Modelling of minimum bias and underlying events

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Expect and observe high multiplicities at the LHC. What are production mechanisms behind this?
What is minimum bias (MB)?

MB ≈ “all events, with no bias from restricted trigger conditions”

\[ \sigma_{\text{tot}} = \sigma_{\text{elastic}} + \sigma_{\text{single-diffractive}} + \sigma_{\text{double-diffractive}} + \cdots + \sigma_{\text{non-diffractive}} \]

Schematically:

Reality: can only observe events with particles in central detector: no universally accepted, detector-independent definition

\[ \sigma_{\text{min-bias}} \approx \sigma_{\text{non-diffractive}} + \sigma_{\text{double-diffractive}} \approx \frac{2}{3} \times \sigma_{\text{tot}} \]
What is underlying event (UE)?

In an event containing a jet pair or another hard process, how much further activity is there, that does not have its origin in the hard process itself, but in other physics processes?

Pedestal effect: the UE contains more activity than a normal MB event does (even discarding diffractive events).

Trigger bias: a jet ”trigger” criterion $E_{\perp jet} > E_{\perp min}$ is more easily fulfilled in events with upwards-fluctuating UE activity, since the UE $E_{\perp}$ in the jet cone counts towards the $E_{\perp jet}$. Not enough!
What is pileup?

\[ \langle n \rangle = \overline{\mathcal{L}} \sigma \]

where \( \overline{\mathcal{L}} \) is machine luminosity per bunch crossing, \( \overline{\mathcal{L}} \sim n_1 n_2 / A \) and \( \sigma \sim \sigma_{\text{tot}} \approx 100 \text{ mb} \).

Current LHC machine conditions \( \Rightarrow \langle n \rangle \) approaching 10.

Pileup introduces no new physics, and is thus not further considered here, but can be a nuisance. However, keep in mind concept of bunches of hadrons leading to multiple collisions.
Cross section for $2 \rightarrow 2$ interactions is dominated by $t$-channel gluon exchange, so diverges like $\frac{d\sigma}{dp_{\perp}^2} \approx \frac{1}{p_{\perp}^4}$ for $p_{\perp} \rightarrow 0$.

Integrate QCD $2 \rightarrow 2$

$qq' \rightarrow qq'$
$q\bar{q} \rightarrow q'\bar{q}'$
$q\bar{q} \rightarrow gg$
$qg \rightarrow qg$
$gg \rightarrow gg$
$gg \rightarrow q\bar{q}$

(with CTEQ 5L PDF's)
What is multiple partonic interactions (MPI)?

Note that \( \sigma_{\text{int}}(p_{\perp\min}) \), the number of \((2 \rightarrow 2 \text{ QCD})\) interactions above \( p_{\perp\min} \), involves integral over PDFs,

\[
\sigma_{\text{int}}(p_{\perp\min}) = \int \int \int_{p_{\perp\min}} d x_1 \, d x_2 \, d p_{\perp}^2 \, f_1(x_1, p_{\perp}^2) \, f_2(x_2, p_{\perp}^2) \, \frac{d \hat{\sigma}}{d p_{\perp}^2}
\]

with \( \int d x \, f(x, p_{\perp}^2) = \infty \), i.e. infinitely many partons.

So half a solution to \( \sigma_{\text{int}}(p_{\perp\min}) > \sigma_{\text{tot}} \) is

**many interactions per event: MPI** (historically MI or MPPI)

\[
\begin{align*}
\sigma_{\text{tot}} & = \sum_{n=0}^{\infty} \sigma_n \\
\sigma_{\text{int}} & = \sum_{n=0}^{\infty} n \sigma_n \\
\sigma_{\text{int}} & > \sigma_{\text{tot}} \iff \langle n \rangle > 1
\end{align*}
\]
If interactions occur independently then **Poissonian statistics**

\[ P_n = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle} \]

but \( n = 0 \Rightarrow \) no event (in many models) and energy–momentum conservation \( \Rightarrow \) large \( n \) suppressed so narrower than Poissonian

MPI is a logical consequence of the composite nature of protons,

\( n_{\text{parton}} \sim \sum_{q,\bar{q},g} \int f(x) \, dx > 3, \) which allows \( \sigma_{\text{int}}(p_{\perp\text{min}}) > \sigma_{\text{tot}}, \)

but what about the limit \( p_{\perp\text{min}} \rightarrow 0? \)
Other half of solution is that perturbative QCD is not valid at small $p_\perp$ since $q, g$ are not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp\text{min}} \simeq \frac{\hbar}{r_p} \approx \frac{0.2 \text{ GeV} \cdot \text{fm}}{0.7 \text{ fm}} \approx 0.3 \text{ GeV} \simeq \Lambda_{\text{QCD}}$$

