(E,Q) < z < z_{\max}(E,Q)$

or $p_\perp > p_{\perp\min} \approx 0.5$ GeV

The Sudakov Form Factor

Conservation of total probability:

$\mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens})$

"multiplicativeness" in "time" evolution:

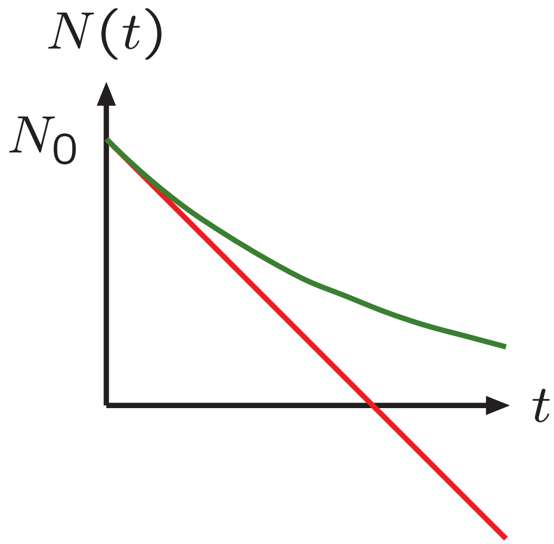
$\mathcal{P}_{\text{nothing}}(0 < t \le T) = \mathcal{P}_{\text{nothing}}(0 < t \le T_1)\, \mathcal{P}_{\text{nothing}}(T_1 < t \le T)$

Subdivide further, with $T_i = (i/n)T$, $0 \le i \le n$:

$$
\begin{aligned}
\mathcal{P}_{\text{nothing}}(0 < t \le T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \le T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} \left(1 - \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right) \\
&= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \le T_{i+1})\right) \\
&= \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
\end{aligned}
$$

$$
\Longrightarrow \quad \mathrm{d}\mathcal{P}_{\text{first}}(T) = \mathrm{d}\mathcal{P}_{\text{something}}(T)\, \exp\left(-\int_0^T \frac{\mathrm{d}\mathcal{P}_{\text{something}}(t)}{\mathrm{d}t}\mathrm{d}t\right)
$$

Example: radioactive decay of nucleus



naively: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN_0 \Rightarrow N(t) = N_0\,(1 - ct)$

depletion: a given nucleus can only decay once

correctly: $\frac{\mathrm{d}N}{\mathrm{d}t} = -cN(t) \Rightarrow N(t) = N_0\,\exp(-ct)$

generalizes to: $N(t) = N_0\,\exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

or: $\frac{\mathrm{d}N(t)}{\mathrm{d}t} = -c(t)\,N_0\,\exp\left(-\int_0^t c(t')\mathrm{d}t'\right)$

sequence allowed: $\text{nucleus}_1 \to \text{nucleus}_2 \to \text{nucleus}_3 \to \ldots$

Correspondingly, with $Q \sim 1/t$ (Heisenberg)

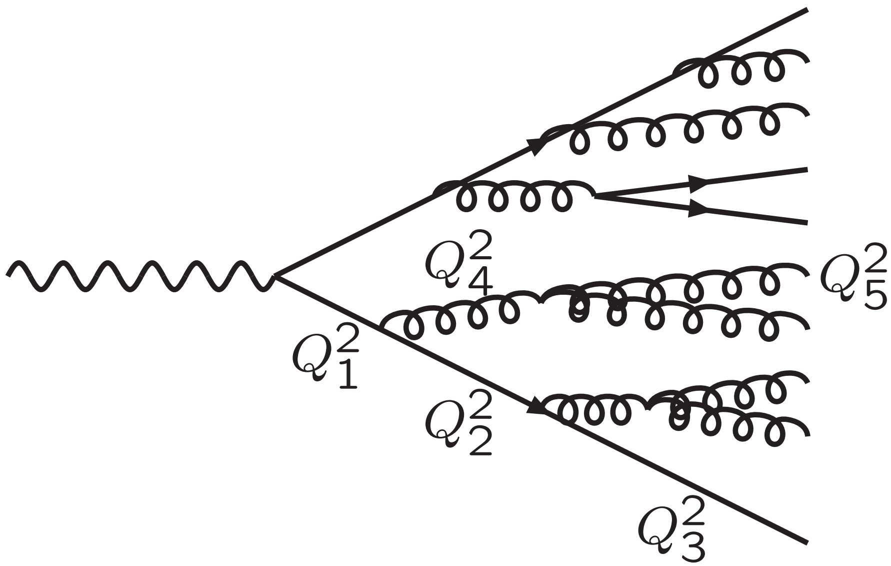
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}P_{a\to bc}(z)\,\mathrm{d}z\;\exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_{\max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int\frac{\alpha_s}{2\pi}P_{a\to bc}(z')\,\mathrm{d}z'\right)$$

