Event Generators for LHC

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1. (today) Introduction and Overview; Parton Showers; Matching Issues
2. (tomorrow) Multiple Interactions; Hadronization; Generators & Conclusions
Event Generator Position

“real life”

Machine $\Rightarrow$ events

produce events

Detector, Data Acquisition

observe & store events

“virtual reality”

Event Generator

Detector Simulation

Event Reconstruction

compare real and simulated data

what is knowable?

Physics Analysis

conclusions, articles, talks, . . .

“quick and dirty”
Event Generator Position

“real life”

Machine ⇒ events
Tevatron, LHC

produce events

“virtual reality”

Event Generator
PYTHIA, HERWIG

observe & store events

Detector, Data Acquisition
ATLAS, CMS, LHC-B, ALICE

Detector Simulation
Geant4, LCG

what is knowable?

Event Reconstruction
CMSSIM, ATHENA

compare real and simulated data

Physics Analysis
ROOT, FastJet

conclusions, articles, talks, …
Why Generators?

- Allow studies of complex multiparticle physics
- Large flexibility in physical quantities that can be addressed
  - Vehicle of ideology to disseminate ideas

Can be used to

- predict event rates and topologies \(\Rightarrow\) estimate feasibility
- simulate possible backgrounds \(\Rightarrow\) devise analysis strategies
- study detector requirements \(\Rightarrow\) optimize detector/trigger design
- study detector imperfections \(\Rightarrow\) evaluate acceptance corrections

Monte Carlo method convenient because Einstein was wrong:
  God does throw dice!

Quantum mechanics: amplitudes \(\Rightarrow\) probabilities
Anything that possibly can happen, will! (but more or less often)
The structure of an event

Warning: schematic only, everything simplified, nothing to scale, …

Incoming beams: parton densities
Hard subprocess: described by matrix elements
Resonance decays: correlated with hard subprocess
Initial-state radiation: spacelike parton showers
Final-state radiation: timelike parton showers
Multiple parton–parton interactions …
... with its initial- and final-state radiation
Beam remnants and other outgoing partons
Everything is connected by colour confinement strings
Recall! Not to scale: strings are of hadronic widths
The strings fragment to produce primary hadrons
Many hadrons are unstable and decay further.
These are the particles that hit the detector
The Monte Carlo method

Want to generate events in as much detail as Mother Nature
   $\implies$ get average and fluctuations right
   $\implies$ make random choices, $\sim$ as in nature

$\sigma_{\text{final state}} = \sigma_{\text{hard process}} P_{\text{tot,hard process}} \rightarrow \text{final state}$

(appropriately summed & integrated over non-distinguished final states)

where $P_{\text{tot}} = P_{\text{res}} P_{\text{ISR}} P_{\text{FSR}} P_{\text{MI}} P_{\text{remnants}} P_{\text{hadronization}} P_{\text{decays}}$

with $P_i = \prod_j P_{ij} = \prod_j \prod_k P_{ijk} = \ldots$ in its turn

$\implies$ divide and conquer

an event with $n$ particles involves $\mathcal{O}(10^n)$ random choices,
(flavour, mass, momentum, spin, production vertex, lifetime, $\ldots$)

LHC: $\sim 100$ charged and $\sim 200$ neutral ($+$ intermediate stages)

$\implies$ several thousand choices
(of $\mathcal{O}(100)$ different kinds)
specialized often best at given task, but need General-Purpose core
Parton Showers

- Final-State (Timelike) Showers
- Initial-State (Spacelike) Showers
- Matching to Matrix Elements

Only towers with $E_T > 0.5$ GeV are shown
Divergences

Emission rate $q \rightarrow qg$ diverges when
- collinear: opening angle $\theta_{qg} \rightarrow 0$
- soft: gluon energy $E_g \rightarrow 0$

Almost identical to $e \rightarrow e \gamma$ ("bremsstrahlung"),
but QCD is non-Abelian so additionally
- $g \rightarrow gg$ similarly divergent
- $\alpha_s(Q^2)$ diverges for $Q^2 \rightarrow 0$
  (actually for $Q^2 \rightarrow \Lambda_{QCD}^2$)

Big probability for one emission $\implies$ also big for several
$\implies$ with ME's need to calculate to high order and with many loops
$\implies$ extremely demanding technically (not solved!), and
involving big cancellations between positive and negative contributions.
Alternative approach: parton showers
The Parton-Shower Approach

\[ 2 \to n = (2 \to 2) \oplus \text{ISR} \oplus \text{FSR} \]

\( Q_1^2 \)
\( Q_2^2 \)
\( Q_3^2 \)
\( Q_4^2 \)

FSR = Final-State Rad.;
timelike shower
\( Q_i^2 \sim m^2 > 0 \) decreasing

ISR = Initial-State Rad.;
spacelike shower
\( Q_i^2 \sim -m^2 > 0 \) increasing

2 \to 2 = hard scattering (on-shell):

\[
\sigma = \int \int \int d^2 x_1 d^2 x_2 d^2 \hat{t} f_i(x_1, Q^2) f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}
\]

Shower evolution is viewed as a probabilistic process,
which occurs with unit total probability:

*the cross section is not directly affected,*
*but indirectly it is, via the changed event shape*
Technical aside: why timelike/spacelike?