...but better replace $r_p$ by (unknown) colour screening length $d$ in hadron:
Regularization of low-$p_\perp$ divergence

so need **nonperturbative regularization for $p_\perp \to 0$**, e.g.

\[
\frac{d\hat{\sigma}}{dp_\perp^2} \propto \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4} \to \frac{\alpha_s^2(p_\perp^2)}{p_\perp^4} \theta(p_\perp - p_{\perp\text{min}}) \quad \text{(simpler)}
\]

or

\[
\frac{\alpha_s^2(p_{\perp 0}^2 + p_\perp^2)}{(p_{\perp 0}^2 + p_\perp^2)^2} \quad \text{(more physical)}
\]

where $p_{\perp\text{min}}$ or $p_{\perp 0}$ are free parameters, empirically of order 2 GeV.

Typically 2 – 3 interactions/event at the Tevatron, 4 – 5 at the LHC, but may be more in “interesting” high-$p_\perp$ ones.
Indirect evidence for multiple interactions – 1

without MPI:

FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low $p_T$ only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.
Indirect evidence for multiple interactions – 2

with MPI included:

FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T_{\text{min}}} = 2.0$ GeV; solid line, $p_{T_{\text{min}}} = 1.6$ GeV; dashed-dotted line, $p_{T_{\text{min}}} = 1.2$ GeV.

FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

Order 4 jets $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > p_{\perp 4}$ and define $\varphi$ as angle between $p_{\perp 1} \mp p_{\perp 2}$ and $p_{\perp 3} \mp p_{\perp 4}$ for AFS/CDF

$|p_{\perp 1} + p_{\perp 2}| \approx 0$
$|p_{\perp 3} + p_{\perp 4}| \approx 0$
\[d\sigma/d\varphi \text{ flat}\]

$|p_{\perp 1} + p_{\perp 2}| \gg 0$
$|p_{\perp 3} + p_{\perp 4}| \gg 0$
\[d\sigma/d\varphi \text{ peaked at } \varphi \approx 0/\pi \text{ for AFS/CDF}\]
AFS 4-jet analysis (pp at 63 GeV; Copenhagen group): observe 6 times Poissonian prediction, with impact parameter expect 3.7 times Poissonian, but big errors ⇒ low acceptance, also UA2

CDF: 3-jet + prompt photon analysis (simplifies)

\[ \sigma_{DPS} = \frac{\sigma_A \sigma_B}{\sigma_{\text{eff}}} \quad \text{for } A \neq B \quad \implies \sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb} \]

Note inverse relationship on \( \sigma_{\text{eff}} \).
Natural scale is \( \sigma_{\text{ND}} \approx 40 \text{ mb} \), so \( \sigma_{\text{eff}} \ll \sigma_{\text{ND}} \)
is strong enhancement relative to naive expectations!
Consistent with (strongly) uneven matter distribution in proton.
Direct observation of multiple interactions – 3

CDF 16 GeV $\gamma/\pi^0 + 3$ Jets

1-Vertex Events

- **Data**
- **Yellow region**: double parton scattering (DPS)
- **The rest**: PYTHIA showers

**Warning:** PYTHIA here used without DPS
D0 results:

\[ \sigma_{\text{eff}} = 15.1 \pm 1.9 \text{ mb} \]

Agreement and precision “too good to be true”; tunes 8 and 4 years old, respectively, and not to this kind of data. More recent tunes have less matter fluctuations, i.e. higher \( \sigma_{\text{eff}} \), so likely to do worse.
Same study also planned for LHC

Selection for DPS delicate balance:

- Showers dominate at large $p_{\perp} \Rightarrow$ too large background
- Multiple interactions dominate at small $p_{\perp}$, but there jet identification difficult

$k_T (R = 0.4), CDF selections$

Pythia 8.108

$pp \rightarrow \gamma + X @ 14$ TeV
All modern general-purpose generators are built on MPI concepts

but details differ, both physics and technology, e.g.

- a single regularized hard component
  or separate hard + soft components
- MPIs generated ordered in $p_{\perp}$ or not
- energy/momentum/flavour conservation
- impact-parameter profile
- colour connection & reconnection strategies
- energy dependence
- ...