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c}\int\int\,\mathrm{d}\mathcal{P}_{a\to bc} \equiv 1$ $\Rightarrow$ *convenient for Monte Carlo*

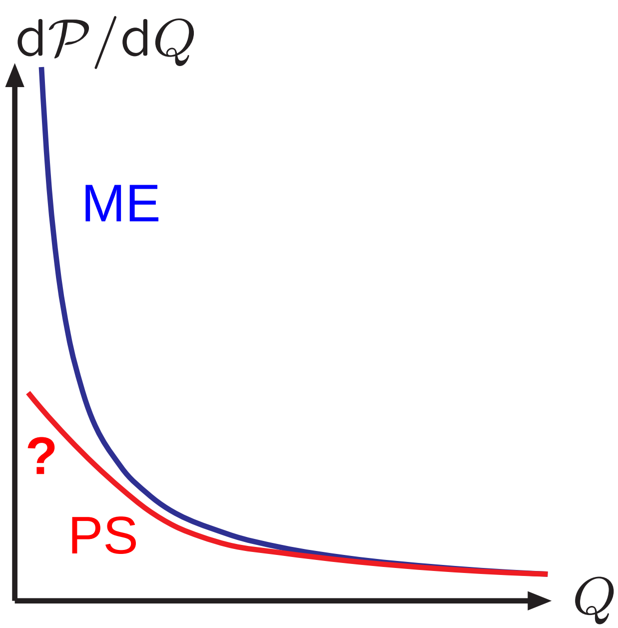
($\equiv 1$ if extended over whole phase space, else possibly nothing happens)



Sudakov form factor provides
"time" ordering of shower:
lower $Q^2 \Longleftrightarrow$ longer times

$Q_1^2 > Q_2^2 > Q_3^2$
$Q_1^2 > Q_4^2 > Q_5^2$
etc.

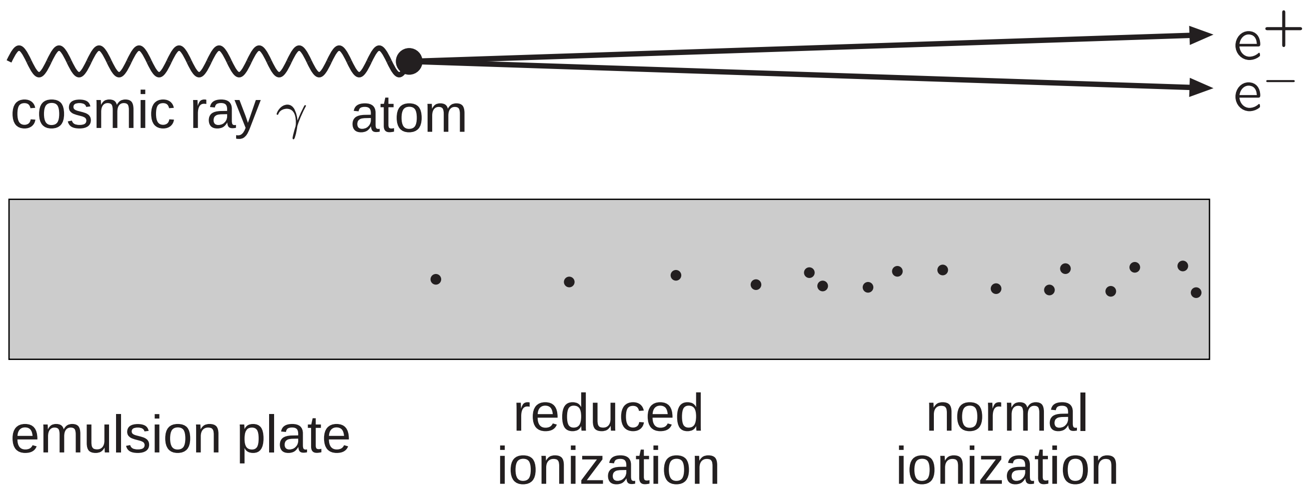
Sudakov regulates singularity for *first* emission . . .