Consider four-momentum conservation in a branching $a \rightarrow b c$

$$\begin{align*}
p_{\perp a} = 0 & \Rightarrow p_{\perp c} = -p_{\perp b} \\
p_+ = E + p_L & \Rightarrow p_{+a} = p_{+b} + p_{+c} \\
p_- = E - p_L & \Rightarrow p_{-a} = p_{-b} + p_{-c}
\end{align*}$$

Define $p_{+b} = z p_{+a}, \ p_{+c} = (1 - z) p_{+a}$

Use $p_+ p_- = E^2 - p_L^2 = m^2 + p_{\perp}^2$

$$\begin{align*}
\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} &= \frac{m_b^2 + p_{\perp b}^2}{zp_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1 - z)p_{+a}} \\
\Rightarrow m_a^2 &= \frac{m_b^2 + p_{\perp}}{z} + \frac{m_c^2 + p_{\perp}}{1 - z} = \frac{m_b^2}{z} + \frac{m_c^2}{1 - z} + \frac{p_{\perp}^2}{z(1 - z)}
\end{align*}$$

Final-state shower: $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_{\perp}^2}{z(1 - z)} > 0 \Rightarrow$ timelike

Initial-state shower: $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_{\perp}^2}{1 - z} < 0 \Rightarrow$ spacelike
Doublecounting

A $2 \rightarrow n$ graph can be “simplified” to $2 \rightarrow 2$ in different ways:

- $g \rightarrow q\bar{q} \oplus qg \rightarrow qg$
- $g \rightarrow gg \oplus gg \rightarrow q\bar{q}$

Do not doublecount: $2 \rightarrow 2 = \text{most virtual} = \text{shortest distance}$

Conflict: theory derivations often assume virtualities strongly ordered; interesting physics often in regions where this is not true!
From Matrix Elements to Parton Showers

\[ e^+ e^- \rightarrow q\bar{q}g \]

\[ x_j = 2E_j/E_{cm} \Rightarrow x_1 + x_2 + x_3 = 2 \]

\[ m_q = 0 : \quad \frac{d\sigma_{ME}}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} dx_1 dx_2 \]

Rewrite for \( x_2 \rightarrow 1 \), i.e. \( q-g \) collinear limit:

\[ 1 - x_2 = \frac{m_{13}^2}{E_{cm}^2} = \frac{Q^2}{E_{cm}^2} \Rightarrow dx_2 = \frac{dQ^2}{E_{cm}^2} \]

\[ x_1 \approx z \Rightarrow dx_1 \approx dz \]

\[ x_3 \approx 1 - z \]

\[ \Rightarrow dP = \frac{d\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{dx_2}{1 - x_2} \frac{4}{3} \frac{x_2^2 + x_1^2}{(1 - x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \frac{4}{3} \frac{1 + z^2}{1 - z} dz \]
Generalizes to DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

\[
d\mathcal{P}_{a \to bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \mathcal{P}_{a \to bc}(z) \, dz
\]

\[
P_{q \to qg} = \frac{4}{3} \frac{1 + z^2}{1 - z}
\]

\[
P_{g \to gg} = 3 \frac{(1 - z(1 - z))^2}{z(1 - z)}
\]

\[
P_{g \to q\bar{q}} = \frac{n_f}{2} (z^2 + (1 - z)^2) \quad (n_f = \text{no. of quark flavours})
\]

Iteration gives final-state parton showers

Need soft/collinear cut-offs to stay away from nonperturbative physics.
Details model-dependent, e.g.
\[Q > m_0 = \min(m_{ij}) \approx 1 \text{ GeV},\]
\[z_{\min}(E, Q) < z < z_{\max}(E, Q)\]
or \[p_{\perp} > p_{\perp\min} \approx 0.5 \text{ GeV}\]
The Sudakov Form Factor

Conservation of total probability:
\[ \mathcal{P}(\text{nothing happens}) = 1 - \mathcal{P}(\text{something happens}) \]

“multiplicativeness” in “time” evolution:
\[ \mathcal{P}_{\text{nothing}}(0 < t \leq T) = \mathcal{P}_{\text{nothing}}(0 < t \leq T_1) \mathcal{P}_{\text{nothing}}(T_1 < t \leq T) \]