In the following PYTHIA, Herwig++, Phojet;
current Sherpa $\approx$ PYTHIA; tomorrow future Sherpa
A Poissonian process is one where “events” (e.g. radioactive decays) can occur uncorrelated in “time” $t$ (or other ordering variable). If the probability for an “event” to occur at “time” $t$ is $P(t)$ then the probability for a first “event” after $t_0 = 0$ at $t_1$ is

$$P(t_1) = P(t_1) \exp \left( - \int_{0}^{t_1} P(t) \, dt \right)$$

and for an $i$’th at $t_i$ is

$$P(t_i) = P(t_i) \exp \left( - \int_{t_{i-1}}^{t_i} P(t) \, dt \right)$$

Example: Sudakov form factor for parton showers, where increasing $t \rightarrow$ decreasing evolution variable $Q$ and “event” $\rightarrow$ parton branchings ... but relevant for MPIs as well ...
For now exclude diffractive (and elastic) topologies, i.e. only model nondiffractive events, with $\sigma_{\text{nd}} \simeq 0.6 \times \sigma_{\text{tot}}$.

Differential probability for interaction at $p_\perp$ is

$$\frac{dP}{dp_\perp} = \frac{1}{\sigma_{\text{nd}}} \frac{d\sigma}{dp_\perp}$$

Average number of interactions naively

$$\langle n \rangle = \frac{1}{\sigma_{\text{nd}}} \int_{0}^{E_{\text{cm}}/2} \frac{d\sigma}{dp_\perp} dp_\perp$$

Require $\geq 1$ interaction in an event or else pass through without anything happening

$$P_{\geq 1} = 1 - P_0 = 1 - \exp(-\langle n \rangle)$$

(Alternatively: allow soft nonperturbative interactions even if no perturbative ones.)
Can pick $n$ from Poissonian and then generate $n$ independent interactions according to $d\sigma/dp_\perp$ (so long as energy left), or better...

**generate interactions in ordered sequence** $p_\perp 1 > p_\perp 2 > p_\perp 3 > \ldots$

- Apply to ordered sequence of decreasing $p_\perp$, starting from $E_{cm}/2$

$$\mathcal{P}(p_\perp = p_\perp i) = \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp_\perp} \exp \left[ - \int_{p_\perp}^{p_\perp (i-1)} \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp'_\perp} dp'_\perp \right]$$

- Use rescaled PDF’s taking into account already used momentum and flavours

$\implies n_{int}$ narrower than Poissonian
So far assumed that all collisions have equivalent initial conditions, but hadrons are extended, e.g. empirical double Gaussian:

\[
\rho_{\text{matter}}(r) = N_1 \exp \left( -\frac{r^2}{r_1^2} \right) + N_2 \exp \left( -\frac{r^2}{r_2^2} \right)
\]

where \( r_2 \neq r_1 \) represents “hot spots”, and overlap of hadrons during collision is

\[
\mathcal{O}(b) = \int \, d^3x \, dt \, \rho_{1,\text{matter}}^{\text{boosted}}(x, t) \rho_{2,\text{matter}}^{\text{boosted}}(x, t)
\]

or electromagnetic form factor:

\[
S_p(b) = \int \frac{d^2k}{2\pi} \frac{\exp(ik \cdot b)}{(1 + k^2/\mu^2)^2}
\]

where \( \mu = 0.71 \text{ GeV} \rightarrow \) free parameter, which gives

\[
\mathcal{O}(b) = \frac{\mu^2}{96\pi} (\mu b)^3 K_3(\mu b)
\]
• Events are distributed in impact parameter $b$
• Average activity at $b$ proportional to $O(b)$
  * central collisions more active $\Rightarrow P_n$ broader than Poissonian
  * peripheral passages normally give no collisions $\Rightarrow$ finite $\sigma_{\text{tot}}$
• Also crucial for *pedestal effect* (more later)
(1) Colour connections:
Each interaction hooks up with colours from beam remnants, but how does the colours in the remnant hook up with each other?

(2) Colour reconnections:
Many interaction “on top of” each other \[ \Rightarrow \] tightly packed partons!
Is there a strict colour memory when partons recede?

Recall: \( N_C = 3 \), not \( N_C = \infty \)!
Energy dependence of $p_{\perp \text{min}}$ and $p_{\perp 0}$

Larger collision energy
$\Rightarrow$ probe parton ($\approx$ gluon) density at smaller $x$
$\Rightarrow$ smaller colour screening length $d$
$\Rightarrow$ larger $p_{\perp \text{min}}$
or $p_{\perp 0}$
$\Rightarrow$ dampened multiplicity rise
Events with hard scale have more underlying activity!
Trigger bias: hard scale $\Rightarrow$ central collision $\Rightarrow$ large UE.
Studied in particular by Rick Field, comparing with CDF data:
http://www.phys.ufl.edu/~rfield/cdf/rdf_talks.html)

- Define the MAX and MIN “transverse” regions on an event-by-event basis with MAX (MIN) having the largest (smallest) density.
“Leading Jet” events correspond to the leading calorimeter jet (MidPoint R = 0.7) in the region $|\eta| < 2$ with no other conditions.