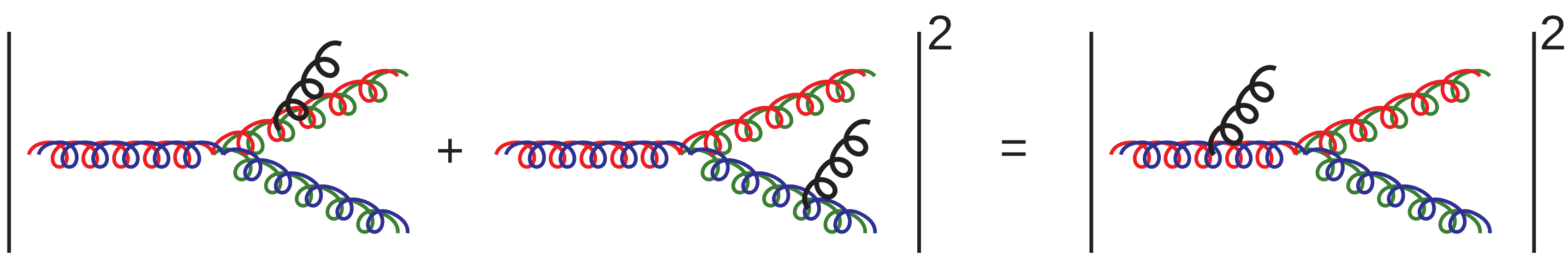
. . . but in limit of *repeated soft*
emissions $\mathrm{q} \to \mathrm{qg}$ (but no $\mathrm{g} \to \mathrm{gg}$)
one obtains the same inclusive
Q emission spectrum as for ME,
i.e. divergent ME spectrum
$\Longleftrightarrow$ infinite number of PS emissions
Proof: as for veto algorithm (what is
probability to have an emission at Q
after 0, 1, 2, 3, . . . previous ones?)

Coherence

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission



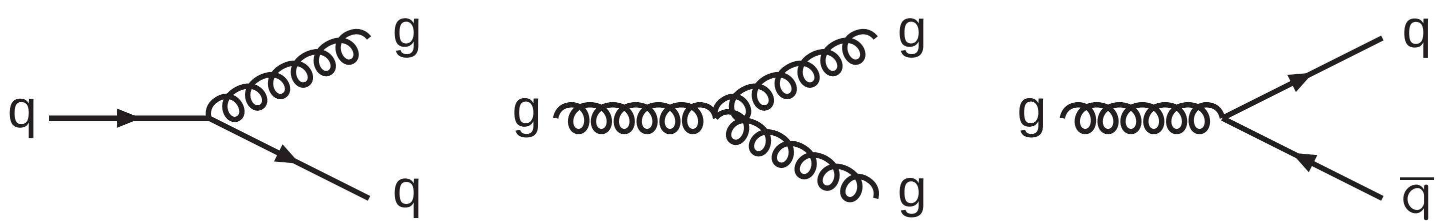
solved by • requiring emission angles to be decreasing

or • requiring transverse momenta to be decreasing

The Common Showering Algorithms (LEP era)

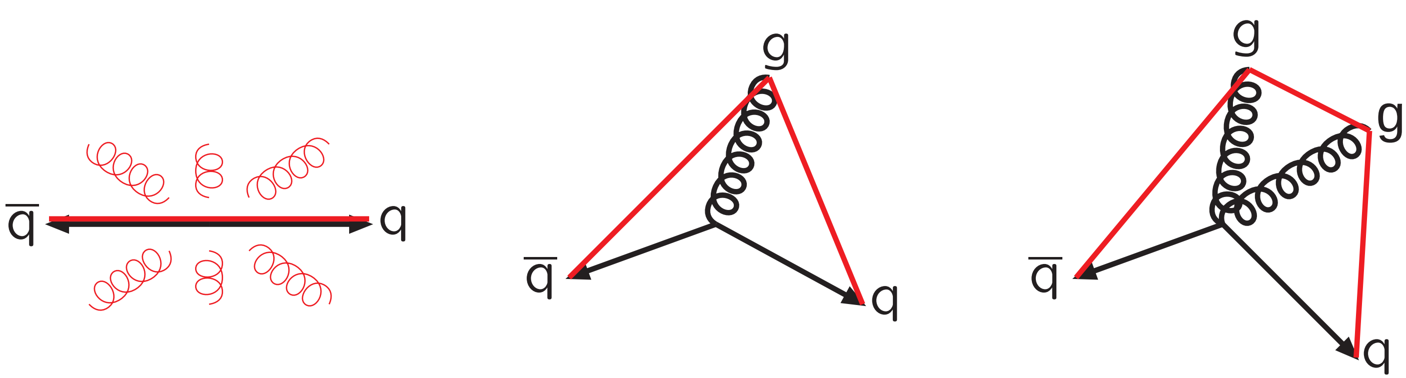
Three main approaches to showering in common use:

Two are based on the standard shower language
of $a \to bc$ successive branchings:



HERWIG: $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$
PYTHIA: $Q^2 = m^2$ (timelike) or $= -m^2$ (spacelike)

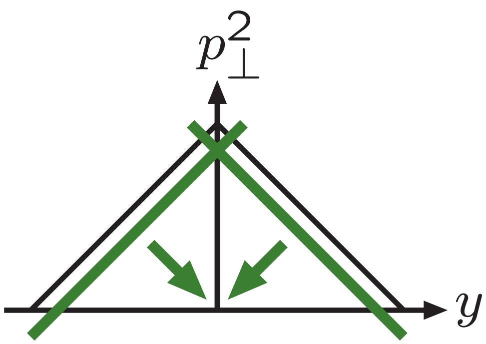
One is based on a picture of dipole emission $ab \to cde$:



ARIADNE: $Q^2 = p_\perp^2$; FSR mainly, ISR is primitive;
there instead LDCMC: sophisticated but complicated

Ordering variables in final-state radiation (LEP era)

PYTHIA: $Q^2 = m^2$



large mass first

$\Rightarrow$ “hardness” ordered

coherence brute force

covers phase space

ME merging simple

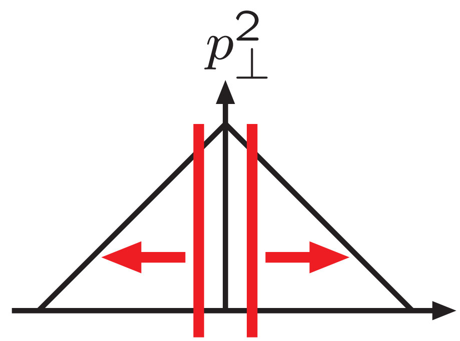
g $\rightarrow$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $m^2 \rightarrow -m^2$

HERWIG: $Q^2 \sim E^2\theta^2$



large angle first

$\Rightarrow$ **hardness not ordered**

coherence inherent

gaps in coverage

ME merging messy

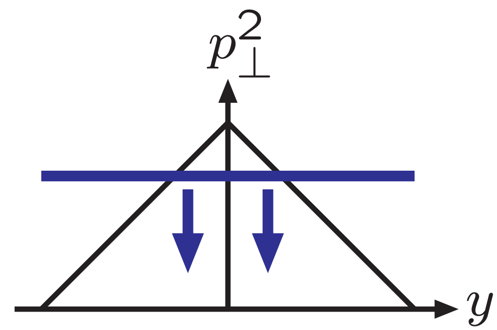
g $\rightarrow$ q$\overline{\text{q}}$ simple