Subdivide further, with \( T_i = (i/n)T, 0 \leq i \leq n \):

\[ \mathcal{P}_{\text{nothing}}(0 < t \leq T) = \lim_{n \to \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{nothing}}(T_i < t \leq T_{i+1}) \]

\[ = \lim_{n \to \infty} \prod_{i=0}^{n-1} \left( 1 - \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \]

\[ = \exp \left( - \lim_{n \to \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{something}}(T_i < t \leq T_{i+1}) \right) \]

\[ = \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \]

\[ \Rightarrow \quad d\mathcal{P}_{\text{first}}(T) = d\mathcal{P}_{\text{something}}(T) \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{something}}(t)}{dt} dt \right) \]
Example: radioactive decay of nucleus

Naively: \( \frac{dN}{dt} = -cN_0 \Rightarrow N(t) = N_0 \ (1 - ct) \)

depletion: a given nucleus can only decay once

correctly: \( \frac{dN}{dt} = -cN(t) \Rightarrow N(t) = N_0 \ \exp(-ct) \)

generalizes to: \( N(t) = N_0 \ \exp \left( -\int_0^t c(t') dt' \right) \)

or: \( \frac{dN(t)}{dt} = -c(t) \ N_0 \ \exp \left( -\int_0^t c(t') dt' \right) \)

sequence allowed: nucleus\_1 \rightarrow \text{nucleus\_2} \rightarrow \text{nucleus\_3} \rightarrow \ldots

Correspondingly, with \( Q \sim 1/t \) (Heisenberg)

\[
dP_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \ P_{a \rightarrow bc}(z) \ dz \ \exp \left( -\sum_{b,c} \int_{Q_2}^{Q_{max}} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z') \ dz' \right)
\]

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that \( \sum_{b,c} \int \int dP_{a \rightarrow bc} \equiv 1 \Rightarrow \text{convenient for Monte Carlo} \)

(\( \equiv 1 \) if extended over whole phase space, else possibly nothing happens)
Sudakov form factor provides “time” ordering of shower: lower $Q^2 \iff$ longer times

$Q^2_1 > Q^2_2 > Q^2_3$  
$Q^2_1 > Q^2_4 > Q^2_5$  
etc.

Sudakov regulates singularity for first emission . . .

. . . but in limit of repeated soft emissions $q \rightarrow qg$  
$(g \rightarrow gg, g \rightarrow q\bar{q}$ not considered)

one obtains the same inclusive $Q$ emission spectrum as for ME,  
i.e. divergent ME spectrum  
$\iff$ infinite number of PS emissions
Coherence

**QED: Chudakov effect (mid-fifties)**

![Diagram of cosmic ray interaction](image)

**QCD: colour coherence for soft gluon emission**

\[
\left| \begin{array}{c} s_1 \\ s_2 \end{array} \right|^2 + \left| \begin{array}{c} s_3 \\ s_4 \end{array} \right|^2 = \left| \begin{array}{c} s_5 \\ s_6 \end{array} \right|^2
\]

solved by

- requiring emission angles to be decreasing
- requiring transverse momenta to be decreasing
The Common Showering Algorithms

Three main approaches to showering in common use:

Two are based on the standard shower language of $a \rightarrow bc$ successive branchings:

HERWIG: $Q^2 \approx E^2(1 - \cos \theta) \approx E^2\theta^2/2$

PYTHIA: $Q^2 = m^2$ (timelike) or $=-m^2$ (spacelike)

One is based on a picture of dipole emission $ab \rightarrow cde$:

ARIADNE: $Q^2 = p_{\perp}^2$; FSR mainly, ISR is primitive; there instead LDCMC: sophisticated but complicated
Ordering variables in final-state radiation

**PYTHIA:** $Q^2 = m^2$
- large mass first
- ⇒ “hardness” ordered
- coherence brute force
- covers phase space
- ME merging simple
- $g \rightarrow q\bar{q}$ simple
- not Lorentz invariant
- no stop/restart
- ISR: $m^2 \rightarrow -m^2$

**HERWIG:** $Q^2 \sim E^2 \theta^2$
- large angle first
- ⇒ hardness not ordered
- coherence inherent
- gaps in coverage
- ME merging messy
- $g \rightarrow q\bar{q}$ simple
- not Lorentz invariant
- no stop/restart
- ISR: $\theta \rightarrow \theta$

**ARIADNE:** $Q^2 = p^2_\perp$
- large $p_\perp$ first
- ⇒ “hardness” ordered
- coherence inherent
- covers phase space
- ME merging simple
- $g \rightarrow q\bar{q}$ messy
- Lorentz invariant
- can stop/restart
- ISR: more messy
Data comparisons

All three algorithms do a reasonable job of describing LEP data, but typically ARIADNE ($p^2$) > PYTHIA ($m^2$) > HERWIG ($\theta$)