“Inclusive 2-Jet Back-to-Back” events are selected to have at least two jets with Jet#1 and Jet#2 nearly “back-to-back” ($\Delta \phi_{12} > 150^\circ$) with almost equal transverse energies ($P_T(jet#2)/P_T(jet#1) > 0.8$) with no other conditions.

“Exclusive 2-Jet Back-to-Back” events are selected to have at least two jets with Jet#1 and Jet#2 nearly “back-to-back” ($\Delta \phi_{12} > 150^\circ$) with almost equal transverse energies ($P_T(jet#2)/P_T(jet#1) > 0.8$) and $P_T(jet#3) < 15$ GeV/c.

“Leading ChgJet” events correspond to the leading charged particle jet (R = 0.7) in the region $|\eta| < 1$ with no other conditions.

“Z-Boson” events are Drell-Yan events with $70 < M(\text{lepton-pair}) < 110$ GeV with no other conditions.
Data at 1.96 TeV on the density of charged particles, dN/dηdϕ, with p_T > 0.5 GeV/c and |η| < 1 for “leading jet” events as a function of the leading jet p_T for the “toward”, “away”, and “transverse” regions. The data are corrected to the particle level (with errors that include both the statistical error and the systematic uncertainty) and are compared with PYTHIA Tune A at the particle level (i.e. generator level).
Data at 1.96 TeV on the density of charged particles, \(dN/d\eta d\phi\), with \(p_T > 0.5\) GeV/c and \(|\eta| < 1\) for "Z-Boson" events as a function of the leading jet \(p_T\) for the "toward", "away", and "transverse" regions. The data are corrected to the particle level (with errors that include both the statistical error and the systematic uncertainty) and are compared with PYTHIA Tune AW at the particle level (i.e. generator level).
Tuned PYTHIA 6.206

“Transverse” $\mathbf{P_T}$ Distribution

Compared the average “transverse” charge particle density ($|\eta|<1$, $P_T>0.5$ GeV) versus $P_T$ (charged jet#1) and the $P_T$ distribution of the “transverse” density, $dN_{\text{chg}}/d\eta d\phi dP_T$ with the QCD Monte-Carlo predictions of two tuned versions of PYTHIA 6.206 ($P_T$ (hard) > 0, CTEQ5L, Set B (PARP(67)=1) and Set A (PARP(67)=4)).
Back-to-Back “Associated” Charged Particle Densities

- Shows the $\Delta \phi$ dependence of the “associated” charged particle density, $dN_{\text{ch}}/d\eta d\phi$, $p_T > 0.5$ GeV/c, $|\eta| < 1$, $PT_{\text{maxT}} > 2.0$ GeV/c (not including $PT_{\text{maxT}}$) relative to $PT_{\text{maxT}}$ (rotated to 180°) and the charged particle density, $dN_{\text{ch}}/d\eta d\phi$, $p_T > 0.5$ GeV/c, $|\eta| < 1$, relative to jet#1 (rotated to 270°) for “back-to-back events” with $30 < E_T(\text{jet#1}) < 70$ GeV.
For $PT_{\text{max}}T > 2.0 \text{ GeV}$ both PYTHIA and HERWIG produce slightly too many “associated” particles in the direction of $PT_{\text{max}}T$!

But HERWIG (without multiple parton interactions) produces too few particles in the direction opposite of $PT_{\text{max}}T$!
PYTHIA implementation – 1

Has gradually evolved from the MPI start in 1985; still older versions in use.

Current version involves (among others):

• MPI ordered in $p_\perp$, and also
• transverse-momentum-ordered parton showers for ISR and FSR.

Allows interleaved evolution for MPI, ISR and FSR:

$$\frac{d\mathcal{P}}{dp_\perp} = \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp_\perp} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp_\perp} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp_\perp} \right)$$

$$\times \exp \left( -\int_{p_\perp}^{p_{\perp\text{max}}} \left( \frac{d\mathcal{P}_{\text{MPI}}}{dp_\perp'} + \sum \frac{d\mathcal{P}_{\text{ISR}}}{dp_\perp'} + \sum \frac{d\mathcal{P}_{\text{FSR}}}{dp_\perp'} \right) \, dp_\perp' \right)$$

ordered in decreasing $p_\perp$ using “Sudakov” trick.

Corresponds to increasing “resolution” of partonic final state: smaller $p_\perp$ fill in details of the basic picture set at larger $p_\perp$. 
PYTHIA implementation – 2

Other aspects in line with previous discussions:

- smooth dampening \( \frac{d\hat{\sigma}}{dp_\perp^2} \propto \frac{1}{(p_{\perp 0}^2 + p_\perp^2)^2} \)
  \( \Rightarrow \) all interactions belong to same “hard” kind

- energy-dependent \( p_{\perp 0} \)

\[
p_{\perp 0}(E_{\text{cm}}) = p_{\perp 0}(E_{\text{cm,ref}}) \left( \frac{E_{\text{cm}}}{E_{\text{cm,ref}}} \right)^k
\]

- matter profile flexible, Gaussian or more spiked
- PDF rescaling for energy/momentum/flavour conservation
- colour connection/reconnection important component
- drift of baryon number by junction topology
Rescattering (optional since 2009)

Same order in $\alpha_s$, $\sim$ same propagators, but one PDF weight less $\Rightarrow$ smaller $\sigma$, and one jet less $\Rightarrow$ 2 $\rightarrow$ 3 QCD radiation background larger

An $x$-dependent proton size (optional since 2011)

$$\rho(r, x) \propto \frac{1}{a^3(x)} \exp \left( -\frac{r^2}{a^2(x)} \right)$$

with

$$a(x) = a_0 \left( 1 + a_1 \ln \frac{1}{x} \right)$$

$a_1 \approx 0.15$ tuned to rise of $\sigma_{ND}$

$a_0$ tuned to value of $\sigma_{ND}$, given PDF, $p_{\perp 0}$, ...
Herwig++ implementation – 1

Old non-MPI Soft Underlying Event thoroughly killed. Jimmy add-on to HERWIG does UE, but not MB.

⇒ Herwig++ first complete alternative:

- number of interactions first picked; thereafter generated unordered in $p_{\perp}$
- interactions uncorrelated, up until energy used up
- force ISR to reconstruct back to gluon after first interaction
- impact parameter by electromagnetic form factor shape, but with tunable width ($\sim$ factor 3 different from em width)
- $p_{\perp\text{min}}$ scale to be tuned energy-by-energy
Key point: two-component model

\[ p_\perp > p_\perp \text{min}: \text{pure perturbation theory (no modification)} \]
\[ p_\perp < p_\perp \text{min}: \text{pure nonperturbative ansatz} \]
Colour reconnection essential to get $dn/d\eta$ correct:
(1) Cut Pomeron (1982)
- Pomeron predates QCD; nowadays $\sim$ glueball tower
- Optical theorem relates $\sigma_{\text{total}}$ and $\sigma_{\text{elastic}}$
- Unified framework of nondiffractive and diffractive interactions
- Purely low-$p_\perp$: only primordial $k_\perp$ fluctuations
- Usually simple Gaussian matter distribution

(2) Extension to large $p_\perp$ (1990)
- distinguish soft and hard Pomerons:
  - soft = nonperturbative, low-$p_\perp$, as above
  - hard = perturbative, “high”-$p_\perp$
- hard based on PYTHIA code, with lower cutoff in $p_\perp
First/most LHC comparisons to old versions of generators, e.g.:
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State of new generators early 2011:

Charged multiplicity, $\sqrt{s} = 7000$ GeV

$1/N_{\text{ev}}dN_{\text{ev}}/dN_{\text{ch}}$

$\langle p_\perp \rangle$ vs $N_{\text{ch}}, \sqrt{s} = 7000$ GeV

State of new generators early 2011:

Transverse $N_{ch}$ density vs $p_\perp$ (leading track), $\sqrt{s} = 7$ TeV

$\frac{d^2 \langle N_{ch} \rangle}{d\eta d\phi}$

Transverse $\sum p_\perp$ density vs $p_\perp$ (leading track), $\sqrt{s} = 7$ TeV

$\frac{d^2 \langle \sum p_\perp \rangle}{d\eta d\phi}$ [GeV]

MPI concept compelling; it has to exist at some level

By now, strong direct evidence, overwhelming indirect

Understanding of MPI crucial for LHC precision physics

Many details uncertain:
- physics and form of $p_{\perp \text{min}}/p_{\perp 0}$ regularization
- non-factorized impact parameter picture
- multiparton densities in incoming hadron
- colour correlations between interactions
- energy dependence $\Rightarrow$ predictivity
- dense-packing of partons and hadrons $\Rightarrow$ collective effects?
- diffraction, forward physics, . . .

Above physics aspects must all be present, and more?
If a model is simple, it is wrong!

So stay tuned for ever more complicated models in the future!