not Lorentz invariant

no stop/restart

ISR: $\theta \rightarrow \theta$

ARIADNE: $Q^2 = p_\perp^2$



large $p_\perp$ first

$\Rightarrow$ “hardness” ordered

coherence inherent

covers phase space

ME merging simple

g $\rightarrow$ q$\overline{\text{q}}$ messy

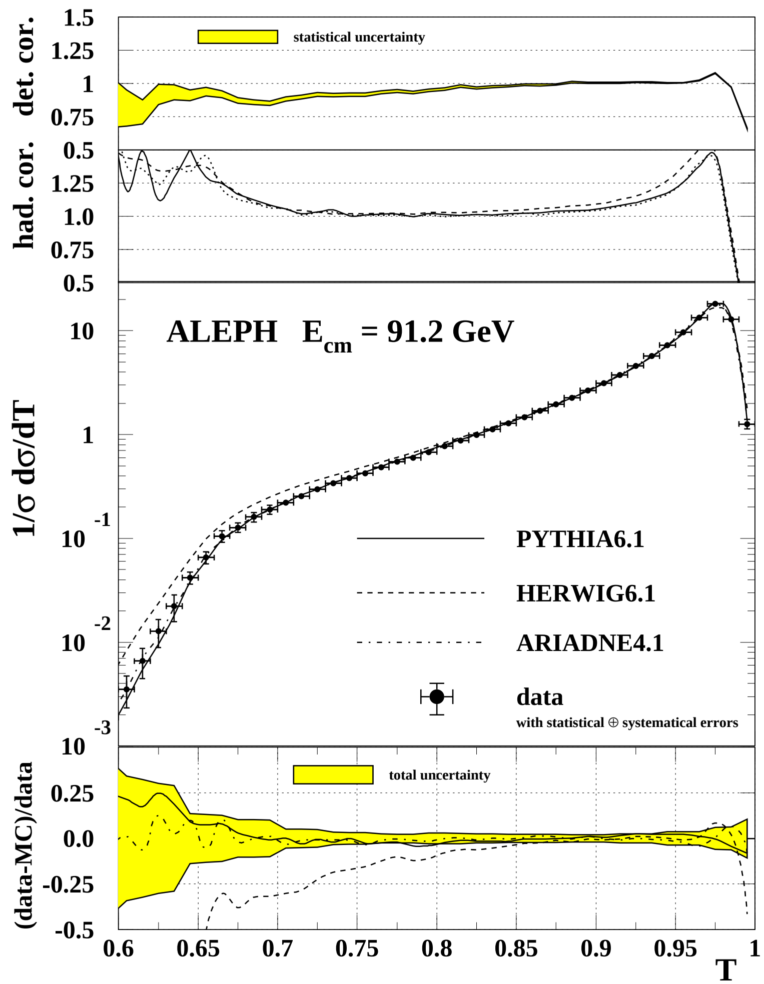
Lorentz invariant

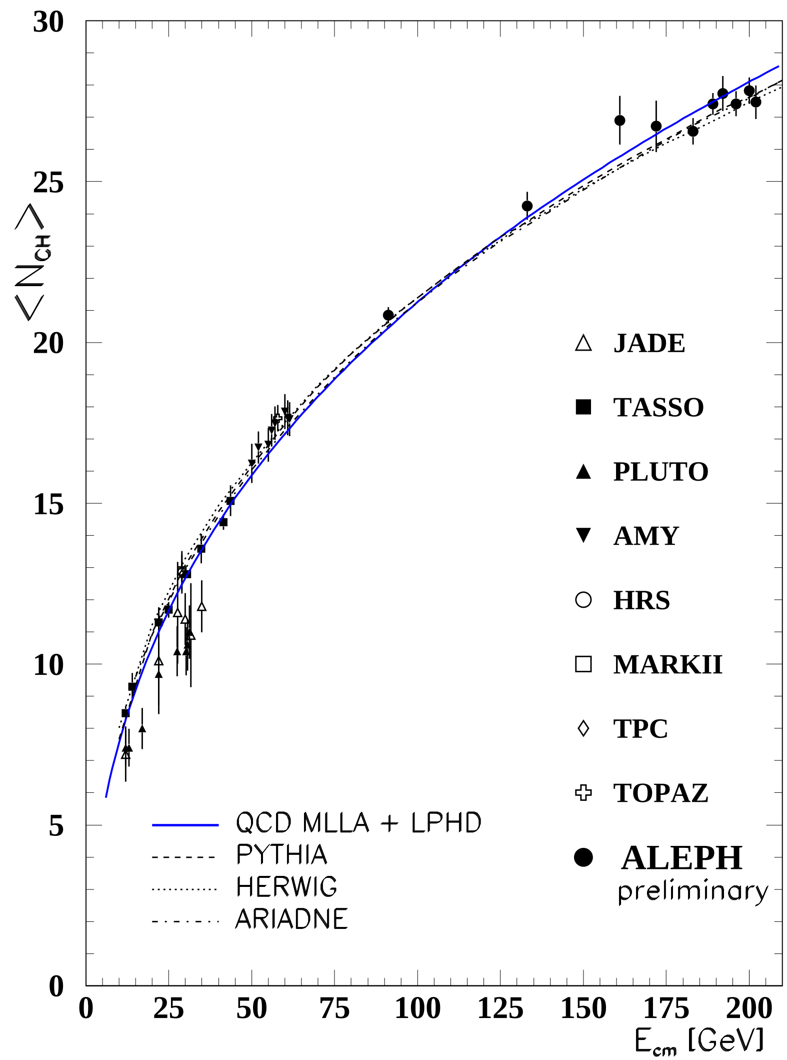
can stop/restart

ISR: more messy

Data comparisons (LEP)

All three algorithms do a reasonable job of describing LEP data, but typically ARIADNE ($p_\perp^2$) > PYTHIA (m^2) > HERWIG (θ)





. . . and programs evolve to do even better . . .

Features of dipole showers

- Quantum coherence on similar grounds for angular and k_T-ordering, typical ordering in dipole showers by $k_\perp$.
- Many new shower formulations in past few years, many (nearly all) based on dipoles in one way or the other.
- Seemingly closer link to NLO calculations: Use subtraction kernels like antennae or Catani-Seymour kernels.
- Typically: First emission fully accounted for.

(borrowed from Frank Krauss)

Survey of existing showering tools

Tools	evolution	AO/Coherence
Ariadne	$k_\perp$-ordered	by construction
Herwig Herwig++	angular ordering improved angular ordering	by construction by construction
Pythia	old: virtuality ordered new: $k_\perp$-ordered	by hand by construction
Sherpa	virtuality ordered (like old Pythia) new: $k_\perp$-ordering	by hand by construction
Vincia	$k_\perp$-ordered	by construction

(borrowed from Frank Krauss)

Leading Log and Beyond

Neglecting Sudakovs, rate of one emission is:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{qg}} \approx \int \frac{\mathrm{d}Q^2}{Q^2} \int \mathrm{d}z \, \frac{\alpha_\mathrm{s}}{2\pi} \frac{4}{3} \frac{1+z^2}{1-z}$$