... and programs evolve to do even better ...
Neglecting Sudakovs, rate of one emission is:

\[
P_{q \to qg} \approx \int \frac{dQ^2}{Q^2} \int dz \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{1 + z^2}{1 - z}
\]

\[
\approx \alpha_s \ln \left( \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} \right) \frac{8}{3} \ln \left( \frac{1 - z_{\text{min}}}{1 - z_{\text{max}}} \right) \sim \alpha_s \ln^2
\]

Rate for \( n \) emissions is of form:

\[
P_{q \to q^n g} \sim (P_{q \to qg})^n \sim \alpha_s^n \ln^{2n}
\]

Next-to-leading log (NLL): inclusion of all corrections of type \( \alpha_s^n \ln^{2n-1} \)

No existing pp/p\( \bar{p} \) generator completely NLL, but

- energy-momentum conservation (and “recoil” effects)
- coherence
- \( 2/(1 - z) \to (1 + z^2)/(1 - z) \)
- scale choice \( \alpha_s(p_T^2) \) absorbs singular terms \( \propto \ln z, \ln(1 - z) \)
  in \( \mathcal{O}(\alpha_s^2) \) splitting kernels \( P_{q \to qg} \) and \( P_{g \to gg} \)

\( \Rightarrow \) far better than naive, analytical LL
Parton Distribution Functions

Hadrons are composite, with time-dependent structure:

\[ f_i(x, Q^2) = \text{number density of partons } i \]
\[ \text{at momentum fraction } x \text{ and probing scale } Q^2. \]

Linguistics (example):

\[ F_2(x, Q^2) = \sum_i e_i^2 x f_i(x, Q^2) \]

structure function parton distributions
Absolute normalization at small $Q_0^2$ unknown.

Resolution dependence by DGLAP:

$$\frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} \frac{\alpha_s}{P_{a \rightarrow bc}} \left( z = \frac{x}{x'} \right)$$

$Q^2 = 4 \text{ GeV}^2$

$Q^2 = 10000 \text{ GeV}^2$
For cross section calculations NLO PDF’s are combined with NLO $\sigma$’s. Gives significantly better description of data than LO can.

But NLO $\Rightarrow$ parton model not valid, e.g. $g(x, Q^2)$ can be negative. Not convenient for LO showers, nor for many LO ME’s.

Recent revived interest in modified LO sets, e.g. by Thorne & Sherstnev: allow $\sum_i \int_0^1 x f_i(x, Q^2) \, dx > 1$; around $\sim 1.15$.

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Initial-State Shower Basics

• Parton cascades in $p$ are continuously born and recombined.
• Structure at $Q$ is resolved at a time $t \sim 1/Q$ before collision.
• A hard scattering at $Q^2$ probes fluctuations up to that scale.
• A hard scattering inhibits full recombination of the cascade.

Event generation could be addressed by **forwards evolution**: pick a complete partonic set at low $Q_0$ and evolve, see what happens.

**Inefficient:**
1) have to evolve and check for *all* potential collisions, but 99.9...% inert
2) impossible to steer the production e.g. of a narrow resonance (Higgs)
**Backwards evolution** is viable and \( \sim \) equivalent alternative: start at hard interaction and trace what happened “before”

Monte Carlo approach, based on *conditional probability*: recast

\[
\frac{df_b(x, Q^2)}{dt} = \sum_a \int_0^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a\to bc}(z)
\]

with \( t = \ln(Q^2/\Lambda^2) \) and \( z = x/x' \) to

\[
dP_b = \frac{df_b}{f_b} = |dt| \sum_a \int dz \frac{x'f_a(x', t)}{xf_b(x, t)} \frac{\alpha_s}{2\pi} P_{a\to bc}(z)
\]

then solve for decreasing \( t \), i.e. backwards in time, starting at high \( Q^2 \) and moving towards lower, with Sudakov form factor \( \exp(-\int dP_b) \)
Ladder representation combines whole event:

\[ Q_{\text{max}}^2 > Q_1^2 > Q_2^2 \sim Q_0^2 \]
\[ Q_{\text{max}}^2 > Q_3^2 > Q_4^2 > Q_5^2 \sim Q_0^2 \]

DGLAP: \[ Q_{\text{max}}^2 > Q_1^2 > Q_2^2 \sim Q_0^2 \]

One possible Monte Carlo order:
1) Hard scattering
2) Initial-state shower from center outwards
3) Final-state showers

cf. previously:
Coherence in spacelike showers

\[ z_1 = \frac{E_3}{E_1} \]
\[ z_3 = \frac{E_5}{E_3} \]
\[ \theta_2 = \theta_{12} \]
\[ \theta_4 = \theta_{14}!! \]

with \( Q^2 = -m^2 = \) spacelike virtuality

- **kinematics only:**
  \[ Q^2_3 > z_1 Q^2_1, \quad Q^2_5 > z_3 Q^2_3, \ldots \]
  i.e. \( Q^2_i \) need not even be ordered

- **coherence of leading collinear singularities:**
  \[ Q^2_5 > Q^2_3 > Q^2_1, \text{ i.e. } Q^2 \text{ ordered} \]

- **coherence of leading soft singularities (more messy):**
  \[ E_3 \theta_4 > E_1 \theta_2, \text{ i.e. } z_1 \theta_4 > \theta_2 \]
  \[ z \ll 1: \quad E_1 \theta_2 \approx p^2_{12} \approx Q^2_3, \quad E_3 \theta_4 \approx p^2_{14} \approx Q^2_5 \]
  i.e. reduces to \( Q^2 \) ordering as above
  \[ z \approx 1: \quad \theta_4 > \theta_2, \text{ i.e. angular ordering of soft gluons} \]
  \[ \implies \text{ reduced phase space} \]
Evolution procedures

**DGLAP**: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi
evolution towards larger $Q^2$ and (implicitly) towards smaller $x$

**BFKL**: Balitsky–Fadin–Kuraev–Lipatov
evolution towards smaller $x$ (with small, unordered $Q^2$)

**CCFM**: Ciafaloni–Catani–Fiorani–Marchesini
interpolation of DGLAP and BFKL

**GLR**: Gribov–Levin–Ryskin
nonlinear equation in dense-packing (saturation) region,
where partons recombine, not only branch
Initial-State Shower Comparison

Two (?) CCFM Generators:
(SMALLX (Marchesini, Webber))
CASCADE (Jung, Salam)
LDC (Gustafson, Lönnblad):
reformulated initial/final rad.
⇒ eliminate non-Sudakov

Test 1) forward (= p direction) jet activity at HERA

\begin{align*}
\text{In In } k^2_\perp & \Rightarrow \text{low-}k_\perp \text{ part unordered}
\end{align*}

\begin{align*}
\ln 1/x & \Rightarrow \text{DGLAP-like increasing } k_\perp
\end{align*}
2) Heavy flavour production

but also explained by DGLAP with leading order pair creation
+ flavour excitation ($\approx$ unordered chains)
+ gluon splitting (final-state radiation)

CCFM requires off-shell ME’s + unintegrated parton densities

$$F(x, Q^2) = \int^{Q^2} \frac{dk^2}{k^2} F(x, k^2) + \text{(suppressed with } k^2 > Q^2)$$

so not ready for prime time in pp
Initial- vs. final-state showers

Both controlled by same evolution equations

\[
dP_{a\to bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) \, dz \cdot (\text{Sudakov})
\]

but

**Final-state showers:**

\(Q^2\) timelike \((\sim m^2)\)

- Decreasing \(E, m^2, \theta\)
- Both daughters \(m^2 \geq 0\)
- Physics relatively simple
  \(\Rightarrow\) "minor" variations:
  \(Q^2,\) shower vs. dipole, \ldots

**Initial-state showers:**

\(Q^2\) spacelike \((\approx -m^2)\)

- Decreasing \(E,\) increasing \(Q^2, \theta\)
- One daughter \(m^2 \geq 0,\) one \(m^2 < 0\)
- Physics more complicated
  \(\Rightarrow\) more formalisms:
  DGLAP, BFKL, CCFM, GLR, \ldots
Future of showers

Showers still evolving:

HERWIG has new evolution variable better suited for heavy particles

\[ \tilde{q}^2 = \frac{q^2}{z^2(1-z)^2} + \frac{m^2}{z^2} \quad \text{for } q \to qg \]

Gives smooth coverage of soft-gluon region, no overlapping regions in FSR phase space, but larger dead region.

PYTHIA has moved (but not yet users?) to \( p_{\perp} \)-ordered showers (borrowing some of ARIADNE dipole approach, but still showers)

\[ p^2_{\perp \text{evol}} = z(1-z)Q^2 = z(1-z)M^2 \text{ for FSR} \]
\[ p^2_{\perp \text{evol}} = (1-z)Q^2 = (1-z)(-M^2) \text{ for ISR} \]

Guarantees better coherence for FSR, hopefully also better for ISR.

However, main evolution is matching to matrix elements
Matrix Elements vs. Parton Showers

ME : Matrix Elements
+ systematic expansion in $\alpha_s$ (‘exact’)
+ powerful for multiparton Born level
+ flexible phase space cuts
  - loop calculations very tough
  - negative cross section in collinear regions
    ⇒ unpredictable jet/event structure
  - no easy match to hadronization

PS : Parton Showers
- approximate, to LL (or NLL)
- main topology not predetermined
  ⇒ inefficient for exclusive states
+ process-generic ⇒ simple multiparton
+ Sudakov form factors/resummation
  ⇒ sensible jet/event structure
+ easy to match to hadronization
$p \perp (1 \text{ jet})$