$$\approx \alpha_\mathrm{s} \ln\left(\frac{Q^2_{\mathrm{max}}}{Q^2_{\mathrm{min}}}\right) \frac{8}{3} \ln\left(\frac{1-z_{\mathrm{min}}}{1-z_{\mathrm{max}}}\right) \sim \alpha_\mathrm{s} \ln^2$$

Rate for n emissions is of form:

$$\mathcal{P}_{\mathrm{q}\to\mathrm{q}n\mathrm{g}} \sim (\mathcal{P}_{\mathrm{q}\to\mathrm{qg}})^n \sim \alpha_\mathrm{s}^n \ln^{2n}$$

Next-to-leading log (NLL): inclusion of *all* corrections of type $\alpha_\mathrm{s}^n \ln^{2n-1}$

No existing generator completely NLL (NLLJET?), but

- energy-momentum conservation (and "recoil" effects)
- coherence
- $2/(1-z) \to (1+z^2)/(1-z)$
- scale choice $\alpha_\mathrm{s}(p_\perp^2)$ absorbs singular terms $\propto \ln z, \ln(1-z)$ in $\mathcal{O}(\alpha_\mathrm{s}^2)$ splitting kernels $P_{\mathrm{q}\to\mathrm{qg}}$ and $P_{\mathrm{g}\to\mathrm{gg}}$
- ...

$\Rightarrow$ far better than naive, analytical LL

Summary Lecture 2

• Hard processes: •

$\star$ Simple ones: probably built-in in PYTHIA/HERWIG $\star$

(SHERPA has complete internal ME generator, HERWIG partial)

$\star$ Multiparton LO: external generator + Les Houches Accord $\star$

$\star$ NLO: not easily related to physical events $\star$

• Parton Showers: •

$\star$ 2 kinds: initial-state and final-state $\star$

$\star$ related to and derived from matrix elements $\star$

$\star$ Sudakov form factor ensures sensible physics $\star$

$\star$ Ordering variable ambiguous: θ, $p_\perp^2$, m^2 $\star$

$\star$ Constraints from coherence arguments, and from data $\star$

$\star$ In state of continuous development $\star$

$\star$ *More to come tomorrow!* $\star$