$p_{T,j} (\text{pp}$ → $tt \bar{j})$

$p_{T,j} \geq 50 \text{ GeV} \quad |h_j| < 5, \quad \Delta R_{jj} > 0.4$

$LHC$: $K_{Pythia} = 1.8$

$\frac{d\sigma}{dp_T} [\text{pb}/\text{GeV}]$

$p_{T,j} \geq 100 \text{ GeV} \quad |h_j| < 5, \quad \Delta R_{jj} > 0.4$

$LHC$: $K_{Pythia} = 1.75$

$\frac{d\sigma}{dp_T} [\text{pb}/\text{GeV}]$

$p \perp (2 \text{ jets})$

$p_{T,j} (\text{pp}$ → $t\bar{t}jj)$

$p_{T,j} \geq 50 \text{ GeV} \quad |h_j| < 5, \quad \Delta R_{jj} > 0.4$

$LHC$: $K_{Pythia} = 1.8$

$\frac{d\sigma}{dp_T} [\text{pb}/\text{GeV}]$

$p_{T,j} \geq 100 \text{ GeV} \quad |h_j| < 5, \quad \Delta R_{jj} > 0.4$

$LHC$: $K_{Pythia} = 1.75$

$\frac{d\sigma}{dp_T} [\text{pb}/\text{GeV}]$

$p \perp (2 \text{ jets})$

$p_{T,j} (\text{pp}$ → $\tilde{g} \tilde{g} jj)$

$p_{T,j} \geq 50 \text{ GeV} \quad |h_j| < 5, \quad \Delta R_{jj} > 0.4$

$LHC$: $K_{Pythia} = 1.8$

$\frac{d\sigma}{dp_T} [\text{pb}/\text{GeV}]$

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$\frac{d\sigma}{dp_T} [\text{pb}/\text{GeV}]$

$p \perp (2 \text{ jets})$

$p_{T,j} (\text{pp}$ → $\tilde{u} L \bar{\tilde{u}} \bar{L} jj)$

$p_{T,j} \geq 50 \text{ GeV} \quad |h_j| < 5, \quad \Delta R_{jj} > 0.4$

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$\frac{d\sigma}{dp_T} [\text{pb}/\text{GeV}]$

$\text{power: } Q_{\text{max}}^2 = s; \quad \text{wimpy: } Q_{\text{max}}^2 = m_\perp^2; \quad \text{tune A: } Q_{\text{max}}^2 = 4m_\perp^2$

$m_t = 175 \text{ GeV}, \quad m_{\tilde{g}} = 608 \text{ GeV}, \quad m_{\tilde{u}L} = 567 \text{ GeV}$

(T. Plehn, D. Rainwater, P. Skands)
Matrix Elements and Parton Showers

Recall complementary strengths:
- ME’s good for well separated jets
- PS’s good for structure inside jets

Marriage desirable! But how?
Problems:
- gaps in coverage?
- doublecounting of radiation?
- Sudakov?
- NLO consistency?

Much work ongoing \(\Rightarrow\) no established orthodoxy

Three main areas, in ascending order of complication:
1) Match to lowest-order nontrivial process — merging
2) Combine leading-order multiparton process — vetoed parton showers
3) Match to next-to-leading order process — MC@NLO
Merging

= cover full phase space with smooth transition ME/PS

Want to reproduce

\[ W_{\text{ME}} = \frac{1}{\sigma(\text{LO})} \frac{d\sigma(\text{LO} + g)}{d(\text{phasespace})} \]

by shower generation + correction procedure

\[ \frac{\text{wanted}}{\hat{W}^{\text{ME}}} = \frac{\text{generated}}{\hat{W}^{\text{PS}}} \frac{\text{correction}}{\hat{W}^{\text{ME}}/W^{\text{PS}}} \]

- Exponentiate ME correction by shower Sudakov form factor:

\[ W^{\text{PS}}_{\text{actual}}(Q^2) = W^{\text{ME}}(Q^2) \exp \left( - \int_{Q^2}^{Q^2_{\text{max}}} W^{\text{ME}}(Q'^2) \, dQ'^2 \right) \]

- Do not normalize \( W^{\text{ME}} \) to \( \sigma(\text{NLO}) \) (error \( \mathcal{O}(\alpha_s^2) \) either way)

\[ 1 + \mathcal{O}(\alpha_s) \quad \int = 1 \]

\[ d\sigma = K \, \sigma_0 \, dW^{\text{PS}} \]

- Normally several shower histories \( \Rightarrow \) \( \sim \) equivalent approaches
Final-State Shower Merging

Merging with $\gamma^*/Z^0 \to q\bar{q}g$ for $m_q = 0$ since long

For $m_q > 0$ pick $Q^2_i = m^2_i - m^2_{i,\text{onshell}}$ as evolution variable since

$$W^{\text{ME}} = \frac{(\ldots)}{Q^2_1 Q^2_2} - \frac{(\ldots)}{Q^4_1} - \frac{(\ldots)}{Q^4_2}$$

Coloured decaying particle also radiates:

\[0 (t) \quad 1 (b) \quad 2 (W^+) \quad 3 (g)\]

\[\Rightarrow \text{can merge PS with generic } a \to b c g \text{ ME}\]

(E. Norrbin & TS, NPB603 (2001) 297)

Subsequent branchings $q \to qg$: also matched to ME, with reduced energy of system
PYTHIA performs merging with generic FSR $a \to bcg$ ME,
in SM: $\gamma^*/Z^0/W^\pm \to q\bar{q}$, $t \to bW^+$, $H^0 \to q\bar{q}$,
and MSSM: $t \to bH^+$, $Z^0 \to q\bar{q}$, $\tilde{q} \to q'W^+$, $H^0 \to q\bar{q}$, $\tilde{q} \to q'H^+$,
$\chi \to q\bar{q}$, $\chi \to q\bar{q}$, $\tilde{q} \to q\chi$, $t \to \tilde{t}\chi$, $\tilde{g} \to q\bar{q}$, $\tilde{q} \to q\tilde{g}$, $t \to \tilde{t}\tilde{g}$
g emission for different
colour, spin and parity:

$R^b_l(y_c)$: mass effects
in Higgs decay:
Initial-State Shower Merging

resummation:
physical $p_{\perp Z}$ spectrum

shower: ditto
+ accompanying jets (exclusive)

Merged with matrix elements for
$q\bar{q} \to (\gamma^*/Z^0/W^\pm)g$ and $qg \to (\gamma^*/Z^0/W^\pm)q'$:

$\left( \frac{W^{\text{ME}}}{W^{\text{PS}}} \right)_{q\bar{q}' \to gW} = \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{s}^2 + m_W^4} \leq 1$

with $Q^2 = -m^2$
and $z = m_W^2/\hat{s}$

$\left( \frac{W^{\text{ME}}}{W^{\text{PS}}} \right)_{qg \to q'W} = \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{(\hat{s} - m_W^2)^2 + m_W^4} < 3$
HERWIG also contains merging, for
• $Z^0 \rightarrow q\bar{q}$
• $t \rightarrow bW^+$
• $q\bar{q} \rightarrow Z^0$
and some more

Special problem:
angular ordering does not cover full phase space; so
(1) fill in “dead zone” with ME
(2) apply ME correction in allowed region

Important for agreement with data:
Vetoed Parton Showers


Generic method to combine ME’s of several different orders to NLL accuracy; will be a ‘standard tool’ in the future

Basic idea:
• consider (differential) cross sections $\sigma_0, \sigma_1, \sigma_2, \sigma_3, \ldots$, corresponding to a lowest-order process (e.g. W or H production), with more jets added to describe more complicated topologies, in each case to the respective leading order
• $\sigma_i, i \geq 1$, are divergent in soft/collinear limits
• absent virtual corrections would have ensured “detailed balance”, i.e. an emission that adds to $\sigma_{i+1}$ subtracts from $\sigma_i$
• such virtual corrections correspond (approximately) to the Sudakov form factors of parton showers
• so use shower routines to provide missing virtual corrections $\Rightarrow$ rejection of events (especially) in soft/collinear regions
Veto scheme:
1) Pick hard process, mixing according to $\sigma_0 : \sigma_1 : \sigma_2 : \ldots$, above some ME cutoff (e.g. all $p_{\perp i} > p_{\perp 0}$, all $R_{ij} > R_0$), with large fixed $\alpha_s s_0$

2) Reconstruct imagined shower history (in different ways)
3) Weight $W_\alpha = \prod_{\text{branchings}} (\alpha_s(k_{\perp i}^2)/\alpha_s s_0) \Rightarrow \text{accept/reject}$

CKKW-L:
4) Sudakov factor for non-emission on all lines above ME cutoff

$$W_{\text{Sud}} = \prod \text{“propagators” } \text{Sudakov}(k_{\perp \text{beg}}^2, k_{\perp \text{end}}^2)$$

4a) CKKW : use NLL Sudakovs
4b) L: use trial showers
5) $W_{\text{Sud}} \Rightarrow \text{accept/reject}$
6) do shower, vetoing emissions above cutoff

MLM:
4) do parton showers
5) (cone-)cluster showered event
6) match partons and jets
7) if all partons are matched, and $n_{\text{jet}} = n_{\text{parton}}$, keep the event, else discard it
CKKW mix of $W + (0, 1, 2, 3, 4)$ partons, hadronized and clustered to jets:

(S. Mrenna, P. Richardson)
Spread of $W + \text{jets}$ rate for different matching schemes + showers, top: Tevatron, bottom: LHC.

**ALPGEN**: MLM + HERWIG

**ARIADNE**: CKKW-L + ARIADNE

**HELAC**: MLM + PYTHIA

**MADEVENT**: MLM/CKKW + PYTHIA

**SHERPA**: CKKW + SHERPA

model variation: $\alpha_S$, cuts, ...

arXiv0706.2569 (Alwall et al.)
Objectives:
• Total rate should be accurate to NLO.
• NLO results are obtained for all observables when (formally) expanded in powers of $\alpha_s$.
• Hard emissions are treated as in the NLO computations.
• Soft/collinear emissions are treated as in shower MC.
• The matching between hard and soft emissions is smooth.
• The outcome is a set of “normal” events, that can be processed further.

Basic scheme (simplified!):
1) Calculate the NLO matrix element corrections to an $n$-body process (using the subtraction approach).
2) Calculate analytically (no Sudakov!) how the first shower emission off an $n$-body topology populates $(n + 1)$-body phase space.
3) Subtract the shower expression from the $(n + 1)$ ME to get the “true” $(n + 1)$ events, and consider the rest of $\sigma_{\text{NLO}}$ as $n$-body.
4) Add showers to both kinds of events.
Disadvantage: not perfect match everywhere, so can lead to events with negative weight, \( \sim 10\% \) when normalized to \( \pm 1 \).

\textbf{MC@NLO in comparison:}

- Superior with respect to “total” cross sections.
- Equivalent to merging for event shapes (differences higher order).
- Inferior to CKKW–L for multijet topologies.

\( \Rightarrow \) pick according to current task and availability.
Later additions: single top, $H^0 W^\pm$, $H^0 Z^0$

<table>
<thead>
<tr>
<th>IPR0C</th>
<th>Process</th>
</tr>
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<tbody>
<tr>
<td>-1350–IL</td>
<td>$H_1 H_2 \rightarrow (Z/\gamma^* \rightarrow) l_{IL} l_{IL} + X$</td>
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<tr>
<td>-1360–IL</td>
<td>$H_1 H_2 \rightarrow (Z \rightarrow) l_{IL} l_{IL} + X$</td>
</tr>
<tr>
<td>-1370–IL</td>
<td>$H_1 H_2 \rightarrow (\gamma^* \rightarrow) l_{IL} l_{IL} + X$</td>
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<tr>
<td>-1460–IL</td>
<td>$H_1 H_2 \rightarrow (W^+ \rightarrow) l_{IL}^+ \bar{\nu}_{IL} + X$</td>
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<tr>
<td>-1470–IL</td>
<td>$H_1 H_2 \rightarrow (W^- \rightarrow) l_{IL}^- \bar{\nu}_{IL} + X$</td>
</tr>
<tr>
<td>-1396</td>
<td>$H_1 H_2 \rightarrow \gamma^* (\rightarrow \sum_i f_i \bar{f}_i) + X$</td>
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<td>$H_1 H_2 \rightarrow Z^0 + X$</td>
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<td>-2880</td>
<td>$H_1 H_2 \rightarrow W^-Z^0 + X$</td>
</tr>
</tbody>
</table>

(Frixione, Webber)

- Works identically to HERWIG: the very same analysis routines can be used
- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- Features a self contained library of PDFs with old and new sets alike
- LHAPDF will also be implemented
These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly.

Solid: MC@NLO
Dashed: HERWIG $\times \frac{\sigma_{NLO}}{\sigma_{LO}}$
Dotted: NLO
POWHEG

Nason; Frixione, Oleari, Ridolfi (e.g. JHEP 0711 (2007) 070)
Alternative to MC@NLO:

\[ d\sigma = \bar{B}(v) d\Phi_v \left[ \frac{R(v, r)}{B(v)} \exp \left( -\int_{p_{\perp}} \frac{R(v, r')}{B(v)} d\Phi'_r \right) d\Phi_r \right] \]

where

\[ \bar{B}(v) = B(v) + V(v) + \int d\Phi_r [R(v, r) - C(v, r)] . \]

and

\( v, d\Phi_v \) Born-level \( n \)-body variables and differential phase space
\( r, d\Phi_r \) extra \( n + 1 \)-body variables and differential phase space
\( B(v) \) Born-level cross section
\( V(v) \) Virtual corrections
\( R(v, r) \) Real-emission cross section
\( C(v, r) \) Counterterms for collinear factorization of parton densities.

Basic idea:
• Pick the real emission with largest \( p_{\perp} \) according to complete ME’s, with NLO normalization.
• Let showers do subsequent evolution downwards from this \( p_{\perp} \) scale.
Relative to MC@NLO:
+ no negative weights (except in regions with extreme virtual corrections)
+ clean separation to shower stage
± optimal for $p_{\perp}$-ordered showers, messy for others
± different higher-order terms
– as of yet fewer processes than MC@NLO

$p_{\perp}$ spectrum of individual t quark and of $t\bar{t}$ pair: