LUND UNIVERSITY

65th Scottish Universities Summer School
in Physics: LHC Physics
St Andrews, Scotland
16 - 29 August 2009

Monte Carlo Tools

Torbjörn Sjöstrand

Lund University

2. (yesterday) Introduction and Overview; Parton Showers

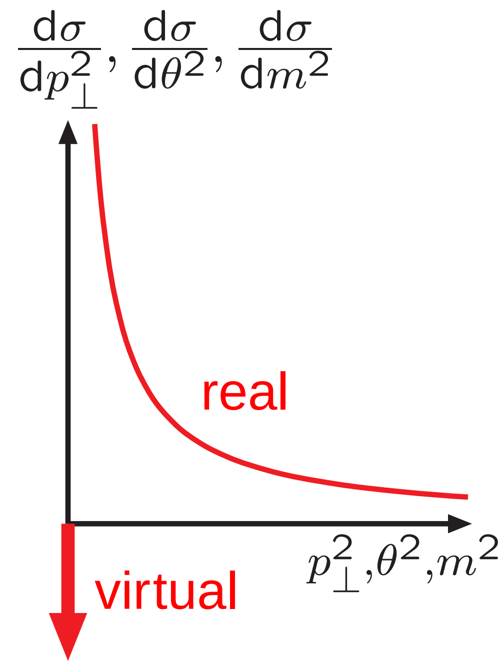
2. (today) Matching Issues; Multiple Parton Interactions

3. (tomorrow) Hadronization; LHC predictions; Generator News

Matrix Elements vs. Parton Showers

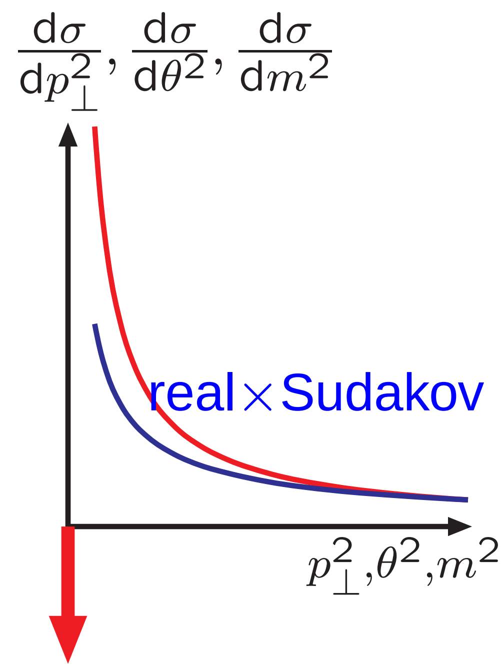
ME : Matrix Elements

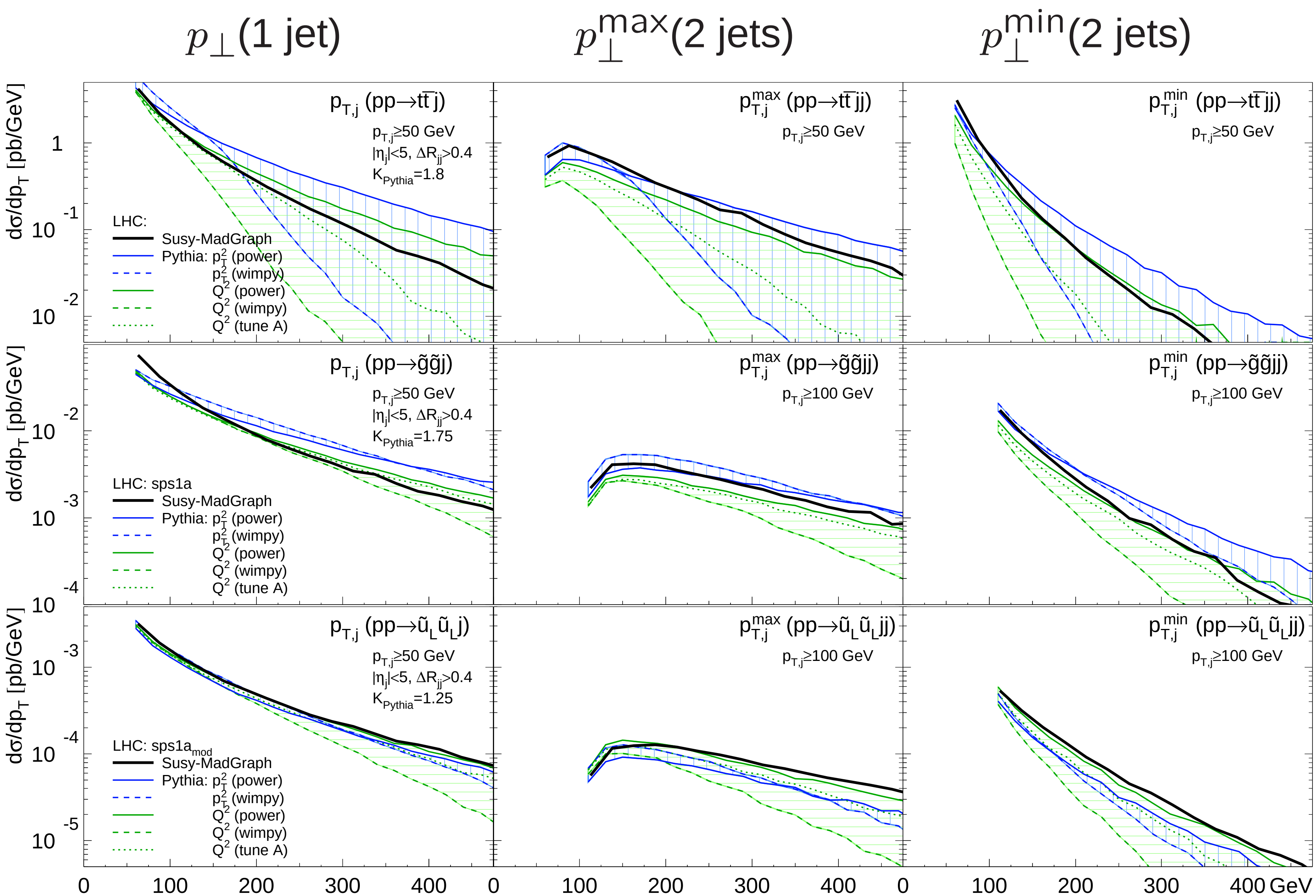
+ systematic expansion in α_s (‘*exact*’)
+ powerful for multiparton Born level
+ flexible phase space cuts
– loop calculations very tough
– negative cross section in collinear regions
  ⇒ unpredictive jet/event structure
– *no easy match to hadronization*



PS : Parton Showers

– approximate, to LL (or NLL)
– main topology not predetermined
  ⇒ inefficient for exclusive states
+ process-generic ⇒ simple multiparton
+ Sudakov form factors/resummation
  ⇒ sensible jet/event structure
+ *easy to match to hadronization*





power: $Q^2_{\rm max} = s$; wimpy: $Q^2_{\rm max} = m_\perp^2$; tune A: $Q^2_{\rm max} = 4m_\perp^2$

$m_{\rm t} = 175$ GeV, $m_{\tilde{\rm g}} = 608$ GeV, $m_{\tilde{\rm u}_L} = 567$ GeV

(T. Plehn, D. Rainwater, P. Skands)

Matrix Elements and Parton Showers

Recall complementary strengths:

- ME's good for well separated jets
- PS's good for structure inside jets

Marriage desirable! But how?

Problems:
- gaps in coverage?
- doublecounting of radiation?
- Sudakov?
- NLO consistency?

Much work ongoing $\Longrightarrow$ no established orthodoxy

Three main areas, in ascending order of complication:

1) Match to lowest-order nontrivial process — merging

2) Combine leading-order multiparton process — vetoed parton showers

3) Match to next-to-leading order process — MC@NLO, POWHEG

Merging

= cover full phase space with smooth transition ME/PS

Want to reproduce $W^{\mathrm{ME}} = \frac{1}{\sigma(\mathrm{LO})} \frac{\mathrm{d}\sigma(\mathrm{LO}+\mathrm{g})}{\mathrm{d(phasespace)}}$

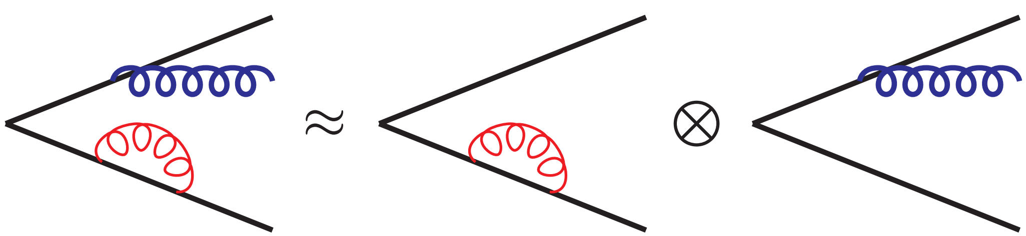
by shower generation + correction procedure

$$\underbrace{W^{\mathrm{ME}}}_{\text{wanted}} = \underbrace{W^{\mathrm{PS}}}_{\text{generated}} \; \overbrace{\frac{W^{\mathrm{ME}}}{W^{\mathrm{PS}}}}^{\text{correction}}$$

- Exponentiate ME correction by shower Sudakov form factor:

$$W^{\mathrm{PS}}_{\mathrm{actual}}(Q^2) = W^{\mathrm{ME}}(Q^2) \exp\left(-\int_{Q^2}^{Q^2_{\mathrm{max}}} W^{\mathrm{ME}}(Q'^2)\, \mathrm{d}Q'^2\right)$$

- Do not normalize W^{ME} to $\sigma(\mathrm{NLO})$ (error $\mathcal{O}(\alpha_s^2)$ either way)



$1 + \mathcal{O}(\alpha_s)$ → K, $\int = 1$ → $\mathrm{d}W^{\mathrm{PS}}$

$$\mathrm{d}\sigma = K \;\sigma_0 \;\mathrm{d}W^{\mathrm{PS}}$$

- Normally several shower histories ⇒ ∼equivalent approaches

Final-State Shower Merging

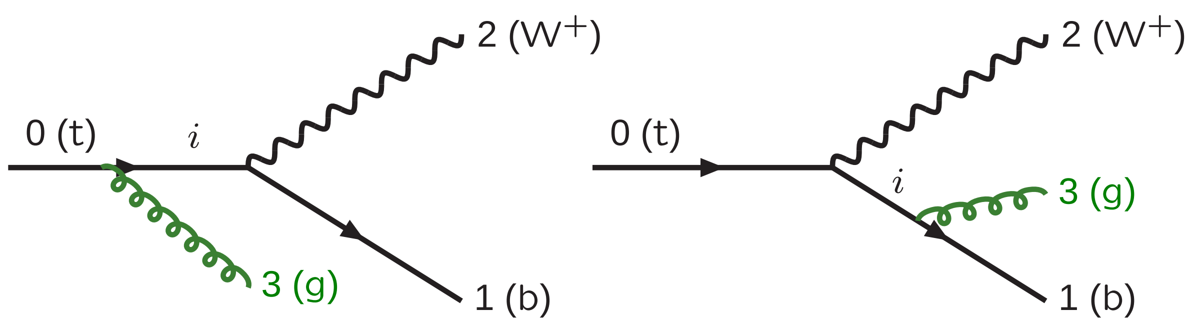
Merging with $\gamma^*/\mathrm{Z}^0 \to \mathrm{q}\overline{\mathrm{q}}\mathrm{g}$ for $m_\mathrm{q} = 0$ since long

(M. Bengtsson & TS, PLB185 (1987) 435, NPB289 (1987) 810)

For $m_\mathrm{q} > 0$ pick $Q_i^2 = m_i^2 - m_{i,\mathrm{onshell}}^2$ as evolution variable since

$$W^{\mathrm{ME}} = \frac{(\ldots)}{Q_1^2 Q_2^2} - \frac{(\ldots)}{Q_1^4} - \frac{(\ldots)}{Q_2^4}$$

Coloured decaying particle also radiates:



ME $\frac{1}{Q_0^2 Q_1^2}$ matches PS $\mathrm{b} \to \mathrm{bg}$

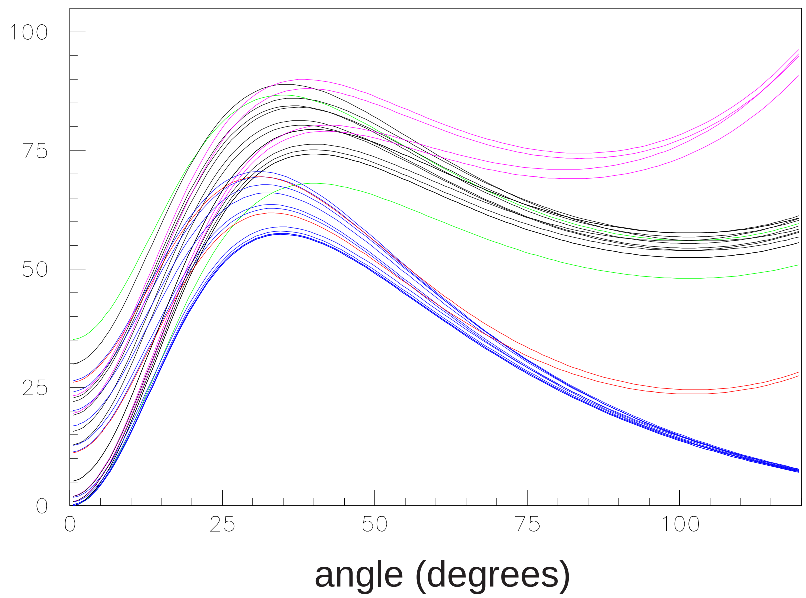
$\Rightarrow$ can merge PS with generic $a \to bc\mathrm{g}$ ME

(E. Norrbin & TS, NPB603 (2001) 297)

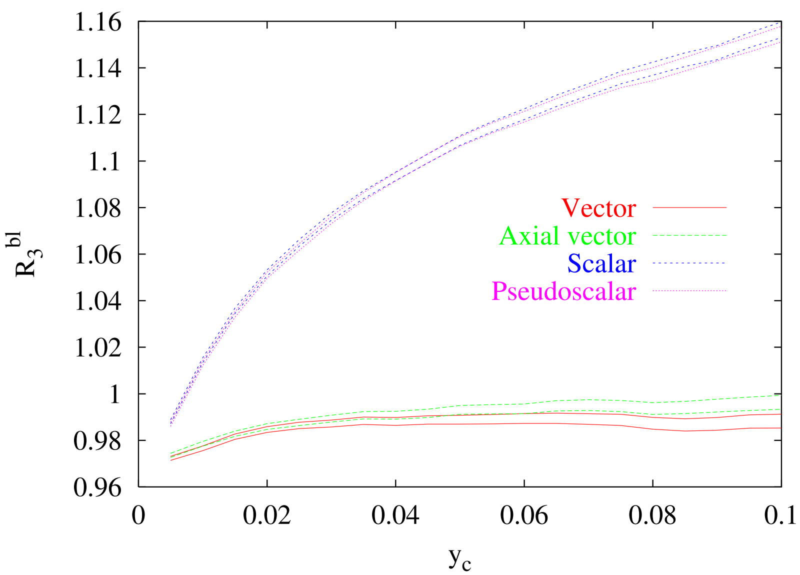
Subsequent branchings $\mathrm{q} \to \mathrm{qg}$: also matched to ME, with reduced energy of system

PYTHIA performs merging with generic FSR $a \to bc$g ME,
in SM: $\gamma^*/\mathrm{Z}^0/\mathrm{W}^\pm \to \mathrm{q\bar{q}}$, $\mathrm{t} \to \mathrm{bW}^+$, $\mathrm{H}^0 \to \mathrm{q\bar{q}}$,
and MSSM: $\mathrm{t} \to \mathrm{bH}^+$, $\mathrm{Z}^0 \to \tilde{\mathrm{q}}\bar{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \tilde{\mathrm{q}}'\mathrm{W}^+$, $\mathrm{H}^0 \to \tilde{\mathrm{q}}\bar{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \tilde{\mathrm{q}}'\mathrm{H}^+$,
$\chi \to \mathrm{q}\bar{\tilde{\mathrm{q}}}$, $\chi \to \mathrm{q}\bar{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \mathrm{q}\chi$, $\mathrm{t} \to \tilde{\mathrm{t}}\chi$, $\tilde{\mathrm{g}} \to \mathrm{q}\bar{\tilde{\mathrm{q}}}$, $\tilde{\mathrm{q}} \to \mathrm{q}\tilde{\mathrm{g}}$, $\mathrm{t} \to \tilde{\mathrm{t}}\tilde{\mathrm{g}}$

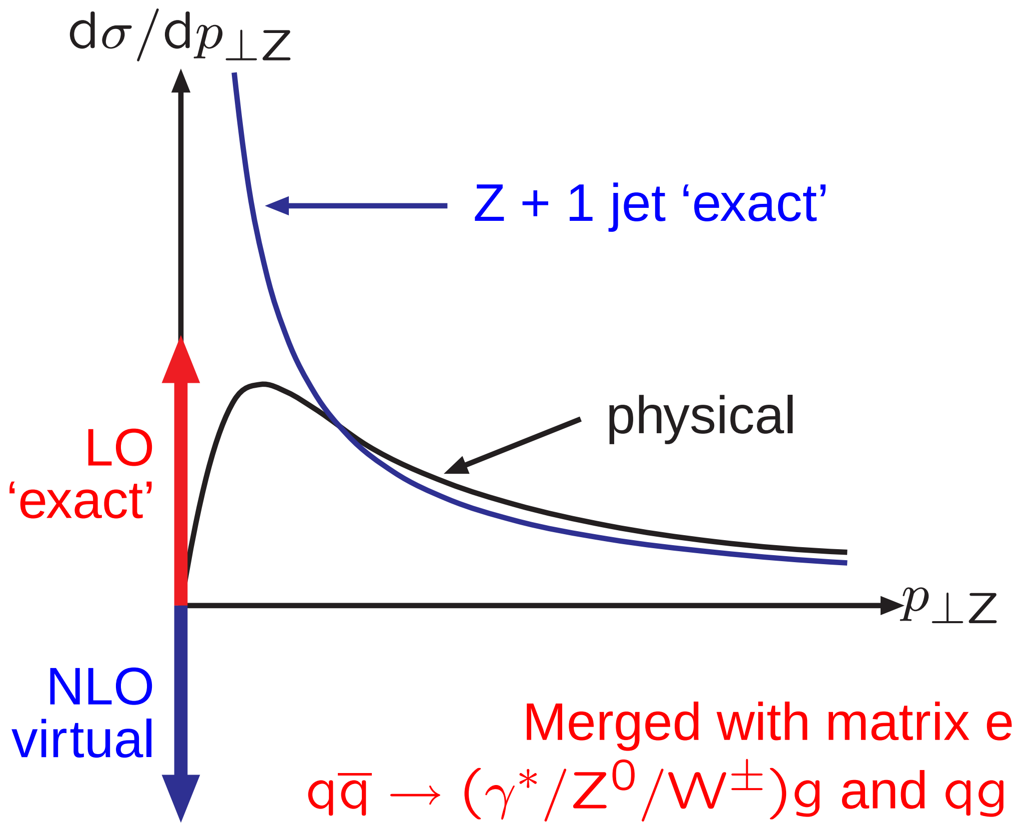
g emission for different colour, spin and parity:



$R_3^{\mathrm{bl}}(y_c)$: mass effects in Higgs decay:



Initial-State Shower Merging



resummation:
physical $p_{\perp Z}$ spectrum

shower: ditto
+ accompanying
jets (exclusive)

Merged with matrix elements for
$q\overline{q} \to (\gamma^*/Z^0/W^\pm)g$ and $qg \to (\gamma^*/Z^0/W^\pm)q'$:

(G. Miu & TS, PLB449 (1999) 313)

$$\left(\frac{W^{\mathrm{ME}}}{W^{\mathrm{PS}}}\right)_{q\overline{q}'\to gW} = \frac{\hat{t}^2 + \hat{u}^2 + 2m_W^2\hat{s}}{\hat{s}^2 + m_W^4} \leq 1$$

$$\left(\frac{W^{\mathrm{ME}}}{W^{\mathrm{PS}}}\right)_{qg\to q'W} = \frac{\hat{s}^2 + \hat{u}^2 + 2m_W^2\hat{t}}{(\hat{s} - m_W^2)^2 + m_W^4} < 3$$

with $Q^2 = -m^2$
and $z = m_W^2/\hat{s}$

Merging in HERWIG

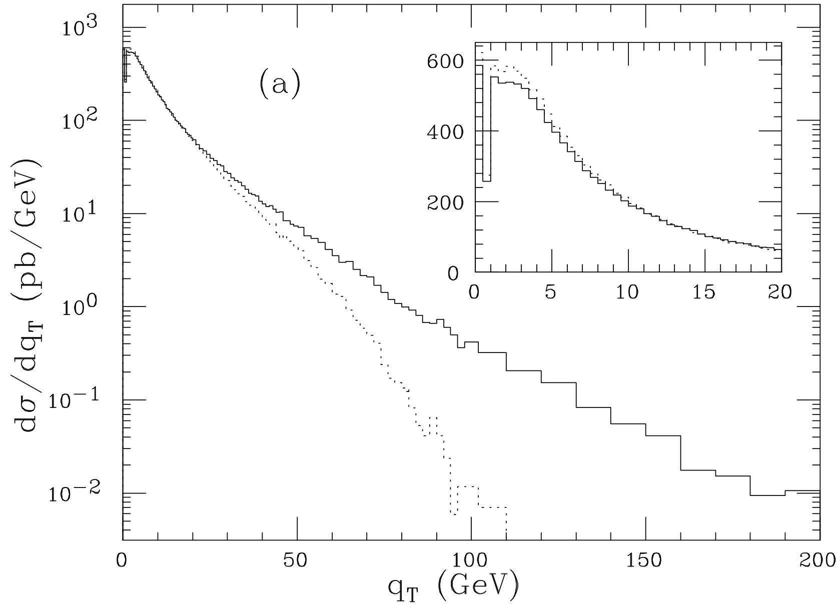
HERWIG also contains merging, for

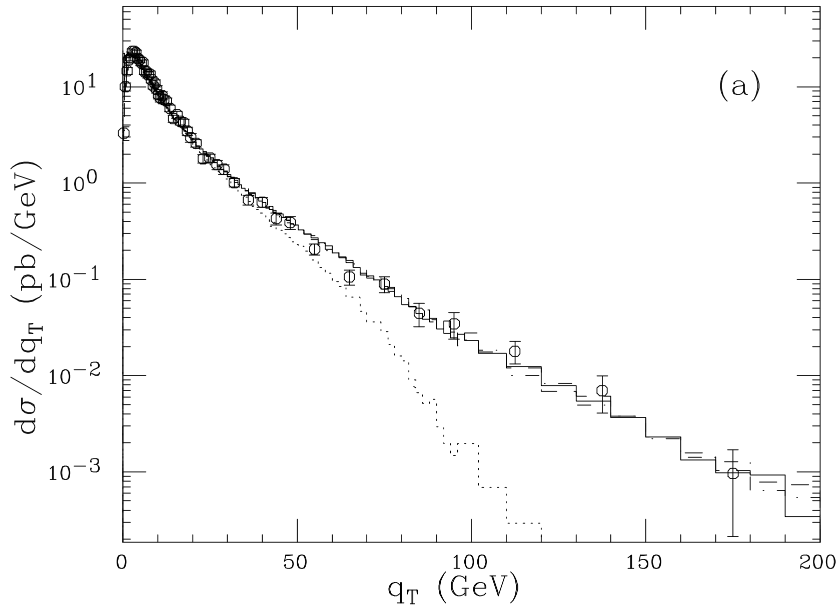
- $Z^0 \to q\bar{q}$
- $t \to bW^+$
- $q\bar{q} \to Z^0$

and some more

Special problem:
angular ordering does not cover full phase space; so
(1) fill in “dead zone” with ME
(2) apply ME correction
in allowed region

Important for agreement with data:





Vetoed Parton Showers

S. Catani, F. Krauss, R. Kuhn, B.R. Webber, JHEP 0111 (2001) 063; L. Lönnblad, JHEP0205 (2002) 046;
F. Krauss, JHEP 0208 (2002) 015; S. Mrenna, P. Richardson, JHEP0405 (2004) 040;
S. Höche et al., hep-ph/0602031

Generic method to combine ME's of several different orders to NLL accuracy; will be a 'standard tool' in the future

Basic idea:

- consider (differential) cross sections $\sigma_0, \sigma_1, \sigma_2, \sigma_3, \ldots$, corresponding to a lowest-order process (e.g. W or H production), with more jets added to describe more complicated topologies, in each case to the respective leading order
- σ_i, $i \geq 1$, are divergent in soft/collinear limits
- absent virtual corrections would have ensured "detailed balance", i.e. an emission that adds to σ_{i+1} subtracts from σ_i
- such virtual corrections correspond (approximately) to the Sudakov form factors of parton showers
- so use shower routines to provide missing virtual corrections $\Rightarrow$ rejection of events (especially) in soft/collinear regions

Veto scheme:

1) Pick hard process, mixing according to $\sigma_0 : \sigma_1 : \sigma_2 : \ldots$,
above some ME cutoff (e.g. all $p_{\perp i} > p_{\perp 0}$, all $R_{ij} > R_0$),
with large fixed α_{s0}

2) Reconstruct imagined shower history (in different ways)

3) Weight $W_\alpha = \prod_{\text{branchings}} (\alpha_s(k_{\perp i}^2)/\alpha_{s0}) \Rightarrow$ accept/reject

CKKW-L:

4) Sudakov factor for non-emission
on all lines above ME cutoff
$W_{\text{Sud}} = \prod_{\text{"propagators"}}$
$\text{Sudakov}(k_{\perp \text{beg}}^2, k_{\perp \text{end}}^2)$

4a) CKKW : use NLL Sudakovs

4b) L: use trial showers

5) $W_{\text{Sud}} \Rightarrow$ accept/reject

6) do shower,
vetoing emissions above cutoff

MLM:

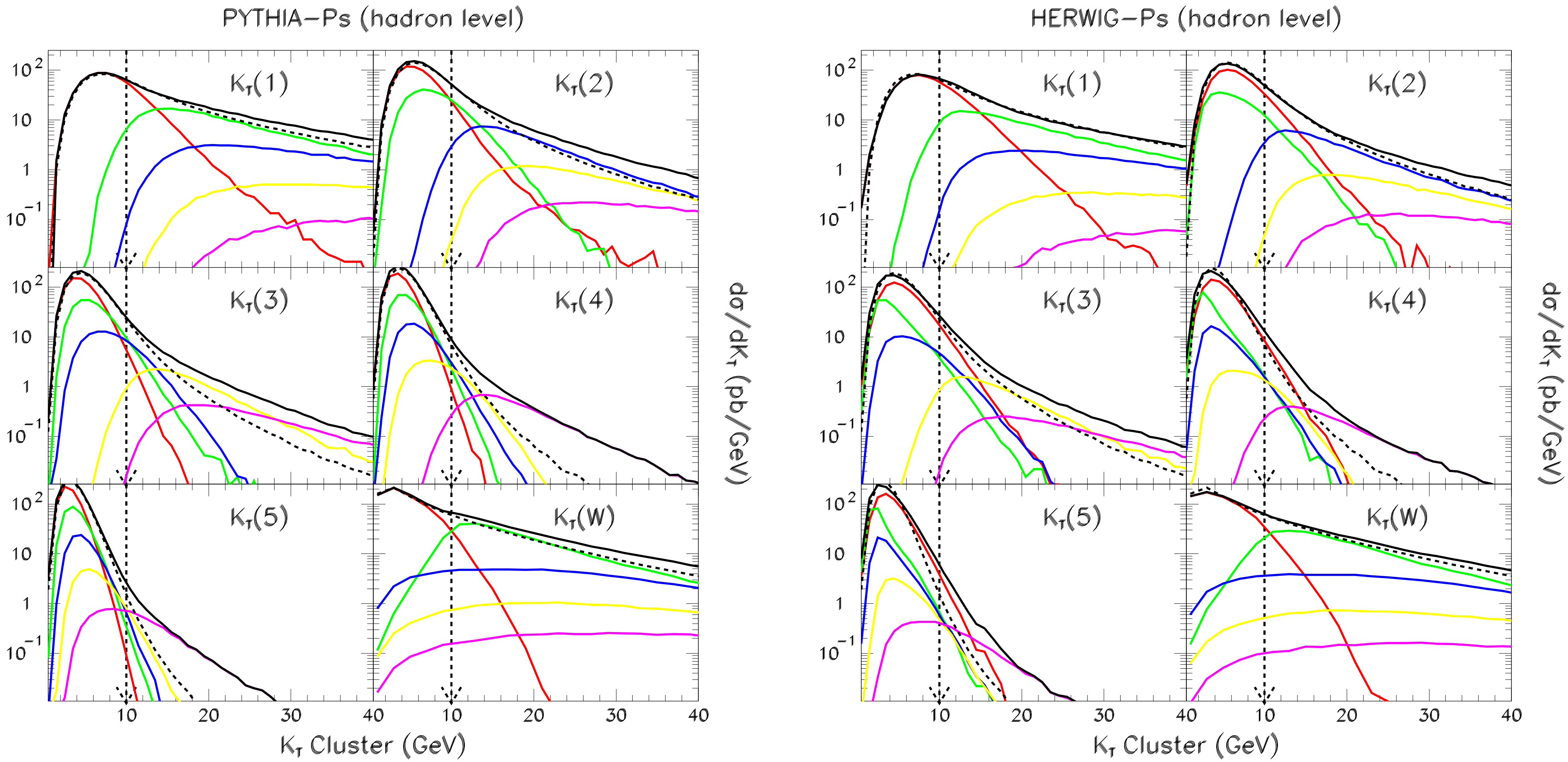
4) do parton showers

5) (cone-)cluster
showered event

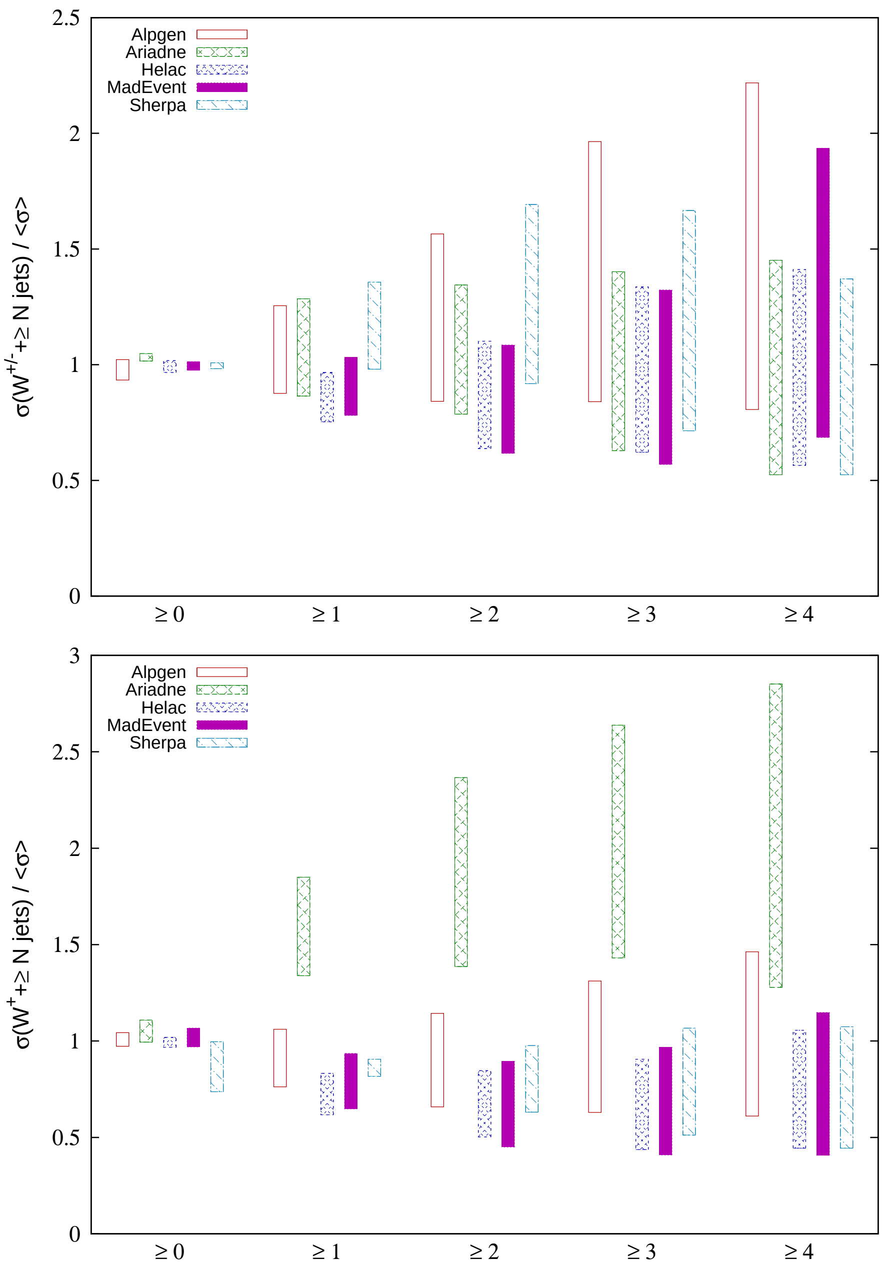
6) match partons and jets

7) if all partons are matched,
and $n_{\text{jet}} = n_{\text{parton}}$,
keep the event,
else discard it

CKKW mix of $W + (0, 1, 2, 3, 4)$ partons, hadronized and clustered to jets:



(S.Mrenna, P. Richardson)



Spread of W + jets rate for different matching schemes + showers,

top: Tevatron,

bottom: LHC.

ALPGEN: MLM + HERWIG

ARIADNE: CKKW-L + ARIADNE

HELAC: MLM + PYTHIA

MADEVENT: MLM/CKKW + PYTHIA

SHERPA: CKKW + SHERPA

model varation: α_S, cuts, . . .

arXiv0706.2569 (Alwall et al.)

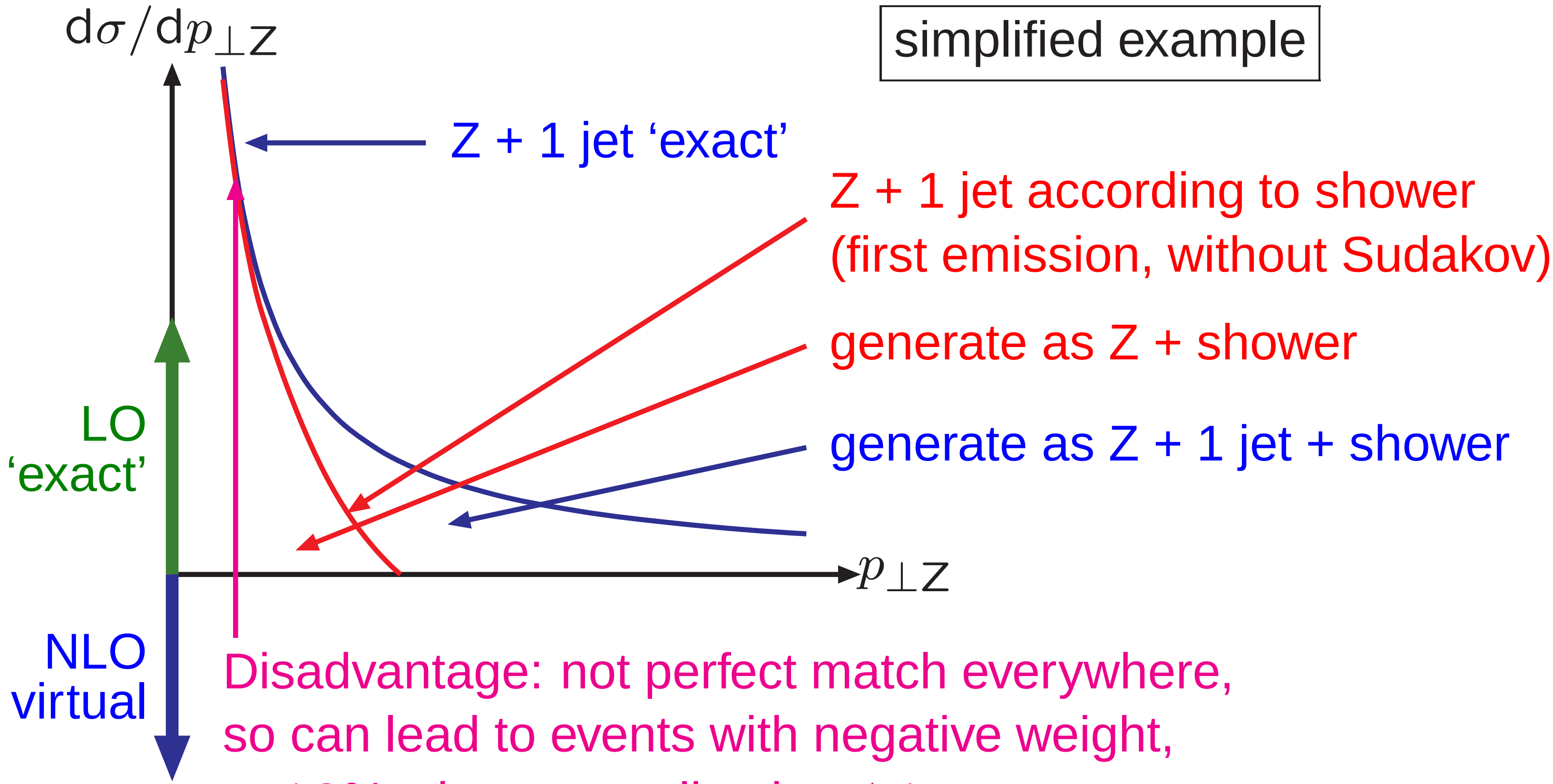
MC@NLO

Objectives:

- Total rate should be accurate to NLO.
- NLO results are obtained for all observables when (formally) expanded in powers of α_S.
- Hard emissions are treated as in the NLO computations.
- Soft/collinear emissions are treated as in shower MC.
- The matching between hard and soft emissions is smooth.
- The outcome is a set of “normal” events, that can be processed further.

Basic scheme (simplified!):

1) Calculate the NLO matrix element corrections to an n-body process (using the subtraction approach).
2) Calculate analytically (no Sudakov!) how the first shower emission off an n-body topology populates $(n+1)$-body phase space.
3) Subtract the shower expression from the $(n+1)$ ME to get the “true” $(n+1)$ events, and consider the rest of $\sigma_{\rm NLO}$ as n-body.
4) Add showers to both kinds of events.



Disadvantage: not perfect match everywhere,
so can lead to events with negative weight,
$\sim$ 10% when normalized to ± 1.

MC@NLO in comparison:

- Superior with respect to “total” cross sections.
- Equivalent to merging for event shapes (differences higher order).
- Inferior to CKKW–L for multijet topologies.

$\Rightarrow$ pick according to current task and availability.

MC@NLO 2.31 [hep-ph/0402116]

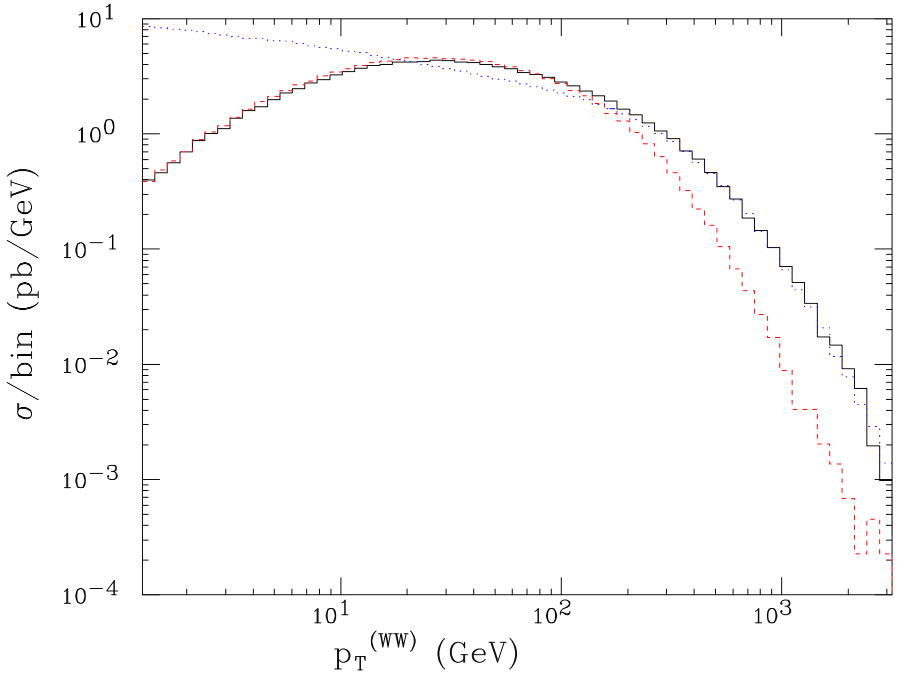
IPROC	Process
–1350–IL	$H_1H_2 \to (Z/\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1360–IL	$H_1H_2 \to (Z \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1370–IL	$H_1H_2 \to (\gamma^* \to) l_{\rm IL}\bar{l}_{\rm IL} + X$
–1460–IL	$H_1H_2 \to (W^+ \to) l^+_{\rm IL}\nu_{\rm IL} + X$
–1470–IL	$H_1H_2 \to (W^- \to) l^-_{\rm IL}\bar{\nu}_{\rm IL} + X$
–1396	$H_1H_2 \to \gamma^*(\to \sum_i f_i\bar{f}_i) + X$
–1397	$H_1H_2 \to Z^0 + X$
–1497	$H_1H_2 \to W^+ + X$
–1498	$H_1H_2 \to W^- + X$
–1600–ID	$H_1H_2 \to H^0 + X$
–1705	$H_1H_2 \to b\bar{b} + X$
–1706	$H_1H_2 \to t\bar{t} + X$
–2850	$H_1H_2 \to W^+W^- + X$
–2860	$H_1H_2 \to Z^0Z^0 + X$
–2870	$H_1H_2 \to W^+Z^0 + X$
–2880	$H_1H_2 \to W^-Z^0 + X$

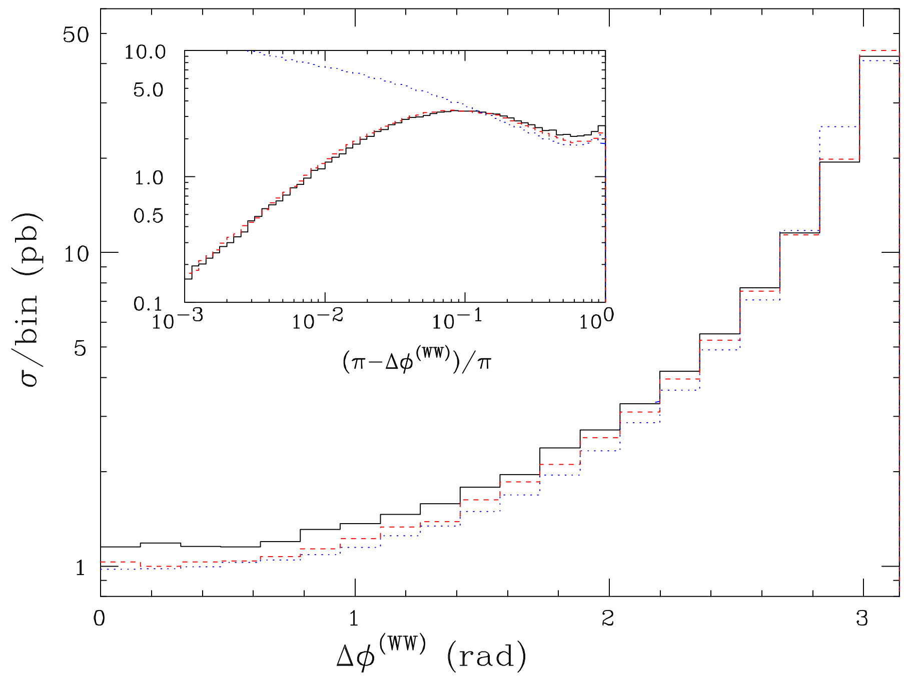
(Frixione, Webber)

- Works identically to HERWIG: the very same analysis routines can be used
- Reads shower initial conditions from an event file (as in ME corrections)
- Exploits Les Houches accord for process information and common blocks
- Features a self contained library of PDFs with old and new sets alike
- LHAPDF will also be implemented

Later additions: single top, $H^0W^\pm$, H^0Z^0, tW, ...

W^+W^- Observables





These correlations are problematic: the soft and hard emissions are both relevant. MC@NLO does well, resumming large logarithms, and yet handling the large-scale physics correctly

Solid: MC@NLO

Dashed: HERWIG$\times\frac{\sigma_{NLO}}{\sigma_{LO}}$

Dotted: NLO

POWHEG

Nason; Frixione, Oleari, Ridolfi (e.g. JHEP **0711** (2007) 070)
Better (?) alternative to MC@NLO:

$$\mathrm{d}\sigma = \bar{B}(v)\mathrm{d}\Phi_v \left[\frac{R(v,r)}{B(v)} \exp\left(-\int_{p_\perp} \frac{R(v,r')}{B(v)} \mathrm{d}\Phi'_r \right) \mathrm{d}\Phi_r \right]$$

where

$$\bar{B}(v) = B(v) + V(v) + \int \mathrm{d}\Phi_r [R(v,r) - C(v,r)] \, .$$

and
$v, \mathrm{d}\Phi_v$ Born-level n-body variables and differential phase space
$r, \mathrm{d}\Phi_r$ extra $n+1$-body variables and differential phase space
$B(v)$ Born-level cross section
$V(v)$ Virtual corrections
$R(v,r)$ Real-emission cross section
$C(v,r)$ Conterterms for collinear factorization of parton densities.

Basic idea:

- Pick the real emission with largest $p_\perp$ according to complete ME's, with NLO normalization.
- Let showers do subsequent evolution downwards from this $p_\perp$ scale.

Relative to MC@NLO:

$+$ no negative weights (except in regions with extreme virtual corrections)

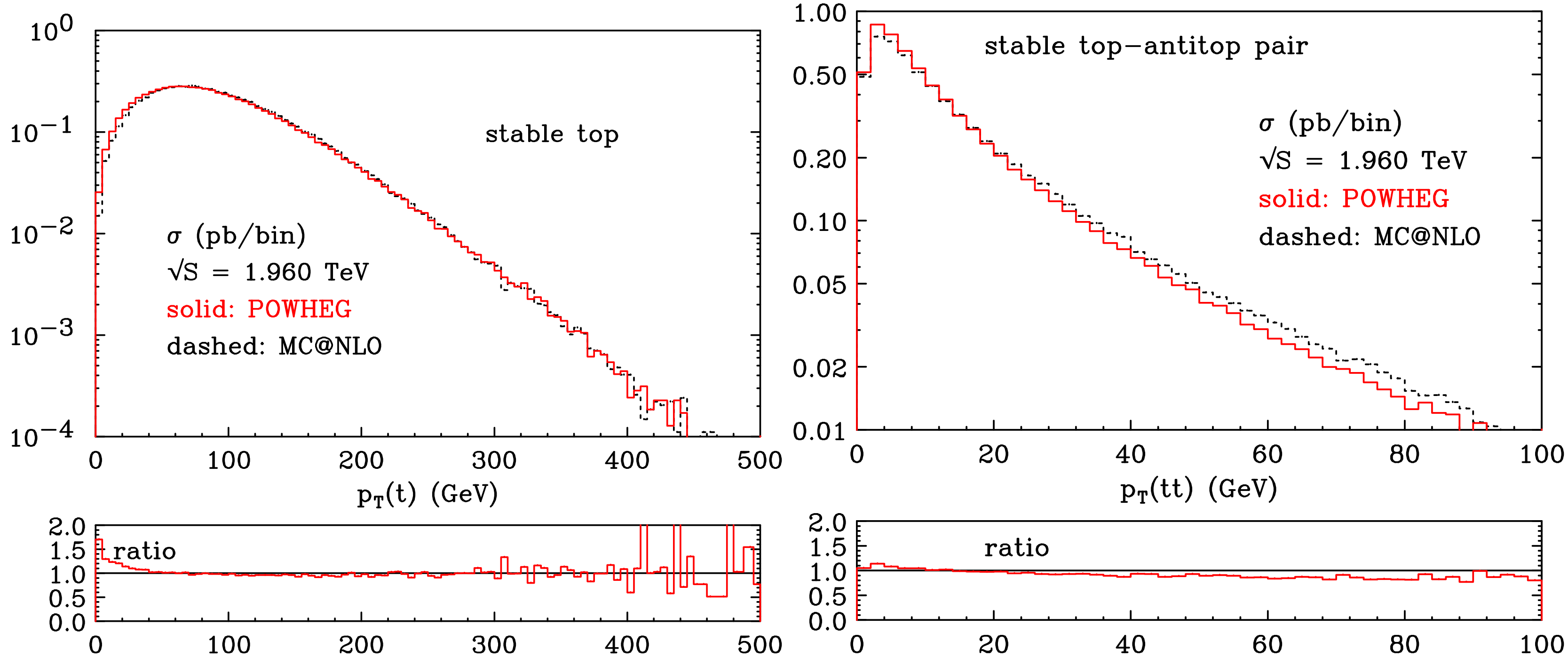
$+$ clean separation to shower stage

$\pm$ optimal for $p_\perp$-ordered showers, messy but manageable for others

$\pm$ different higher-order terms

$-$ as of yet fewer processes than MC@NLO

$p_\perp$ spectrum of individual $\mathtt{t}$ quark and of $\mathtt{t\bar{t}}$ pair:

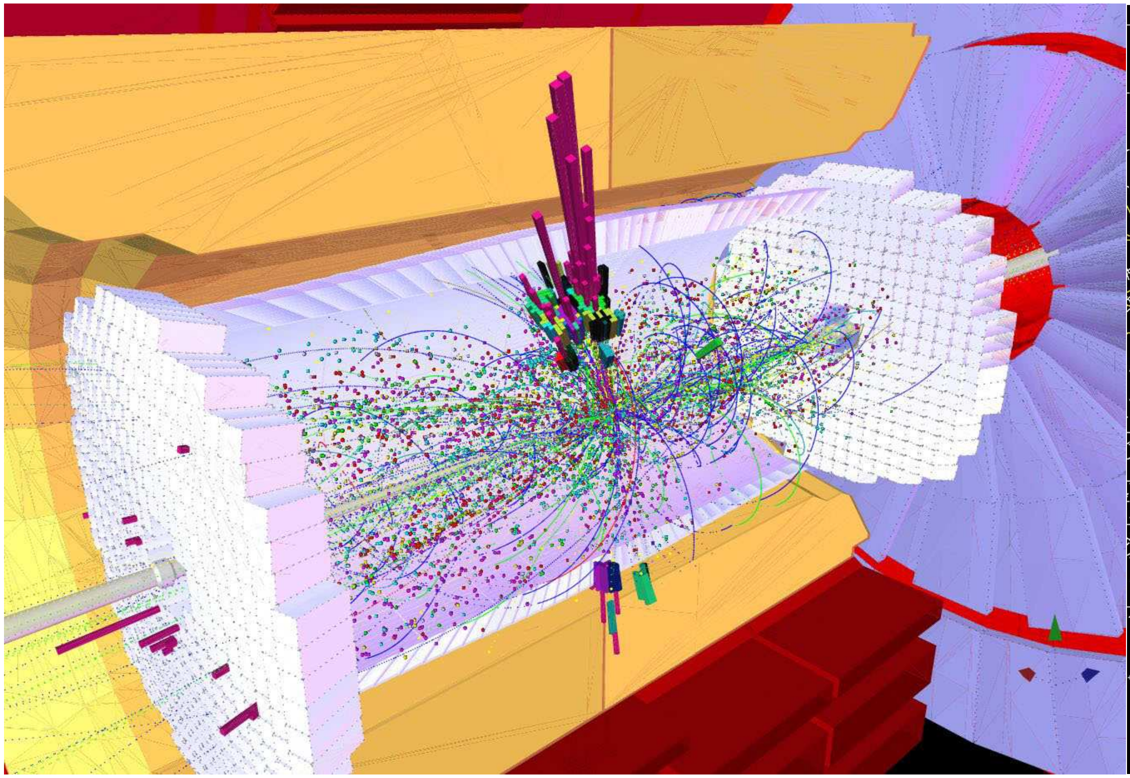


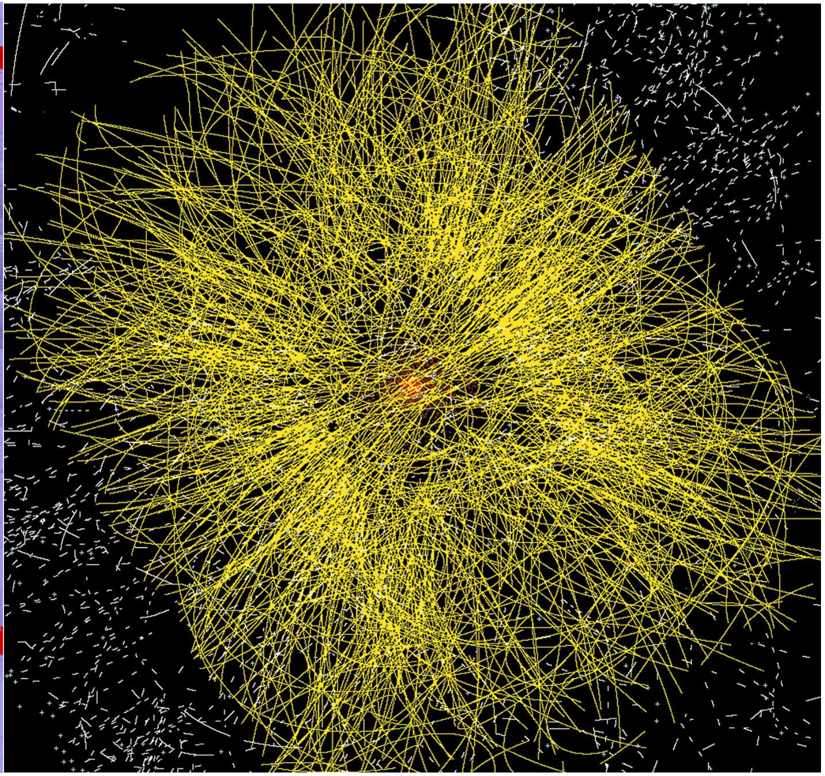
Status of POWHEG

Up to now, the following processes have been implemented in POWHEG:

- $hh \to ZZ$ (Ridolfi, P.N., 2006)
- $e^+e^- \to \mathrm{hadrons}$, (Latunde-Dada,Gieseke,Webber, 2006),
  $e^+e^- \to t\bar{t}$, including top decays at NLO (Latunde-Dada,2008),
- $hh \to Q\bar{Q}$ (Frixione, Ridolfi, P.N., 2007)
- $hh \to Z/W$ (Alioli, Oleari, Re, P.N., 2008;)
  (Hamilton,Richardson,Tully, 2008;)
- $hh \to H$ (gluon fusion) (Alioli, Oleari, Re, P.N., 2008; Herwig++)
- $hh \to H$, $hh \to HZ/W$ (Hamilton,Richardson,Tully, 2009;)
- $hh \to t + X$ (single top) NEW (Alioli, Oleari, Re, P.N., 2009)
- $hh \to Z + \mathrm{jet}$, Very preliminary (Alioli, Oleari, Re, P.N., 2009)
- The POWHEG BOX, Very preliminary, (Alioli, Oleari, Re, P.N., 2009)

Underlying Events and Minimum Bias

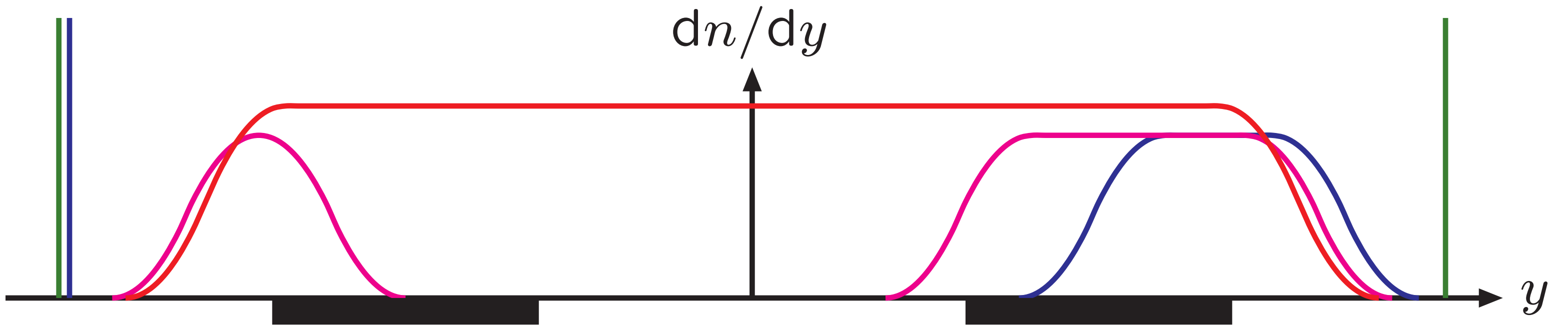




What is minimum bias?

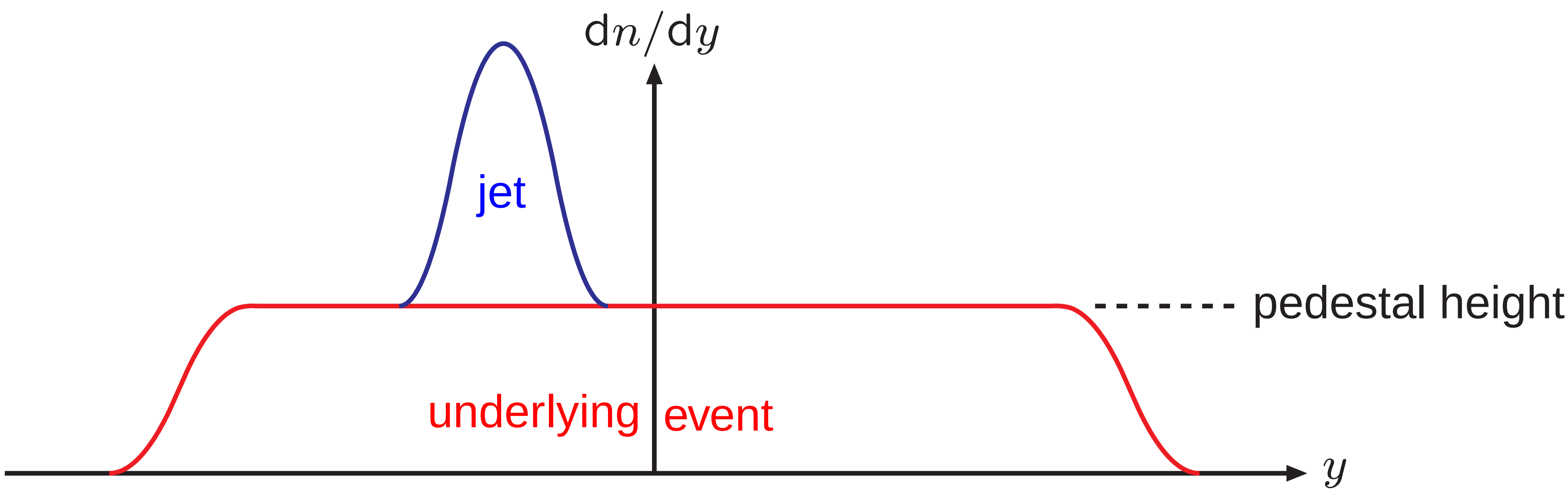
≈ “all events, with no bias from restricted trigger conditions”

$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{elastic}} + \sigma_{\mathrm{single-diffractive}} + \sigma_{\mathrm{double-diffractive}} + \cdots + \sigma_{\mathrm{non-diffractive}}$



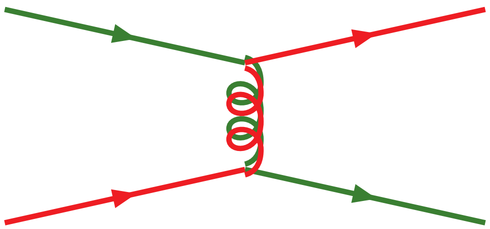
reality: $\sigma_{\mathrm{min-bias}} \approx \sigma_{\mathrm{non-diffractive}} + \sigma_{\mathrm{double-diffractive}} \approx 2/3 \times \sigma_{\mathrm{tot}}$

What is underlying event?



What is multiple (partonic) interactions?

Cross section for $2 \to 2$ interactions is dominated by t-channel gluon exchange, so diverges like $\mathrm{d}\hat{\sigma}/\mathrm{d}p_\perp^2 \approx 1/p_\perp^4$ for $p_\perp \to 0$.



integrate QCD $2 \to 2$

$\mathrm{qq}' \to \mathrm{qq}'$

$\mathrm{q\overline{q}} \to \mathrm{q}'\overline{\mathrm{q}}'$

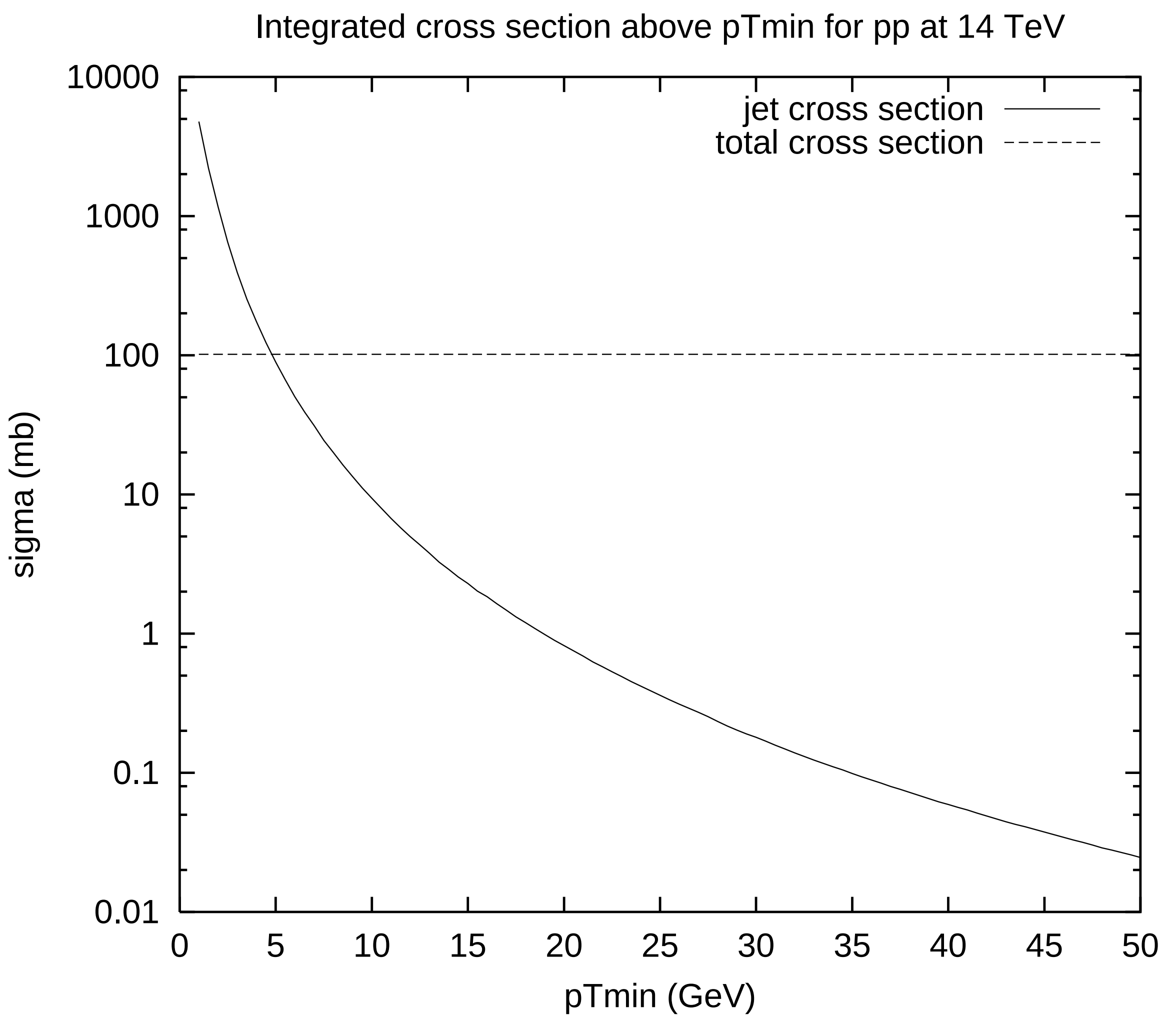
$\mathrm{q\overline{q}} \to \mathrm{gg}$

$\mathrm{qg} \to \mathrm{qg}$

$\mathrm{gg} \to \mathrm{gg}$

$\mathrm{gg} \to \mathrm{q\overline{q}}$

with CTEQ 5L PDF's



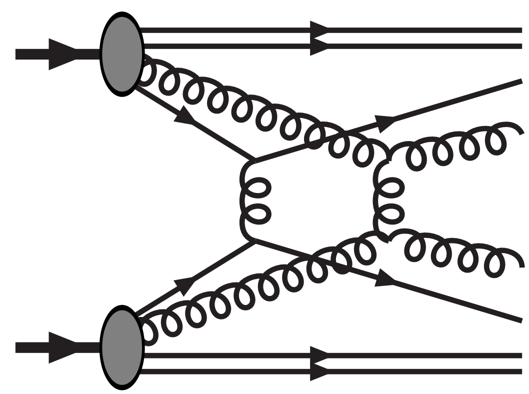
$$\sigma_{\text{int}}(p_{\perp\text{min}}) = \iiint_{p_{\perp\text{min}}} \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}p_\perp^2\, f_1(x_1, p_\perp^2)\, f_2(x_2, p_\perp^2)\, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2}$$

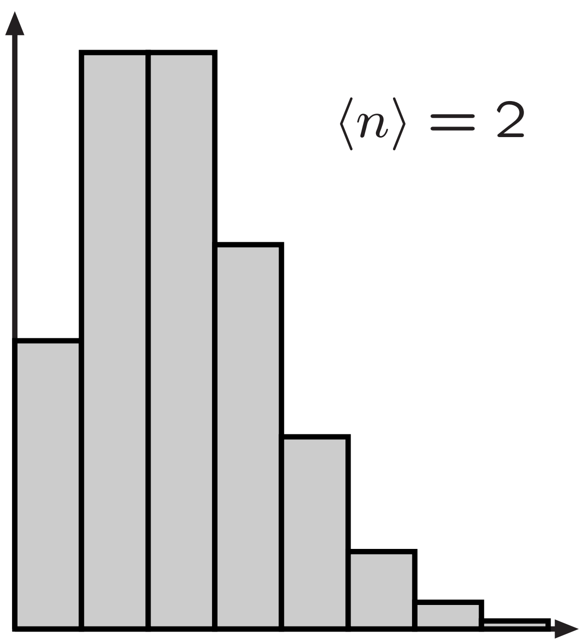
Half a solution to $\sigma_{\text{int}}(p_{\perp\text{min}}) > \sigma_{\text{tot}}$: many interactions per event

$$\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$$

$$\sigma_{\text{int}} = \sum_{n=0}^{\infty} n\,\sigma_n$$

$$\sigma_{\text{int}} > \sigma_{\text{tot}} \Longleftrightarrow \langle n \rangle > 1$$





If interactions occur independently

then **Poissonian statistics**

$$\mathcal{P}_n = \frac{\langle n \rangle^n}{n!}\, e^{-\langle n \rangle}$$

but energy–momentum conservation

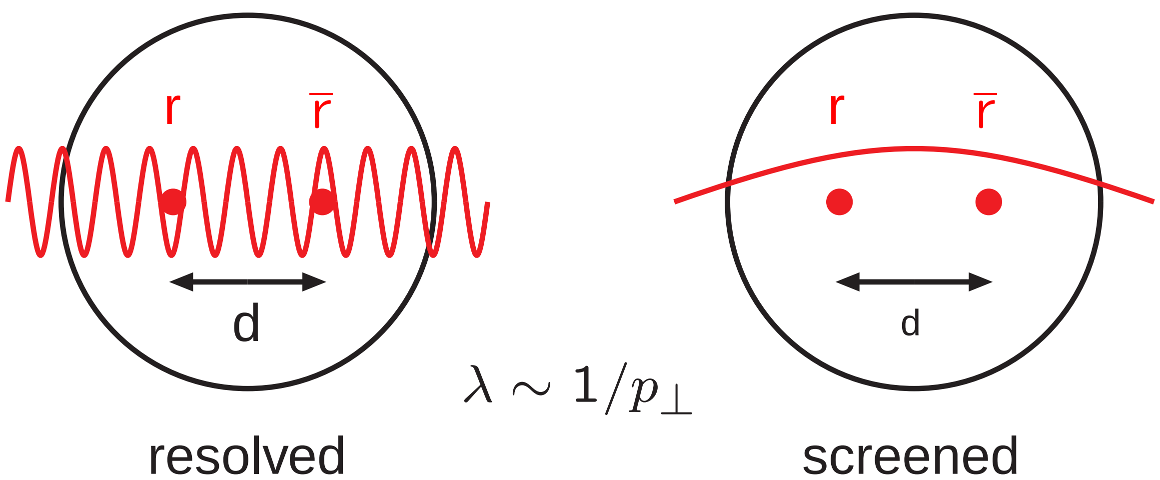
$\Rightarrow$ large n suppressed

Other half of solution:
perturbative QCD not valid at small $p_\perp$ since q, g not asymptotic states (confinement!).

Naively breakdown at

$$p_{\perp\mathrm{min}} \simeq \frac{\hbar}{r_\mathrm{p}} \approx \frac{0.2\ \mathrm{GeV}\cdot\mathrm{fm}}{0.7\ \mathrm{fm}} \approx 0.3\ \mathrm{GeV} \simeq \Lambda_\mathrm{QCD}$$

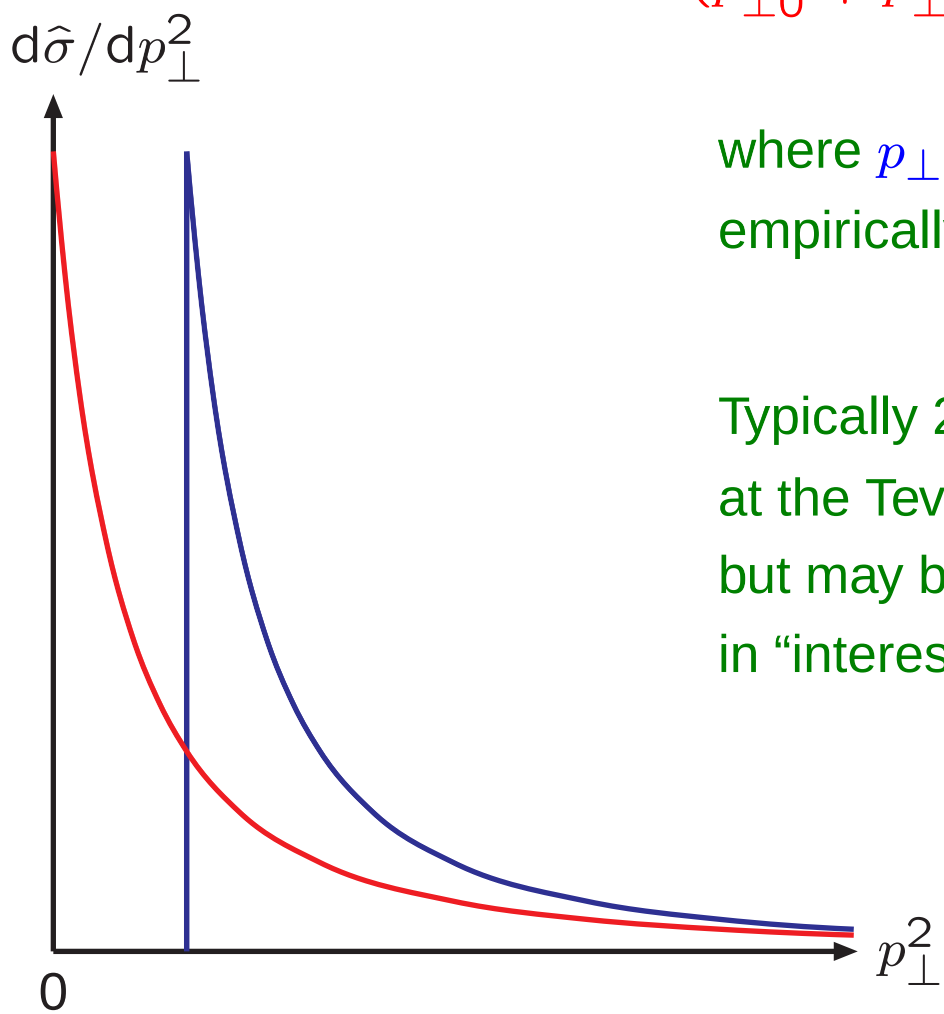
. . . but better replace r_p by (unknown) colour screening length d in hadron



so modify

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}p_\perp^2} \propto \frac{\alpha_\mathrm{s}^2(p_\perp^2)}{p_\perp^4} \to \frac{\alpha_\mathrm{s}^2(p_\perp^2)}{p_\perp^4}\theta\left(p_\perp - p_{\perp\mathrm{min}}\right) \quad \text{(simpler)}$$

$$\text{or} \to \frac{\alpha_\mathrm{s}^2(p_{\perp 0}^2 + p_\perp^2)}{(p_{\perp 0}^2 + p_\perp^2)^2} \quad \text{(more physical)}$$



where $p_{\perp\mathrm{min}}$ or $p_{\perp 0}$ are free parameters, empirically of order **2 GeV**

Typically 2 – 3 interactions/event at the Tevatron, 4 – 5 at the LHC, but may be more in “interesting” high-$p_\perp$ ones.

Basic generation of multiple (partonic) interactions

- For now exclude diffractive (and elastic) topologies, i.e. only model nondiffractive events, with $\sigma_{\mathrm{nd}} \simeq 0.6 \times \sigma_{\mathrm{tot}}$
- Differential probability for interaction at $p_\perp$ is

$$\frac{\mathrm{d}P}{\mathrm{d}p_\perp} = \frac{1}{\sigma_{\mathrm{nd}}} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp}$$

- Average number of interactions naively

$$\langle n \rangle = \frac{1}{\sigma_{\mathrm{nd}}} \int_0^{E_{\mathrm{cm}}/2} \frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp} \mathrm{d}p_\perp$$

- Require ≥ 1 interaction in an event or else pass through without anything happening

$$P_{\geq 1} = 1 - P_0 = 1 - \exp(-\langle n \rangle)$$

(Alternatively: allow soft nonperturbative interactions even if no perturbative ones.)

Can pick n from Poissonian and then generate n independent interactions according to $\mathrm{d}\sigma/\mathrm{d}p_\perp$ (so long as energy left), or better...

...generate interactions in ordered sequence $p_{\perp 1} > p_{\perp 2} > p_{\perp 3} > \ldots$

- recall "Sudakov" trick used e.g. for parton showers:
  if probability for something to happen at "time" t is $P(t)$
  and happenings are uncorrelated in time (Poissonian statistics)
  then the probability for a *first* happening after 0 at t_1 is

$$\mathcal{P}(t_1) = P(t_1)\ \exp\left(-\int_0^{t_1} P(t)\,\mathrm{d}t\right)$$

  and for an i'th at t_i is

$$\mathcal{P}(t_i) = P(t_i)\ \exp\left(-\int_{t_{i-1}}^{t_i} P(t)\,\mathrm{d}t\right)$$

- Apply to ordered sequence of decreasing $p_\perp$, starting from $E_{\mathrm{cm}}/2$

$$\mathcal{P}(p_\perp = p_{\perp i}) = \frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}\sigma}{\mathrm{d}p_\perp} \exp\left[-\int_{p_\perp}^{p_{\perp(i-1)}} \frac{1}{\sigma_{\mathrm{nd}}}\frac{\mathrm{d}\sigma}{\mathrm{d}p'_\perp}\mathrm{d}p'_\perp\right]$$

- Use rescaled PDF's taking into account already used momentum
  $\Longrightarrow n_{\mathrm{int}}$ narrower than Poissonian

Impact parameter dependence

So far assumed that all collisions have equivalent initial conditions, but hadrons are extended,

e.g. empirical double Gaussian:

$$\rho_{\mathrm{matter}}(r) = N_1 \exp\left(-\frac{r^2}{r_1^2}\right) + N_2 \exp\left(-\frac{r^2}{r_2^2}\right)$$

where $r_2 \neq r_1$ represents "hot spots", and overlap of hadrons during collision is

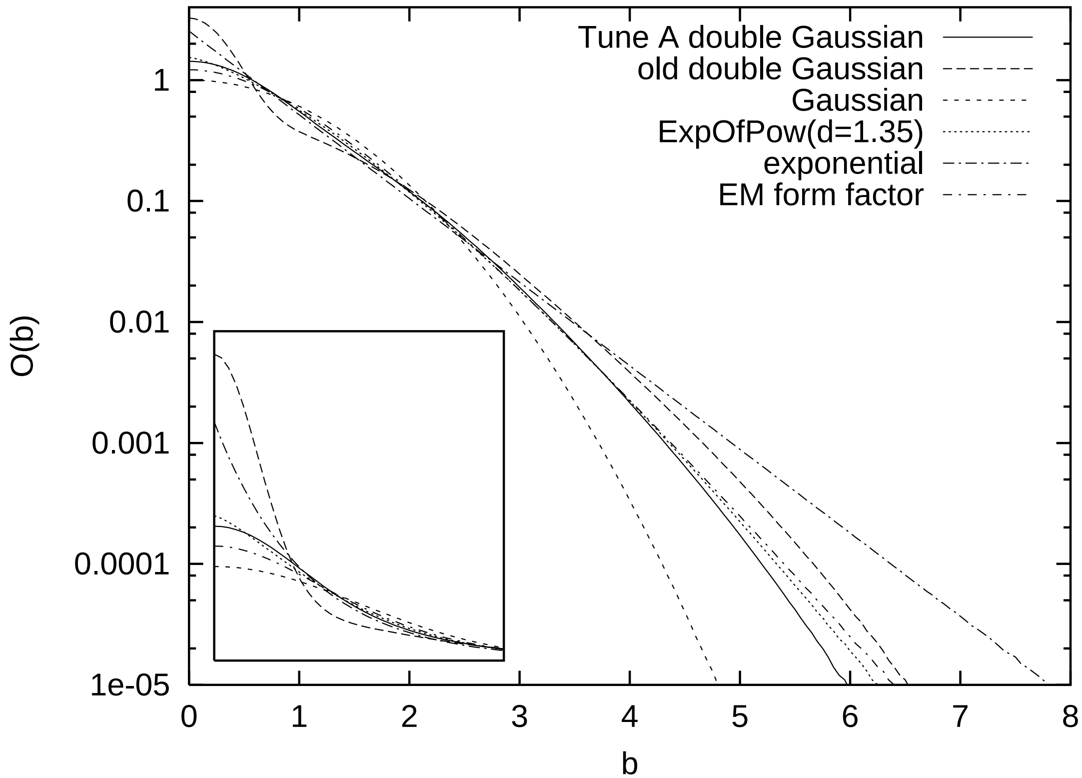
$$\mathcal{O}(b) = \int \mathrm{d}^3\mathbf{x}\,\mathrm{d}t\; \rho_{1,\mathrm{matter}}^{\mathrm{boosted}}(\mathbf{x},t)\rho_{2,\mathrm{matter}}^{\mathrm{boosted}}(\mathbf{x},t)$$

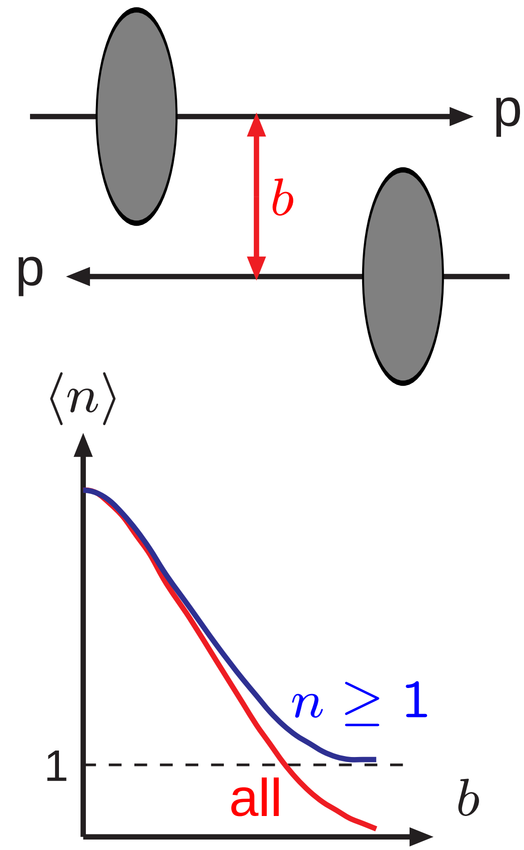
or electromagnetic form factor:

$$S_{\mathrm{p}}(\mathbf{b}) = \int \frac{\mathrm{d}^2\mathbf{k}}{2\pi}\,\frac{\exp(i\mathbf{k}\cdot\mathbf{b})}{(1+\mathbf{k}^2/\mu^2)^2}$$

where $\mu = 0.71$ GeV $\rightarrow$ free parameter, which gives

$$O(b) = \frac{\mu^2}{96\pi}\,(\mu b)^3\,K_3(\mu b)$$





- Events are distributed in impact parameter b
- Average activity at b proportional to $\mathcal{O}(b)$
  - ⋆ central collisions more active $\Rightarrow \mathcal{P}_n$ broader than Poissonian
  - ⋆ peripheral passages normally give no collisions at all $\Rightarrow$ finite σ_{tot}
- Also crucial for *pedestal effect* (more later)

PYTHIA implementation

(1) Simple scenario (1985):

first model for event properties based on perturbative multiple interactions

no longer used (no impact-parameter dependence)

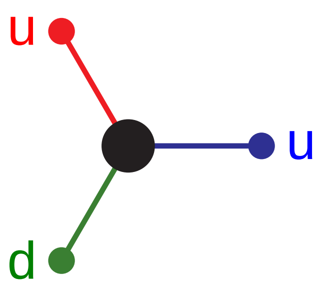
(2) Impact-parameter-dependence (1987):

still in frequent use (Tune A, Tune DWT, ATLAS tune, . . .)

- double Gaussian matter distribution,
- interactions ordered in decreasing $p_\perp$,
- PDF's rescaled for momentum conservation,
- *but* no showers for subsequent interactions and simplified flavours

(3) Improved handling of PDFs and beam remnants (2004)

- Trace flavour content of remnant, including baryon number (junction)
- Study colour (re)arrangement among outgoing partons (ongoing!)
- Allow radiation for all interactions

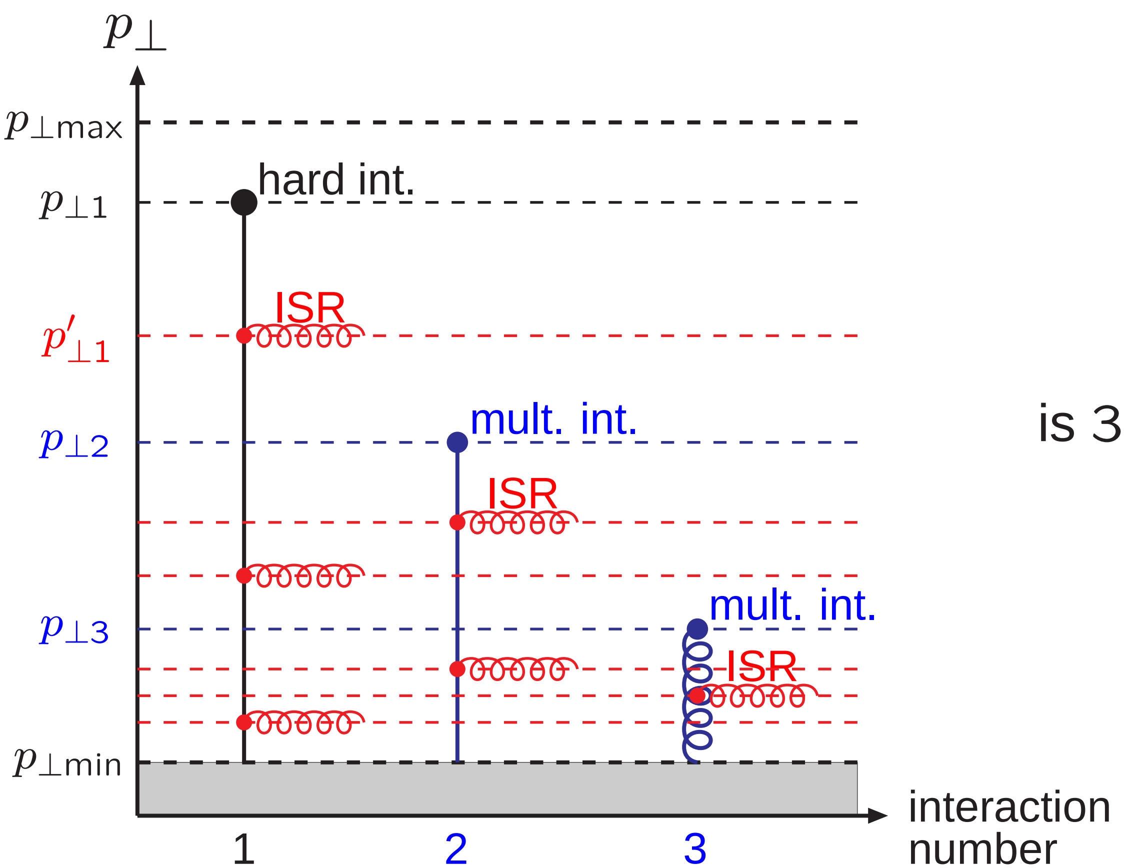


(4) Evolution interleaved with ISR (2004)

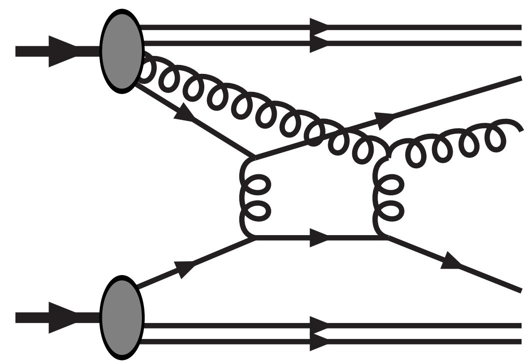
- Transverse-momentum-ordered showers

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}p_\perp} = \left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{MI}}}{\mathrm{d}p_\perp} + \sum \frac{\mathrm{d}\mathcal{P}_{\mathrm{ISR}}}{\mathrm{d}p_\perp}\right) \exp\left(-\int_{p_\perp}^{p_{\perp i-1}} \left(\frac{\mathrm{d}\mathcal{P}_{\mathrm{MI}}}{\mathrm{d}p'_\perp} + \sum \frac{\mathrm{d}\mathcal{P}_{\mathrm{ISR}}}{\mathrm{d}p'_\perp}\right) \mathrm{d}p'_\perp\right)$$

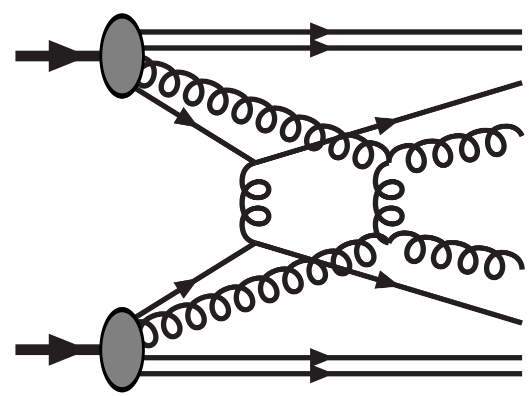
with ISR sum over all previous MI



(5) Rescattering (in progress)

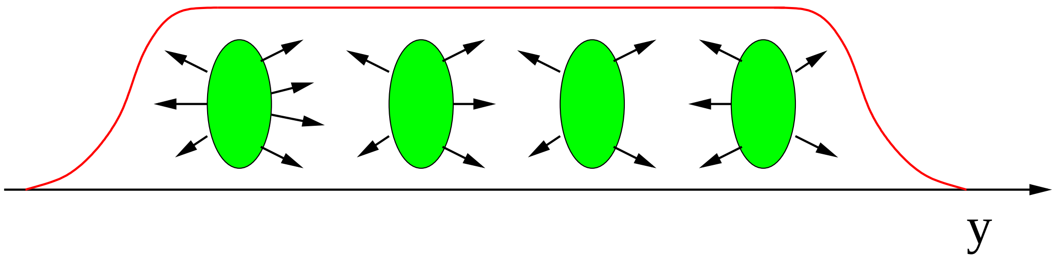


is 3 → 3 instead of 4 → 4:



HERWIG implementation

(1) Soft Underlying Event (1988), based on UA5 Monte Carlo



- Distribute a ($\sim$ negative binomial) number of clusters independently in rapidity and transverse momentum according to parametrization/extrapolation of data
- modify for overall energy/momentum/flavour conservation
- no minijets; correlations only by cluster decays

(2) Jimmy (1995; HERWIG add-on; part of HERWIG++)

- only model of underlying event, not of minimum bias
- similar to PYTHIA (2) above; but details different
- matter profile by electromagnetic form factor (with tuned size)
- no $p_{\perp}$-ordering of emissions, no rescaling of PDF: abrupt stop when (if) run out of energy

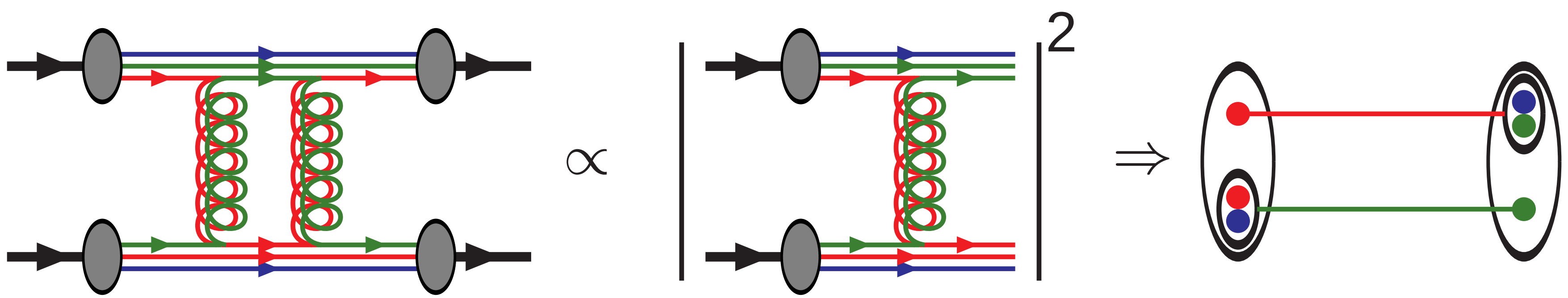
(3) Ivan (2002, code not public; part of HERWIG++)

- also handles minimum bias
- soft and hard multiple interactions together fill whole $p_{\perp}$ range

PhoJet (& relatives) implementation

(1) Cut Pomeron (1982)

- Pomeron predates QCD; nowadays $\sim$ glueball tower
- Optical theorem relates σ_{total} and $\sigma_{\mathrm{elastic}}$



- Unified framework of nondiffractive and diffractive interactions
- Purely low-$p_\perp$: only primordial $k_\perp$ fluctuations
- Usually simple Gaussian matter distribution

(2) Extension to large $p_\perp$ (1990)

- distinguish soft and hard Pomerons (cf. Ivan):
  soft = nonperturbative, low-$p_\perp$, as above
  hard = perturbative, “high”-$p_\perp$
- hard based on PYTHIA code, with lower cutoff in $p_\perp$

without multiple interactions

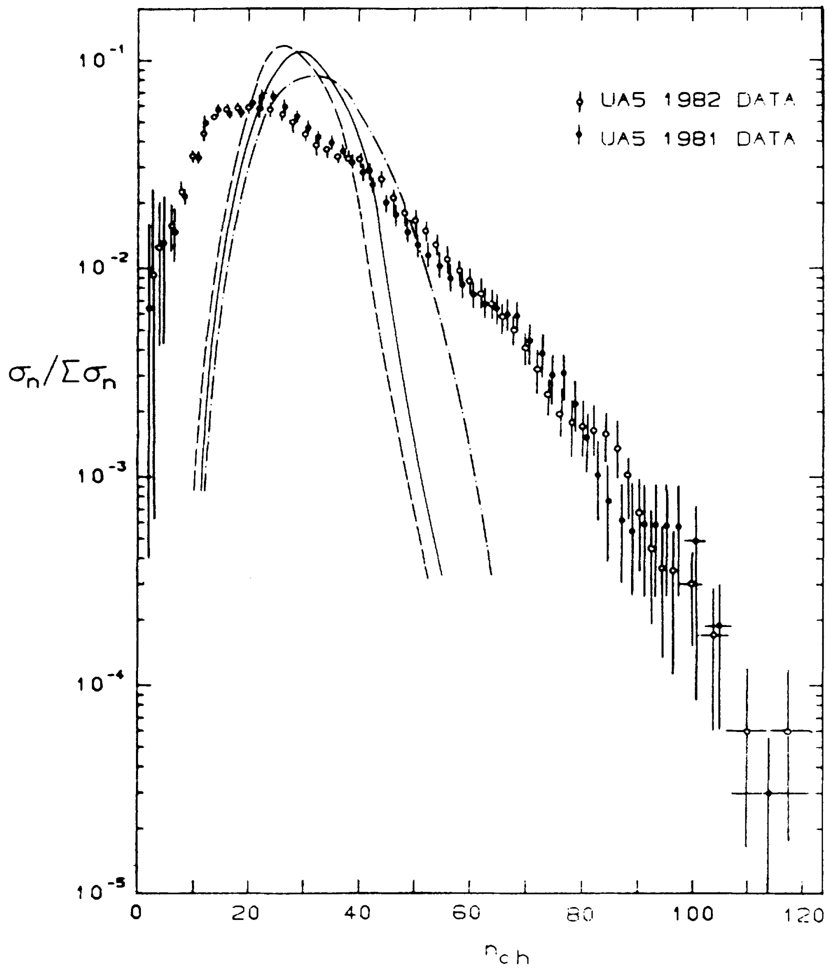


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

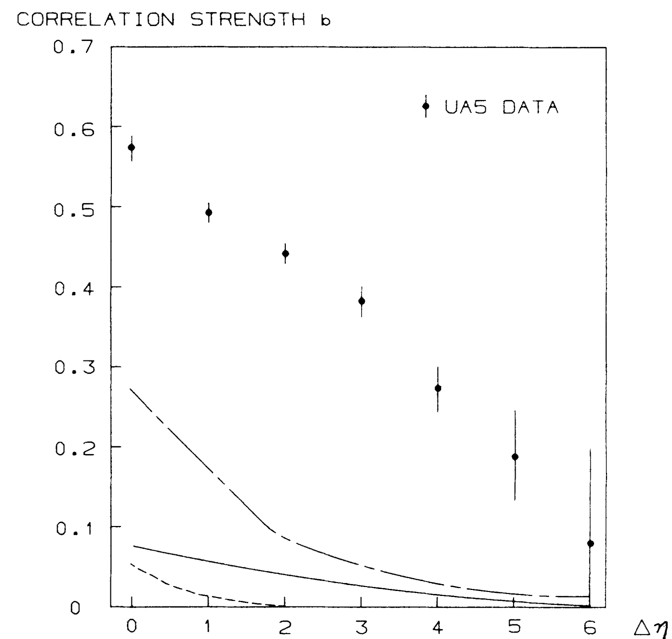


FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.

with multiple interactions

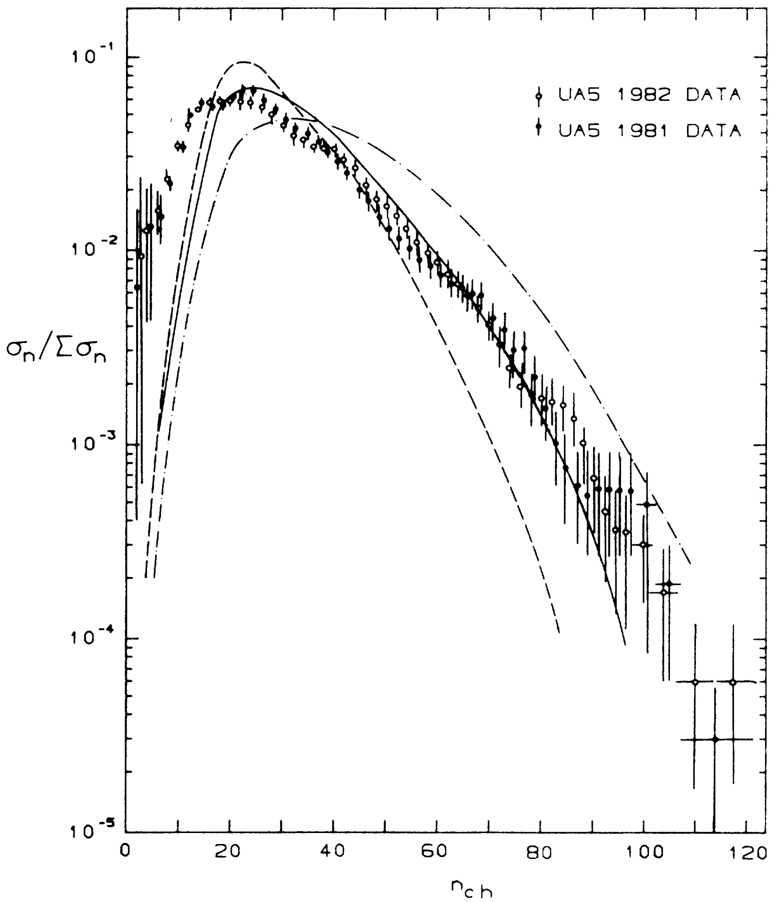


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\min}=2.0$ GeV; solid line, $p_{T\min}=1.6$ GeV; dashed-dotted line, $p_{T\min}=1.2$ GeV.

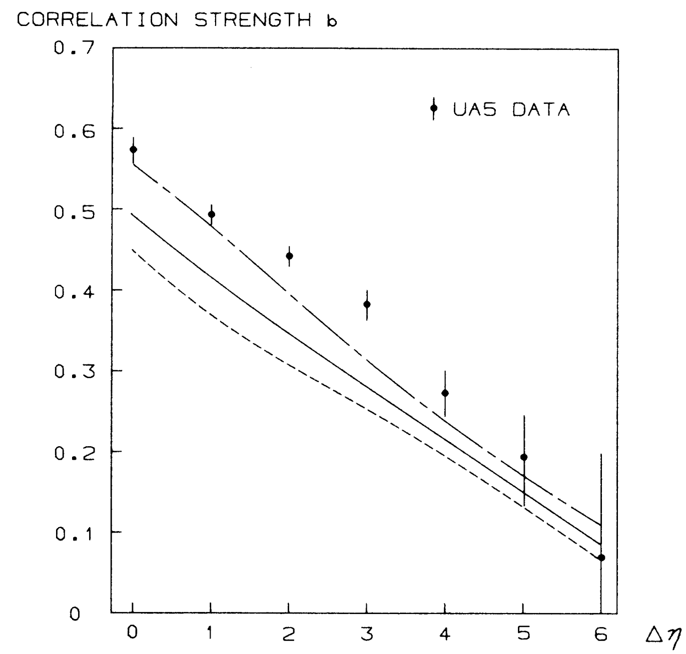


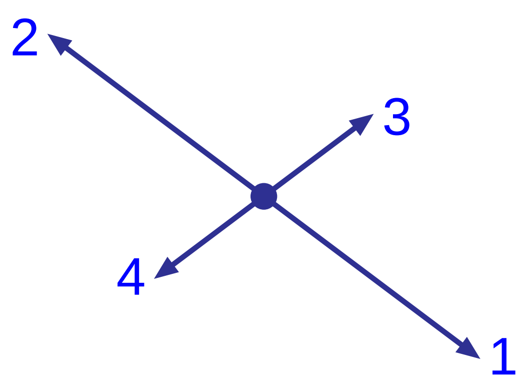
FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

Direct observation of multiple interactions

Five studies: AFS (1987), UA2 (1991), CDF (1993, 1997), D0 (2009)

Order 4 jets $\mathbf{p}_{\perp 1} > \mathbf{p}_{\perp 2} > \mathbf{p}_{\perp 3} > \mathbf{p}_{\perp 4}$ and define φ
as angle between $\mathbf{p}_{\perp 1} \mp \mathbf{p}_{\perp 2}$ and $\mathbf{p}_{\perp 3} \mp \mathbf{p}_{\perp 4}$ for AFS/CDF

Double Parton Scattering

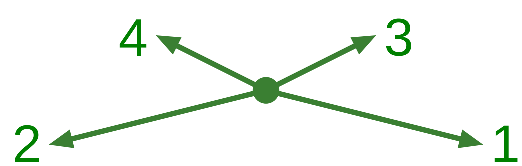


$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \approx 0$

$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \approx 0$

$\mathrm{d}\sigma/\mathrm{d}\varphi$ flat

Double BremsStrahlung

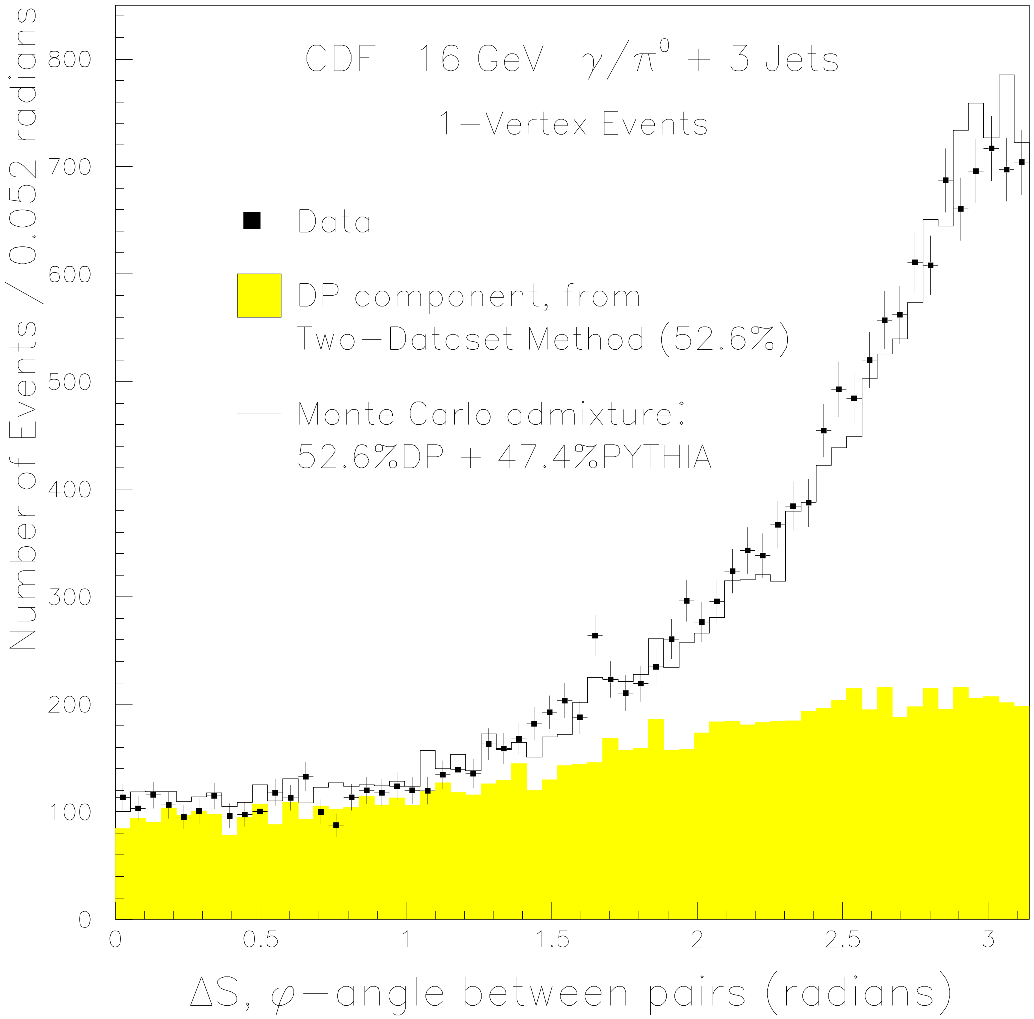


$|\mathbf{p}_{\perp 1} + \mathbf{p}_{\perp 2}| \gg 0$

$|\mathbf{p}_{\perp 3} + \mathbf{p}_{\perp 4}| \gg 0$

$\mathrm{d}\sigma/\mathrm{d}\varphi$ peaked at $\varphi \approx 0/\pi$ for AFS/CDF

AFS 4-jet analysis (pp at 63 GeV): observe 6 times Poissonian prediction,
with impact parameter expect 3.7 times Poissonian,
but big errors $\Rightarrow$ low acceptance, also UA2



CDF 3-jet + prompt photon analysis

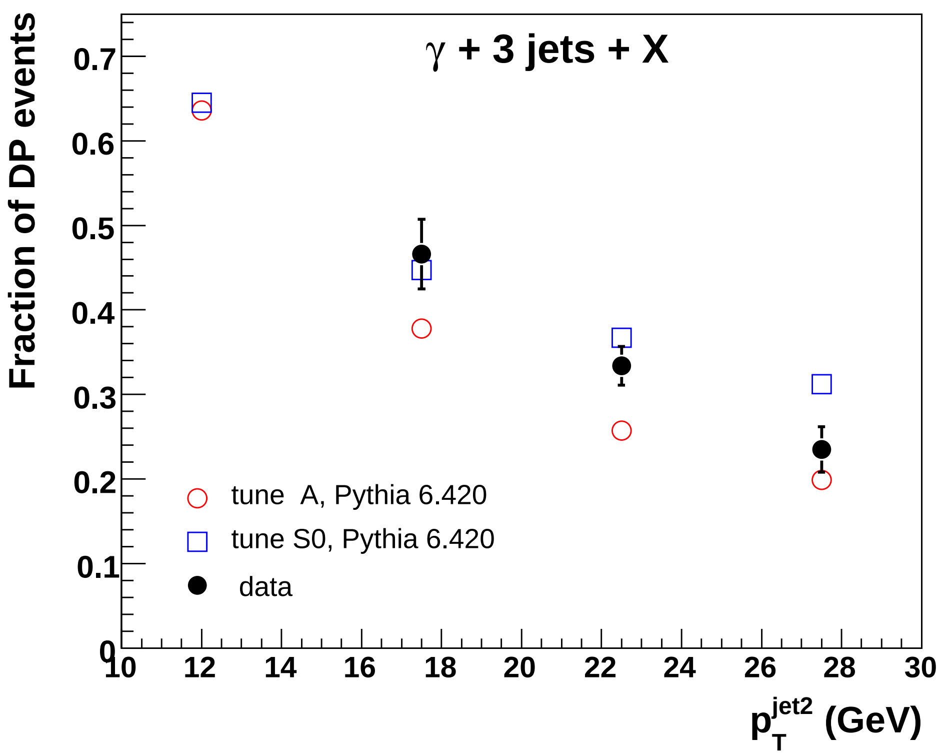
Yellow region = double parton scattering (DPS)

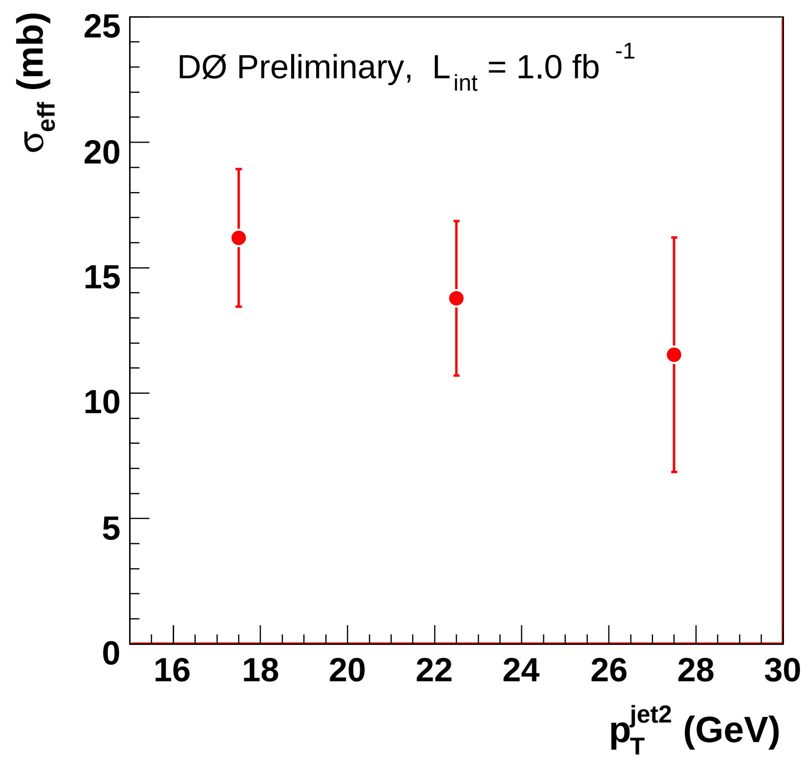
The rest = PYTHIA showers

$$\sigma_{\rm DPS} = \frac{\sigma_A \sigma_B}{\sigma_{\rm eff}} \quad \text{for } A \neq B \quad \Longrightarrow \sigma_{\rm eff} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}$$

Strong enhancement relative to naive expectations!

Preliminary D0 results:





$$\sigma_{\text{eff}} = 15.1 \pm 1.9 \text{ mb}$$

agreement and precision “too good to be true”;

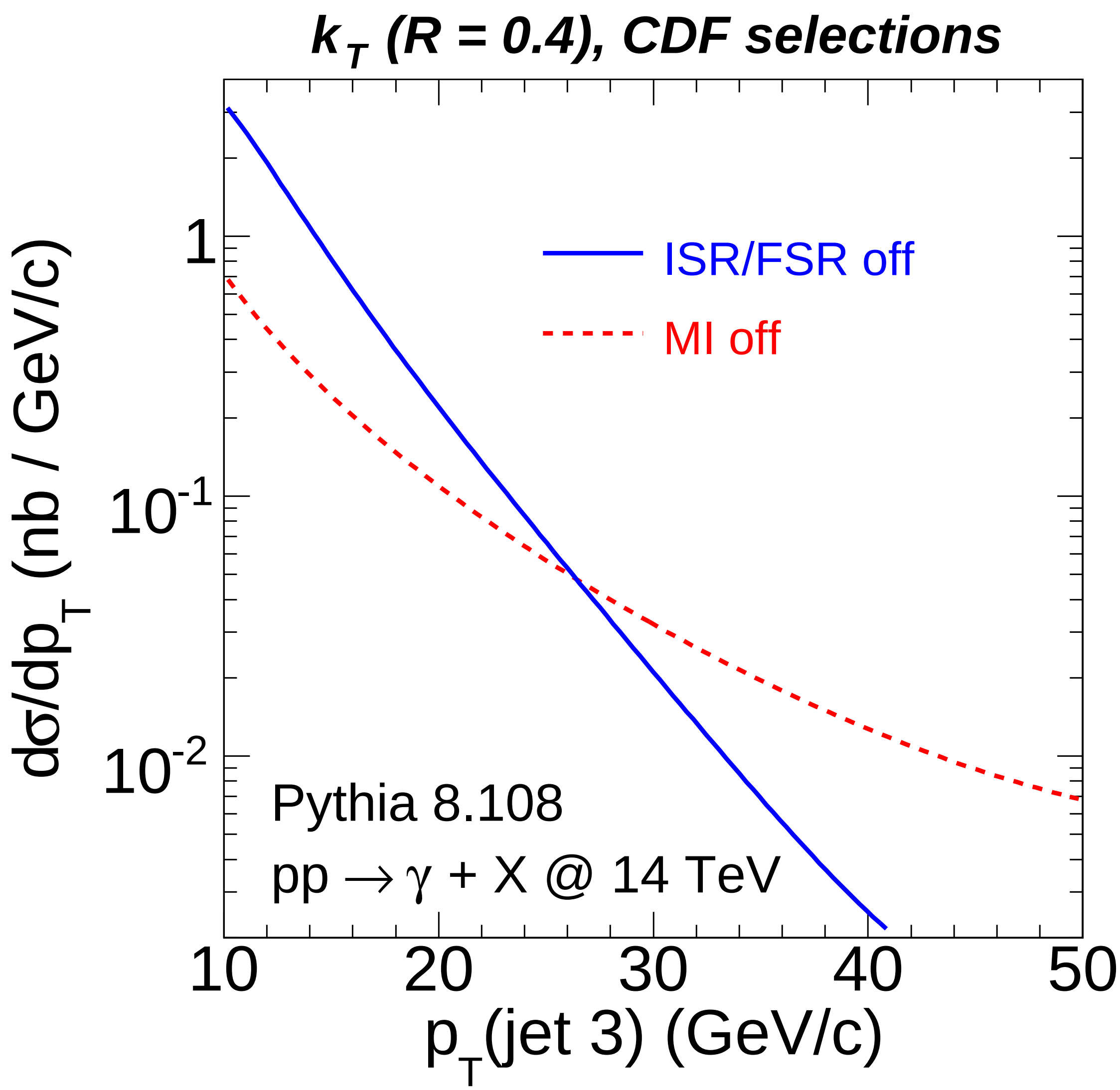
tunes 7 and 3 years old, respectively, and not to this kind of data

Same study also planned for LHC

Selection for DPS delicate balance:

showers dominate at large $p_\perp$ $\Rightarrow$ too large background

multiple interactions dominate at small $p_\perp$, but there jet identification difficult



Jet pedestal effect

Events with hard scale (jet, W/Z, ...) have more underlying activity!

Events with n interactions have n chances that one of them is hard,

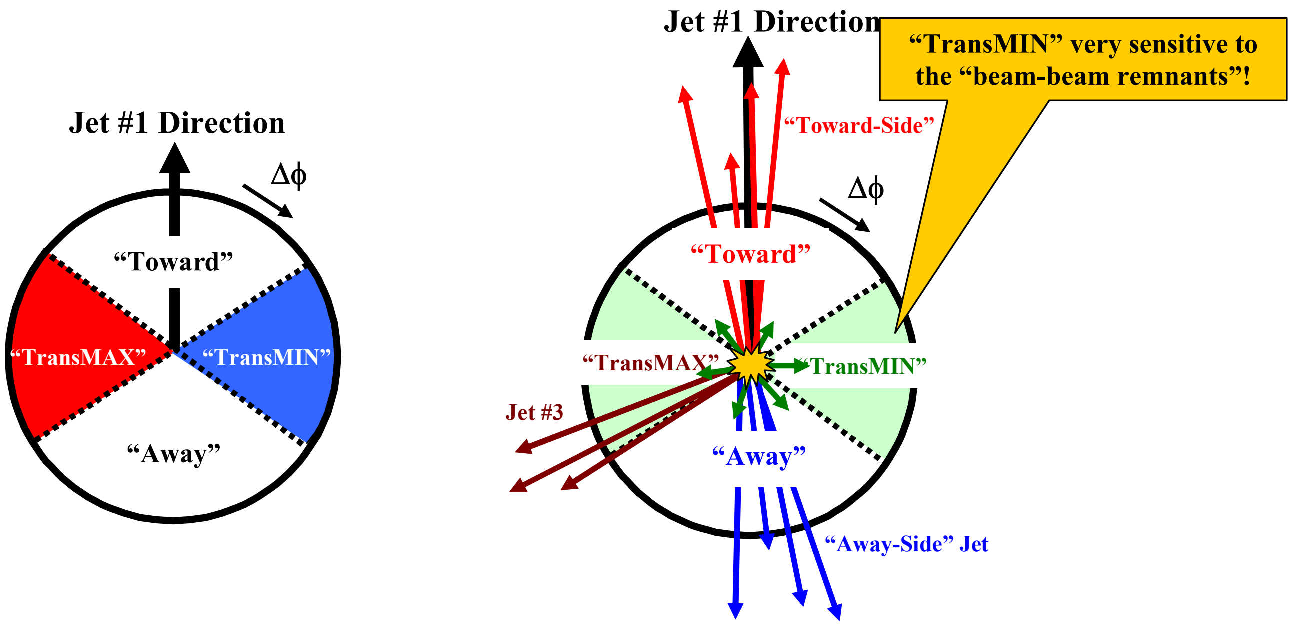
so "trigger bias": hard scale $\Rightarrow$ central collision

$\Rightarrow$ more interactions $\Rightarrow$ larger underlying activity.

Centrality effect saturates at $p_{\perp \mathrm{hard}} \sim 10$ GeV.

Studied in detail by Rick Field, comparing with CDF data:

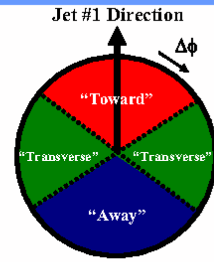
"MAX/MIN Transverse" Densities

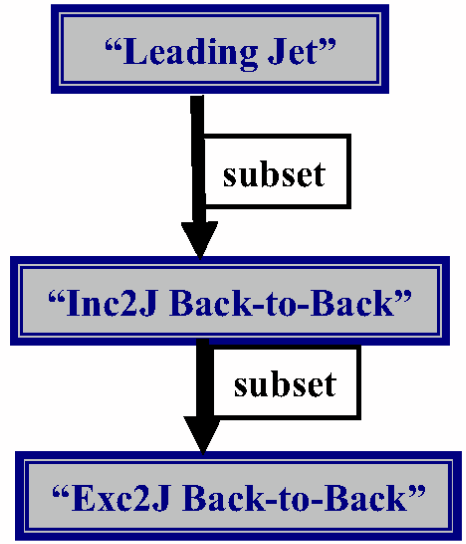


- **Define the MAX and MIN "transverse" regions on an event-by-event basis with MAX (MIN) having the largest (smallest) density.**

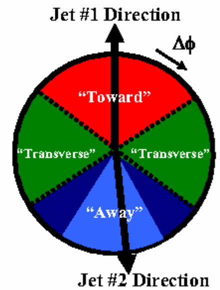
Event Topologies

- **"Leading Jet"** events correspond to the leading calorimeter jet (MidPoint R = 0.7) in the region $|\eta| < 2$ with no other conditions.

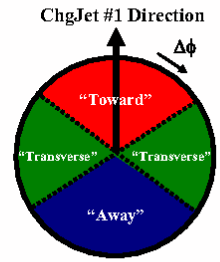




- **"Inclusive 2-Jet Back-to-Back"** events are selected to have at least two jets with Jet#1 and Jet#2 nearly "back-to-back" ($\Delta\phi_{12} > 150^o$) with almost equal transverse energies (P_T(jet#2)/P_T(jet#1) > 0.8) with no other conditions .

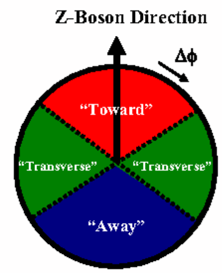


- **"Exclusive 2-Jet Back-to-Back"** events are selected to have at least two jets with Jet#1 and Jet#2 nearly "back-to-back" ($\Delta\phi_{12} > 150^o$) with almost equal transverse energies (P_T(jet#2)/P_T(jet#1) > 0.8) and P_T(jet#3) < 15 GeV/c.



"Charged Jet"

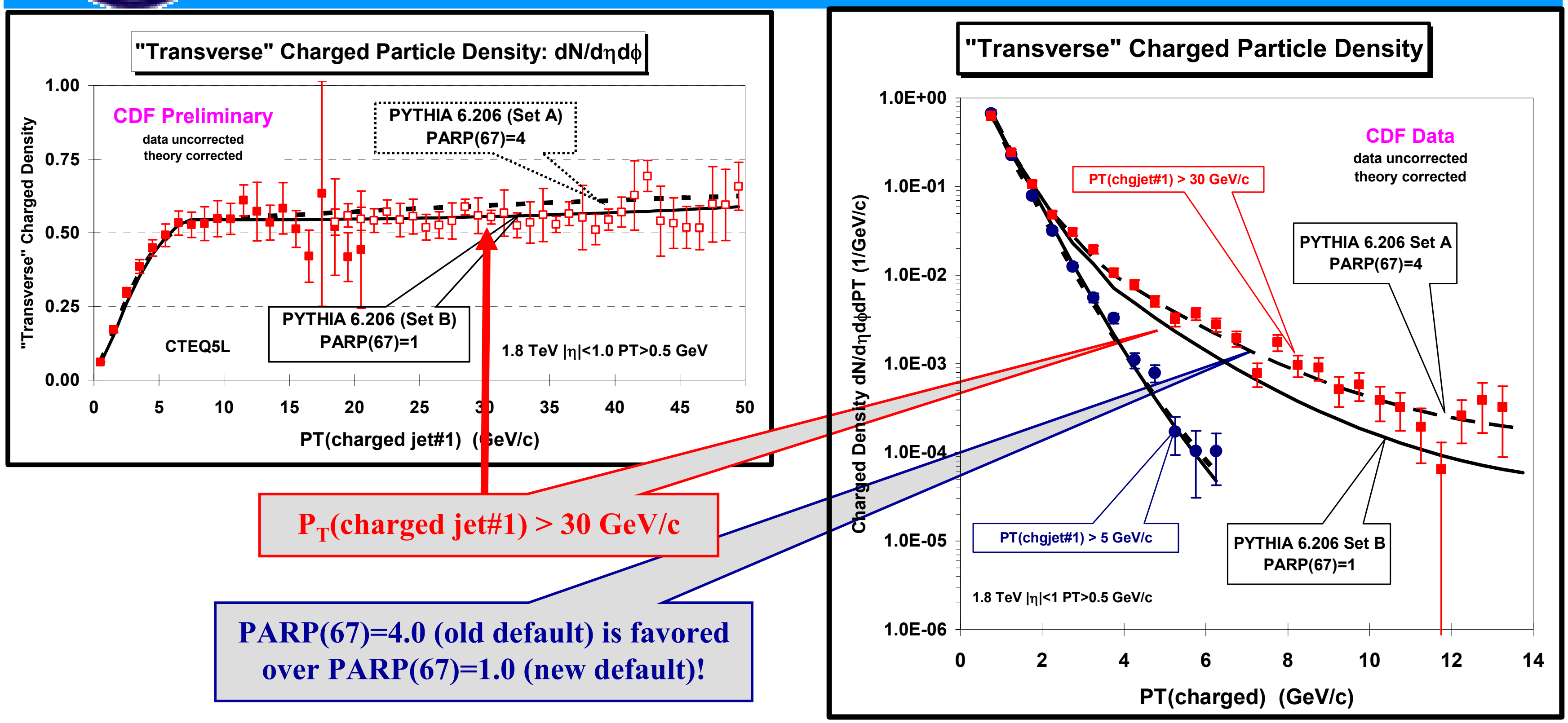
- **"Leading ChgJet"** events correspond to the leading charged particle jet (R = 0.7) in the region $|\eta| < 1$ with no other conditions.



Z-Boson

- **"Z-Boson"** events are Drell-Yan events with 70 < M(lepton-pair) < 110 GeV with no other conditions.

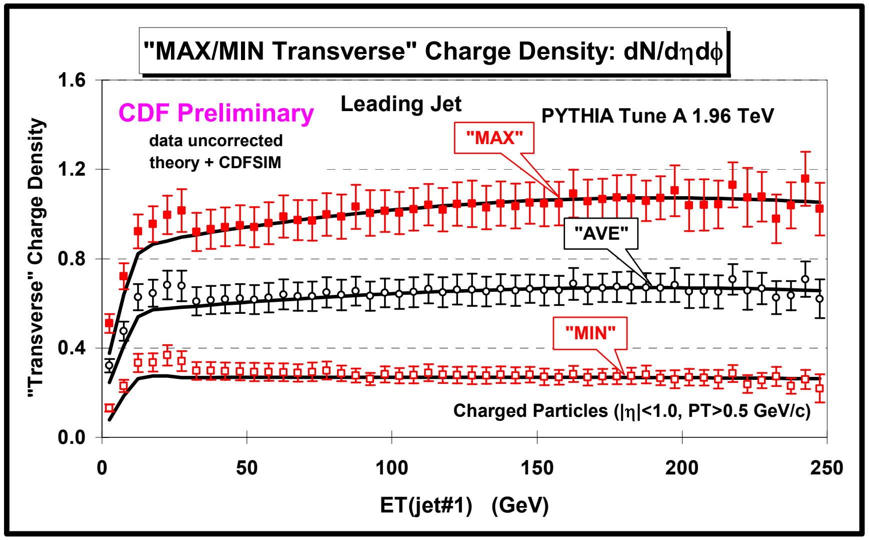
Tuned PYTHIA 6.206 "Transverse" P_T Distribution

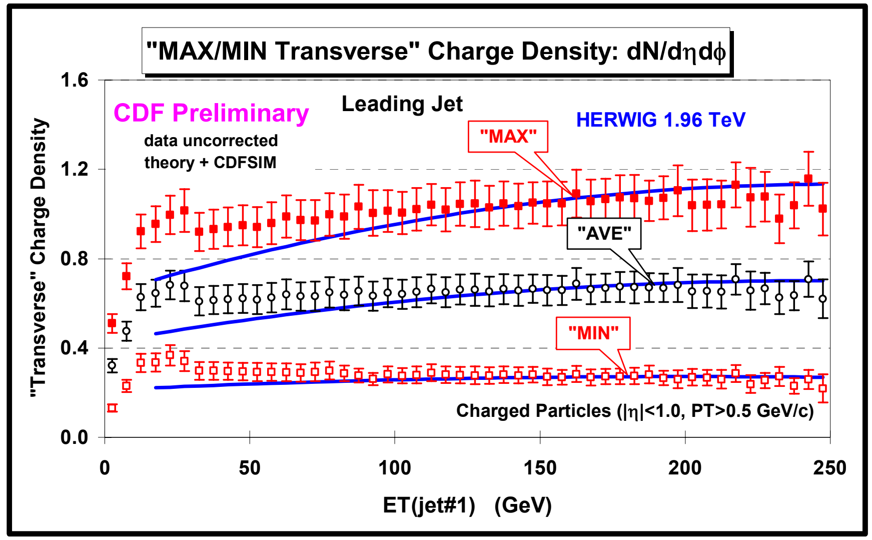


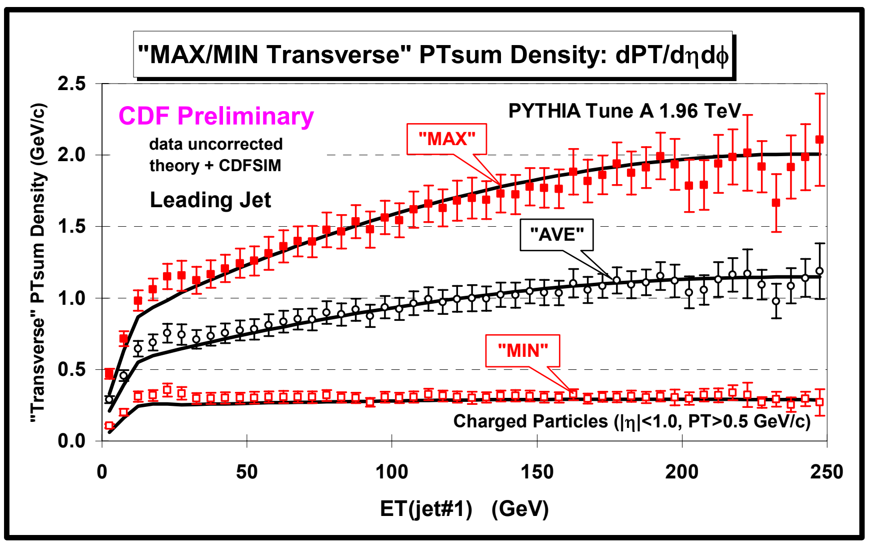
- **Compares the average "transverse" charge particle density (|η|<1, P_T>0.5 GeV) versus P_T(charged jet#1) and the P_T distribution of the "transverse" density, $dN_{chg}/d\eta d\phi dP_T$ with the QCD Monte-Carlo predictions of two tuned versions of PYTHIA 6.206 (P_T(hard) > 0, CTEQ5L, Set B (PARP(67)=1) and Set A (PARP(67)=4)).**

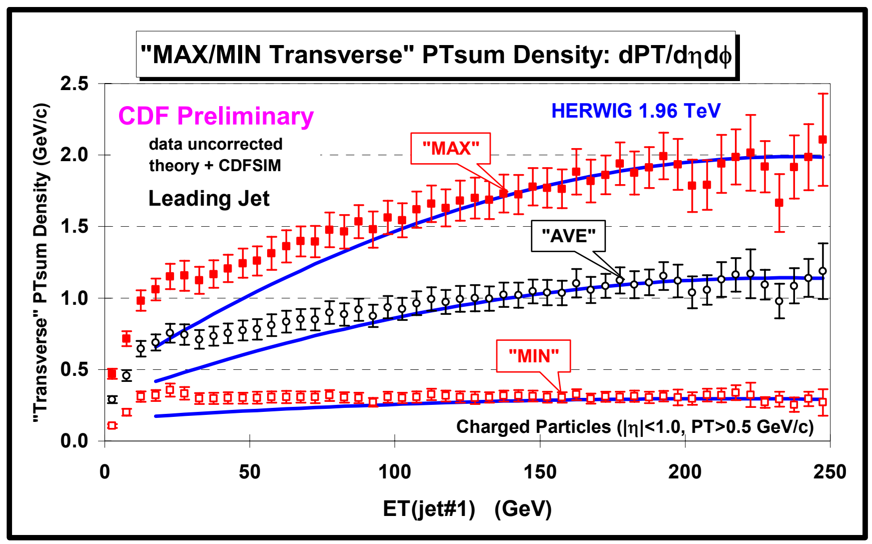
Leading Jet: “MAX & MIN Transverse” Densities

PYTHIA Tune A HERWIG



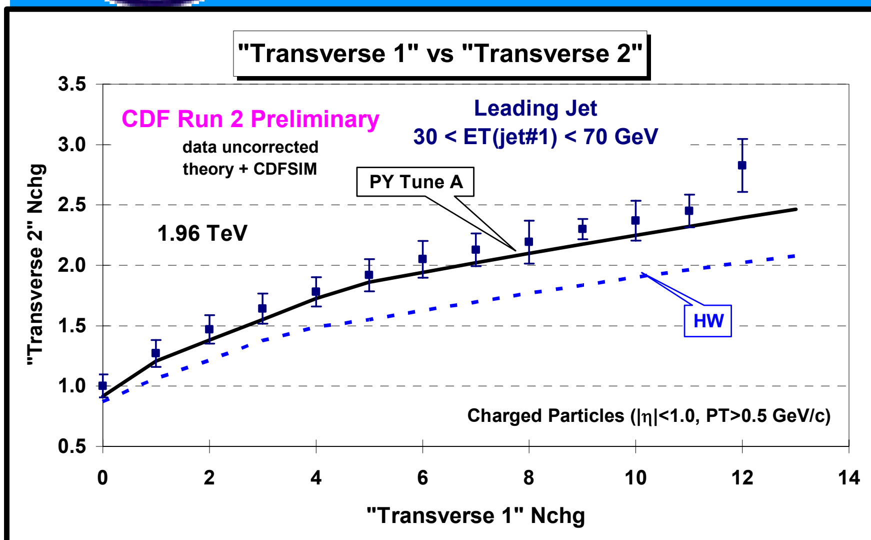


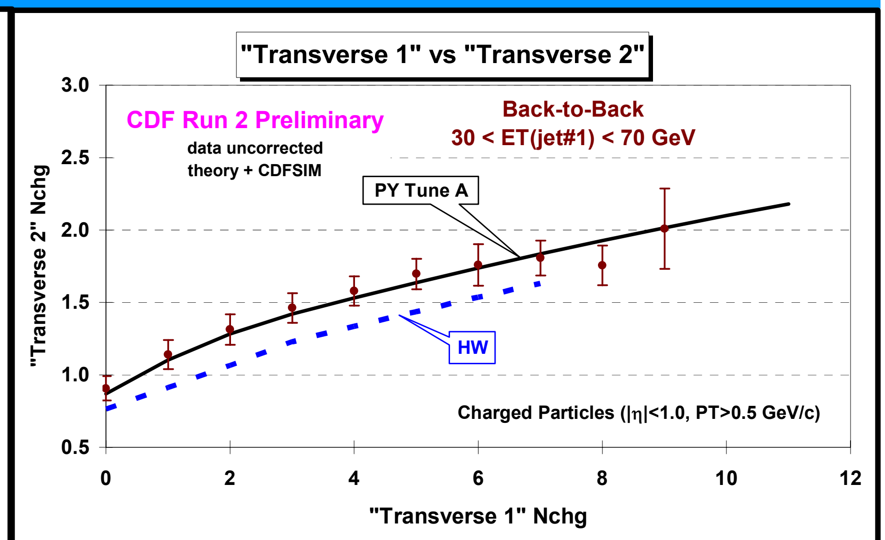


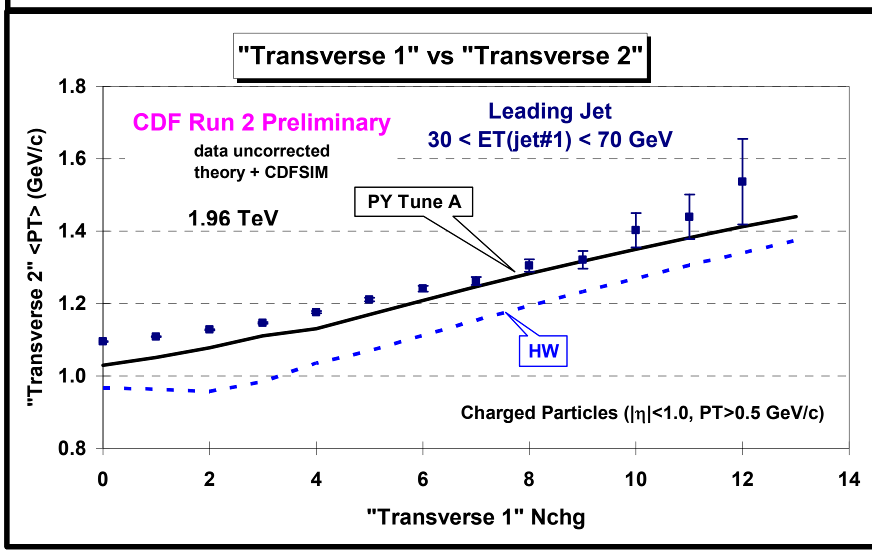


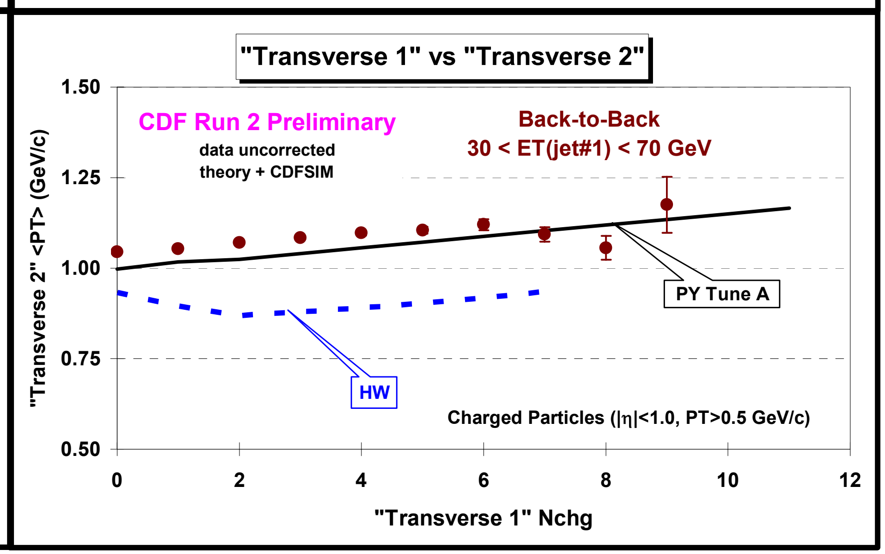
Charged particle density and PTsum density for “leading jet” events versus E_T(jet#1) for PYTHIA Tune A and HERWIG.

"Transverse 1" Region vs "Transverse 2" Region

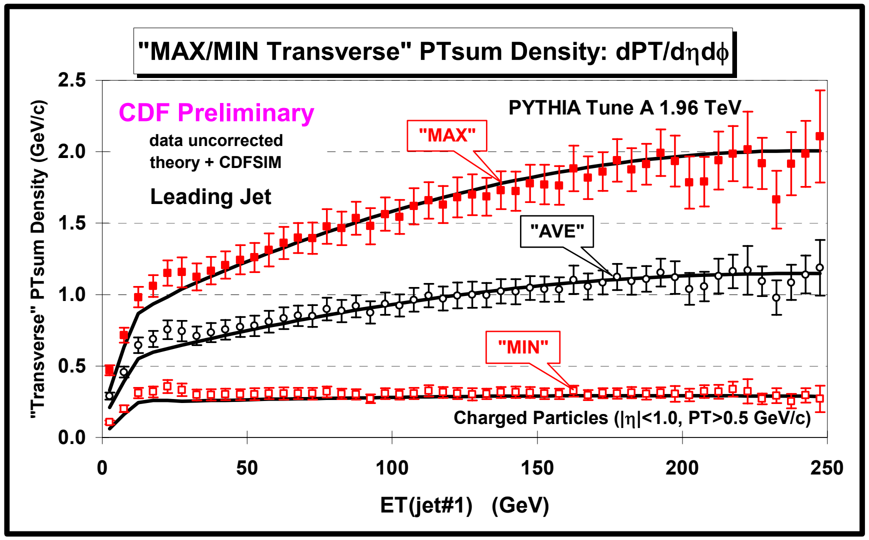


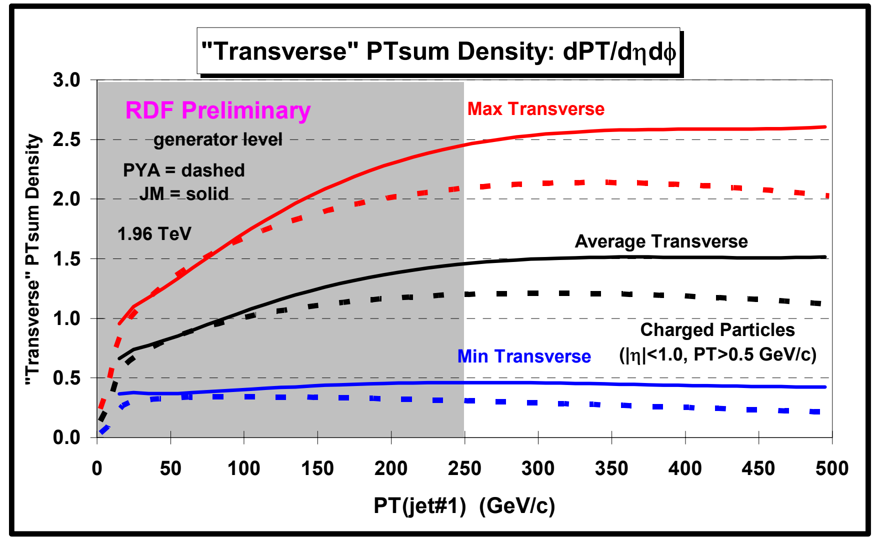






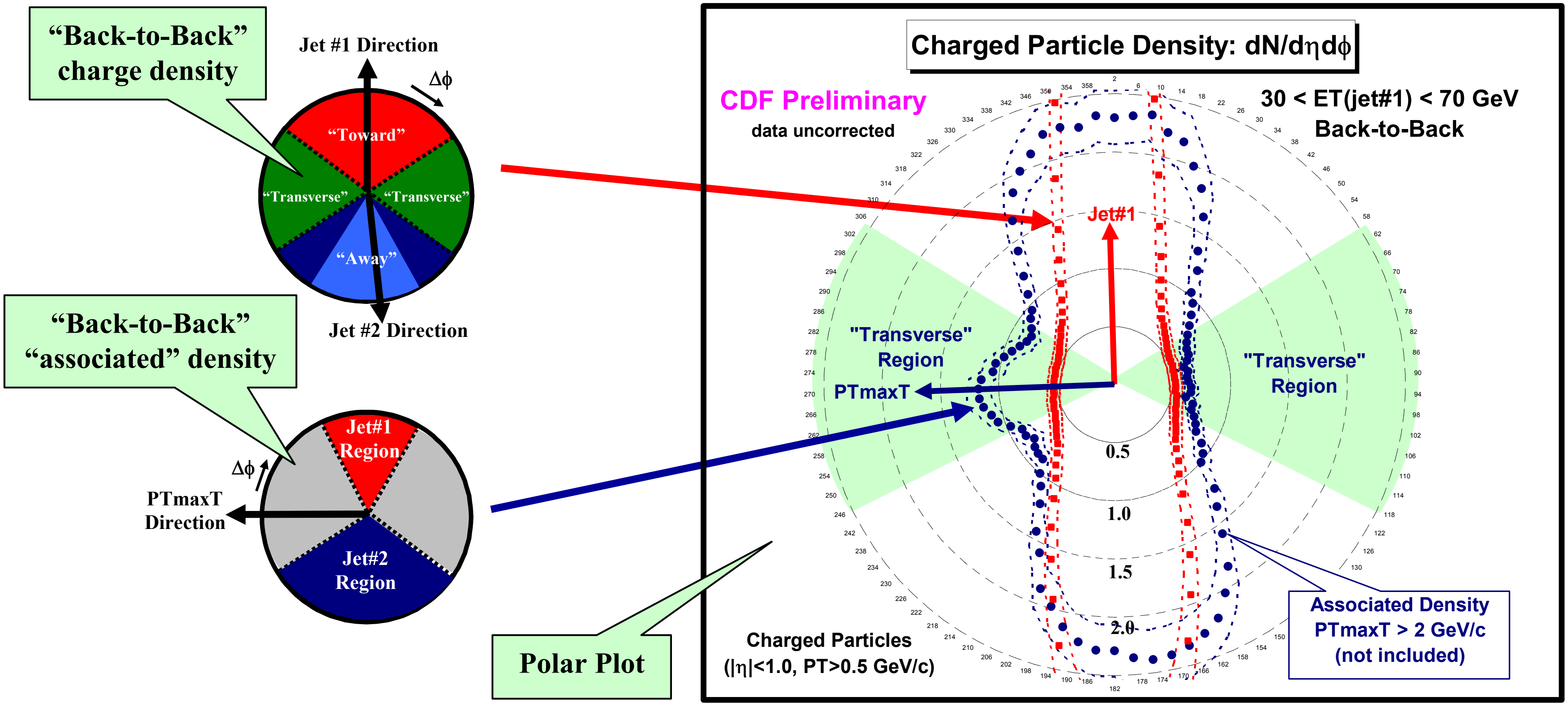
PYTHIA Tune A vs JIMMY: "Transverse Region"





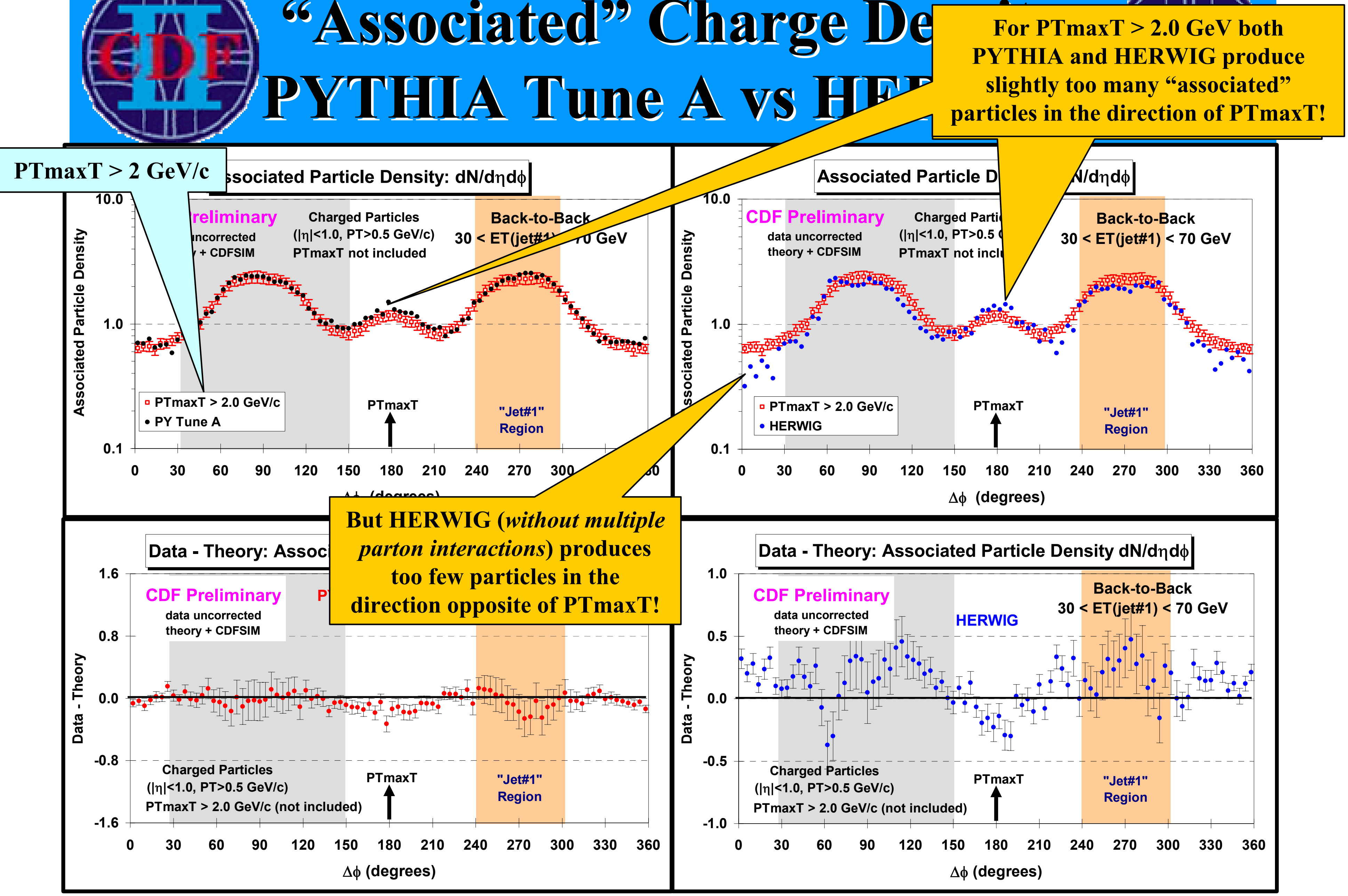
- (*left*) Run 2 data for charged *scalar* PTsum density (|η|<1, p_T>0.5 GeV/c) in the MAX/MIN/AVE "transverse" region versus P_T(jet#1) compared with PYTHIA Tune A (*after CDFSIM*).
- (*right*) Shows the generator level predictions of PYTHIA Tune A (*dashed*) and JIMMY ($P_T$min=1.8 GeV/c) for charged *scalar* PTsum density (|η|<1, p_T>0.5 GeV/c) in the MAX/MIN/AVE "transverse" region versus P_T(jet#1).
- The tuned JIMMY now agrees with PYTHIA for P_T(jet#1) < 100 GeV but produces much more activity than PYTHIA Tune A (and the data?) in the "transverse" region for P_T(jet#1) > 100 GeV!

Back-to-Back "Associated" Charged Particle Densities



- Shows the **Δϕ dependence of the "associated" charged particle density**, $dN_{chg}/d\eta d\phi$, $p_T > 0.5$ GeV/c, $|\eta| < 1$, PTmaxT > 2.0 GeV/c (*not including PTmaxT*) relative to PTmaxT (rotated to 180°) and the charged particle density, $dN_{chg}/d\eta d\phi$, $p_T > 0.5$ GeV/c, $|\eta| < 1$, relative to jet#1 (rotated to 270°) for **"back-to-back events" with $30 < E_T(\text{jet\#1}) < 70$ GeV**.

“Associated” Charge De
PYTHIA Tune A vs HE



CDF Run 1 $P_T(Z)$

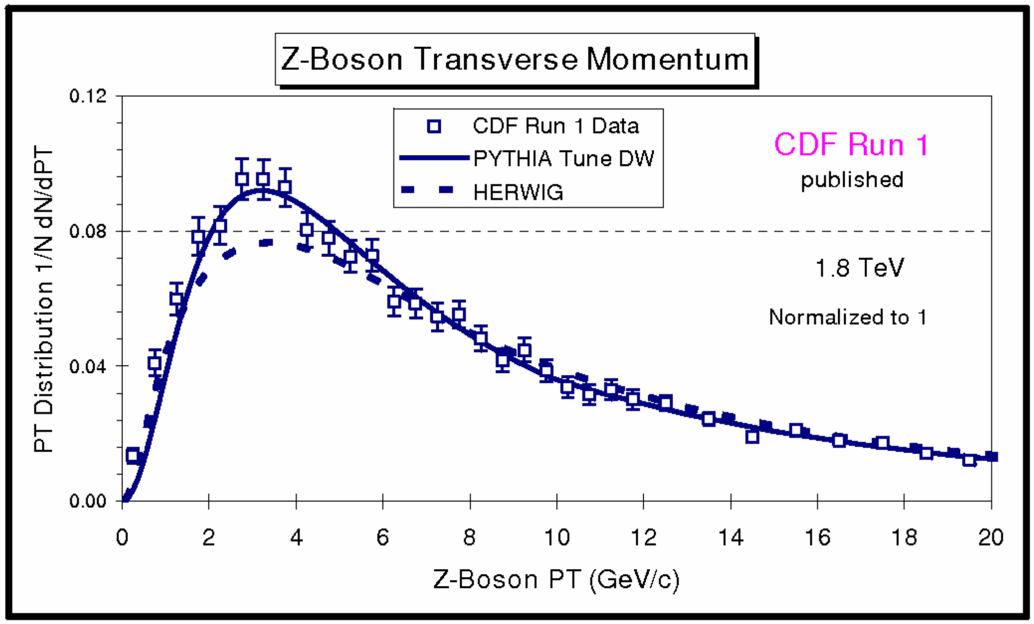
PYTHIA 6.2 CTEQ5L

UE Parameters

ISR Parameters

Intrensic KT

Parameter	Tune DW	Tune AW
MSTP(81)	1	1
MSTP(82)	4	4
PARP(82)	1.9 GeV	2.0 GeV
PARP(83)	0.5	0.5
PARP(84)	0.4	0.4
PARP(85)	1.0	0.9
PARP(86)	1.0	0.95
PARP(89)	1.8 TeV	1.8 TeV
PARP(90)	0.25	0.25
PARP(62)	1.25	1.25
PARP(64)	0.2	0.2
PARP(67)	2.5	4.0
MSTP(91)	1	1
PARP(91)	2.1	2.1
PARP(93)	15.0	[illegible]

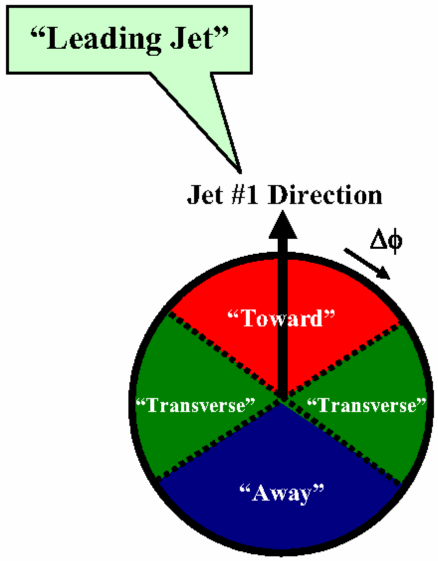


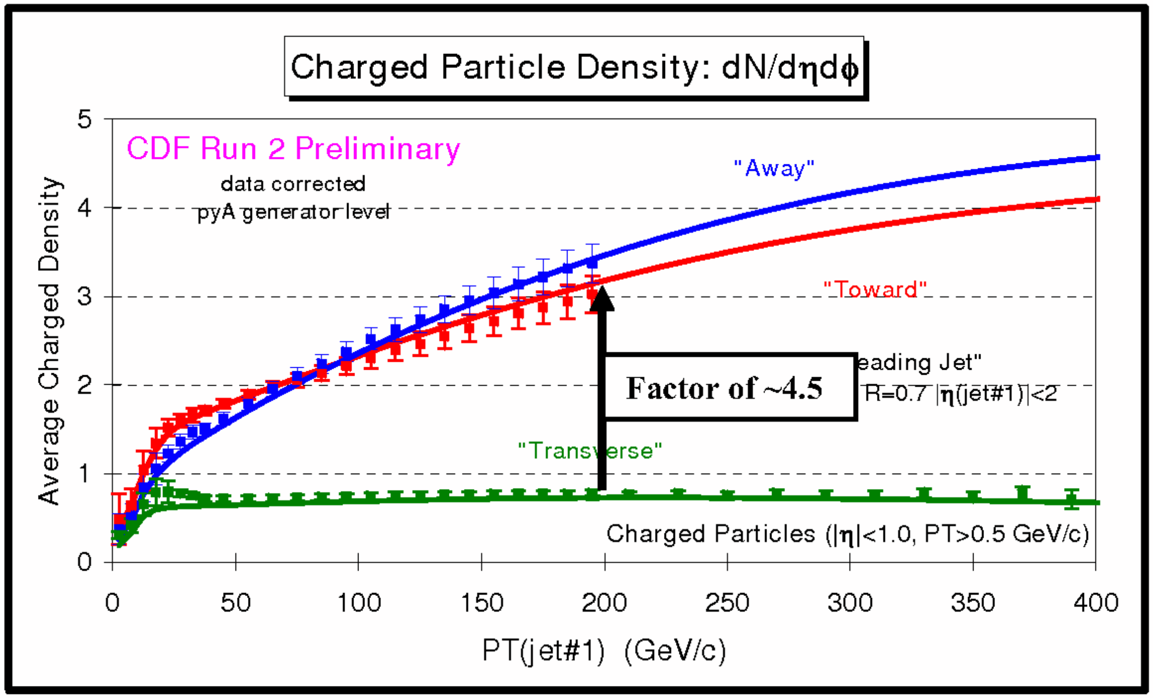
➡ **Shows the Run 1 Z-boson p_T distribution ($<p_T(Z)> \approx 11.5$ GeV/c) compared with PYTHIA Tune DW, and HERWIG.**

Tune DW uses D0's perfered value of PARP(67)!

Tune DW has a lower value of PARP(67) and slightly more MPI!

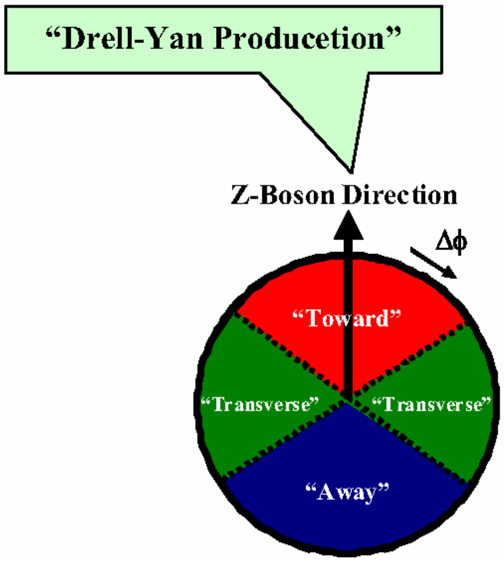
"Towards", "Away", "Transverse"

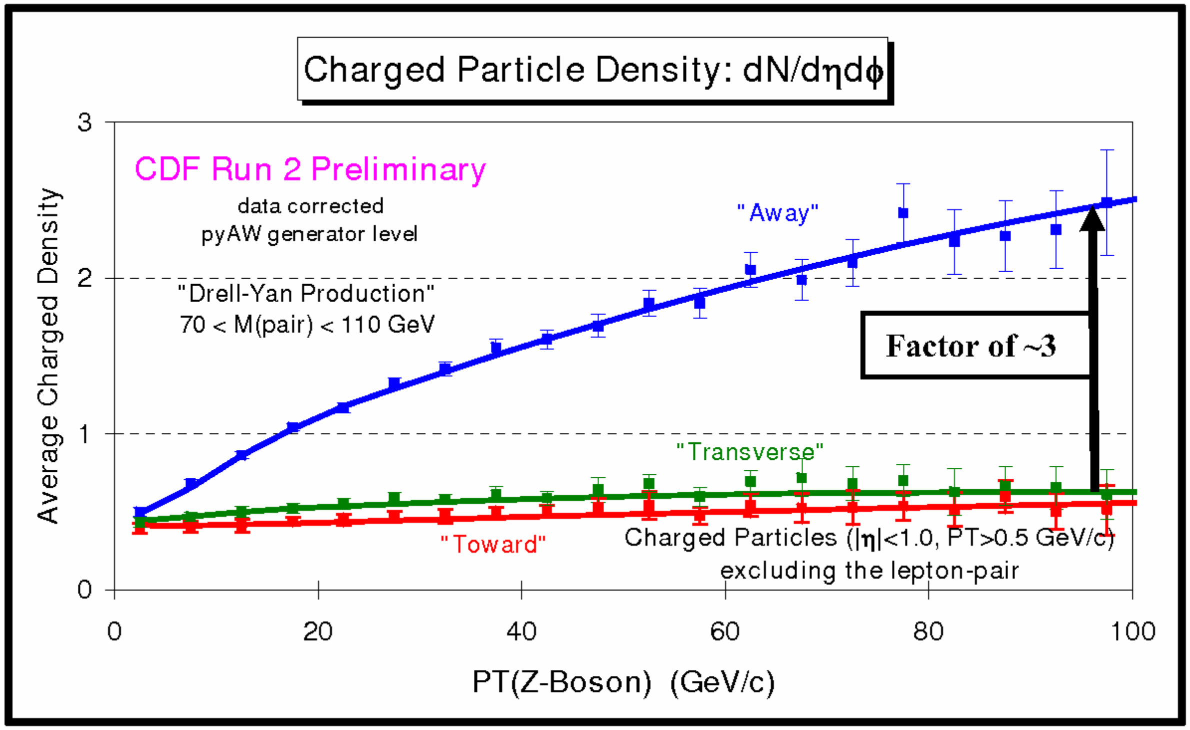




- Data at 1.96 TeV on the density of charged particles, dN/dηdφ, with p_T > 0.5 GeV/c and |η| < 1 for "leading jet" events as a function of the leading jet p_T for the "toward", "away", and "transverse" regions. The data are corrected to the particle level (*with errors that include both the statistical error and the systematic uncertainty*) and are compared with PYTHIA Tune A at the particle level (*i.e.* generator level).

"Towards", "Away", "Transverse"



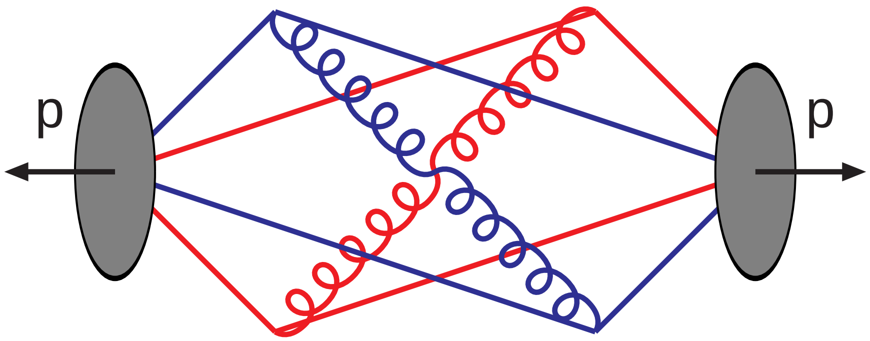


- **Data at 1.96 TeV on the density of charged particles, dN/dηdϕ, with $p_T > 0.5$ GeV/c and $|\eta| < 1$ for "Z-Boson" events as a function of the leading jet p_T for the "toward", "away", and "transverse" regions. The data are corrected to the particle level (*with errors that include both the statistical error and the systematic uncertainty*) and are compared with PYTHIA Tune AW at the particle level (*i.e.* generator level).**

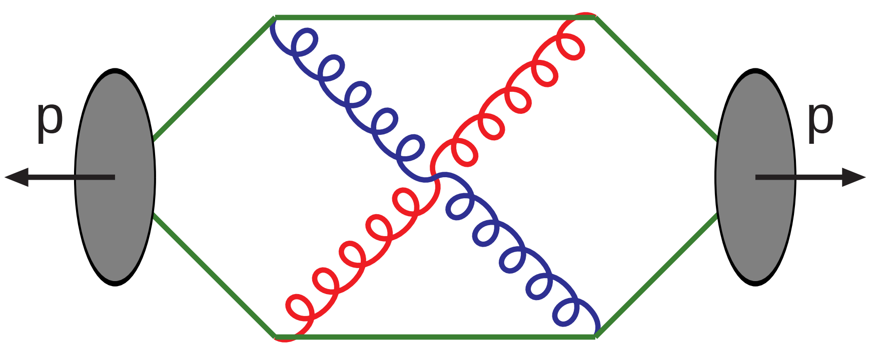
Deepak Kar's Thesis

Colour correlations

$\langle p_\perp \rangle (n_{\text{ch}})$ is very sensitive to colour flow



long strings to remnants $\Rightarrow$ much n_{ch}/interaction $\Rightarrow \langle p_\perp \rangle (n_{\text{ch}}) \sim$ flat



short strings (more central) $\Rightarrow$ less n_{ch}/interaction $\Rightarrow \langle p_\perp \rangle (n_{\text{ch}})$ rising

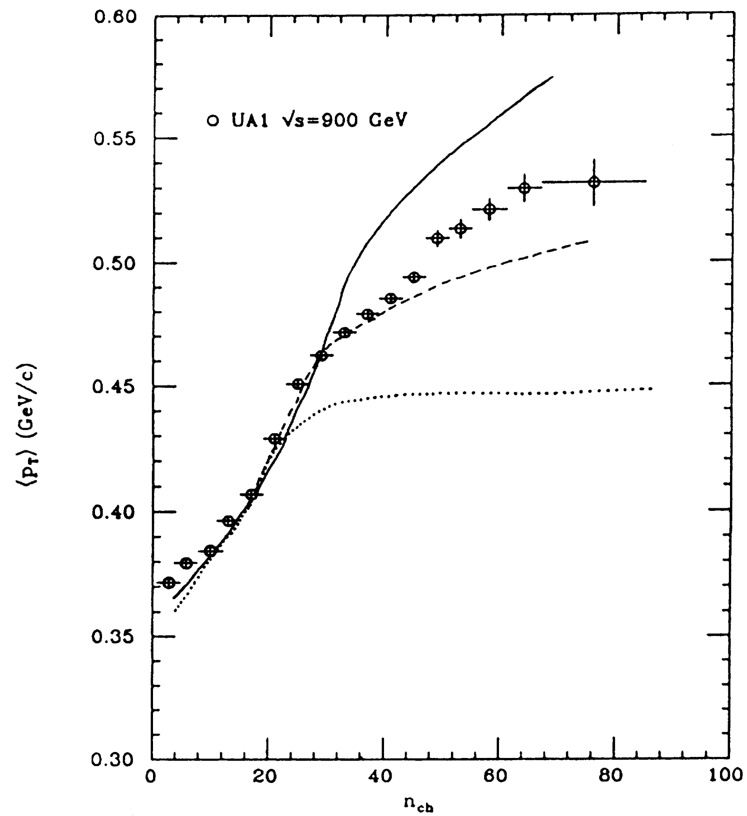
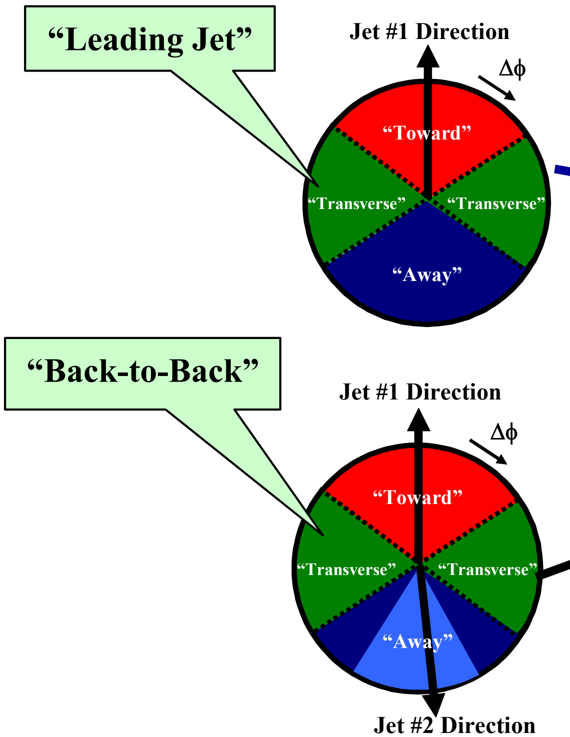
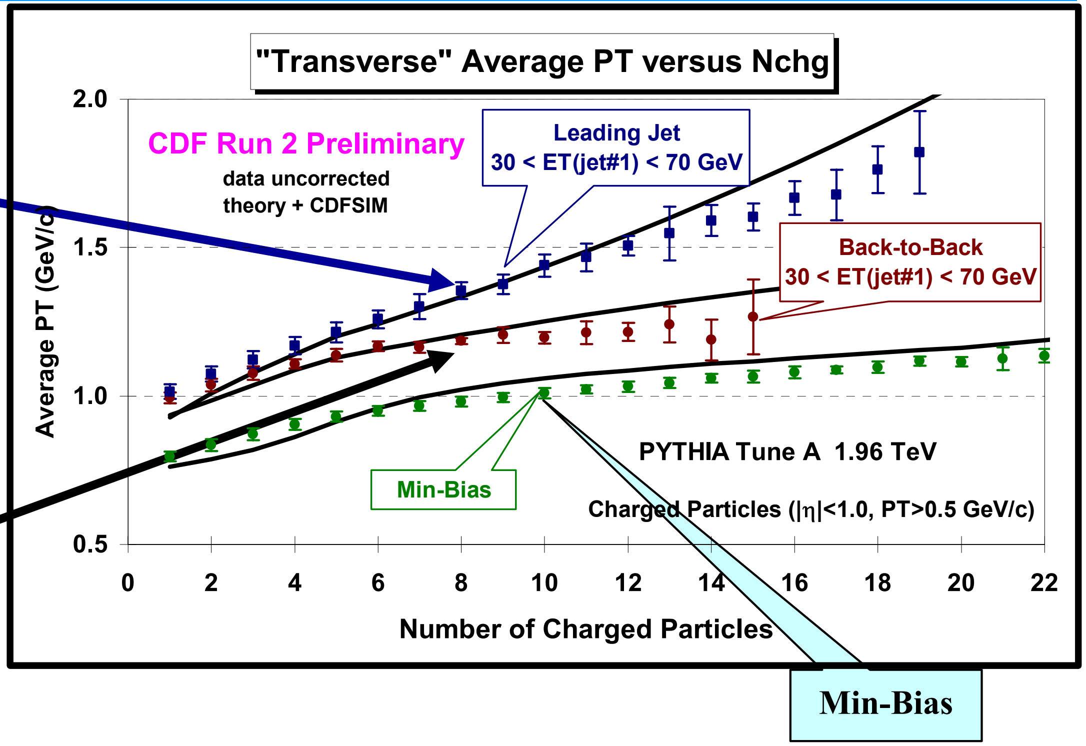


FIG. 27. Average transverse momentum of charged particles in $|\eta| < 2.5$ as a function of the multiplicity. UA1 data points (Ref. 49) at 900 GeV compared with the model for different assumptions about the nature of the subsequent (nonhardest) interactions. Dashed line, assuming $q\bar{q}$ scatterings only; dotted line, gg scatterings with "maximal" string length; solid line gg scatterings with "minimal" string length.

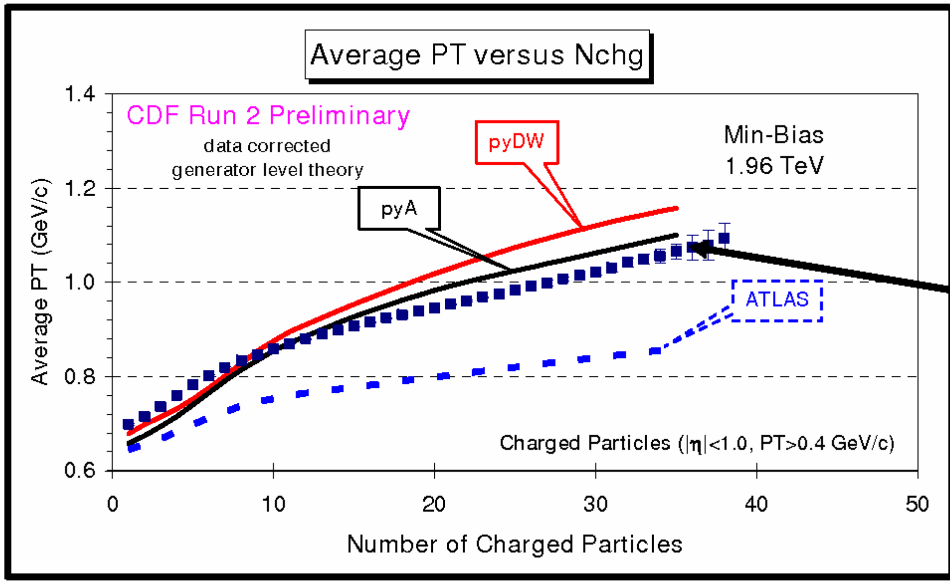
"Transverse" $<p_T>$ versus "Transverse" Nchg

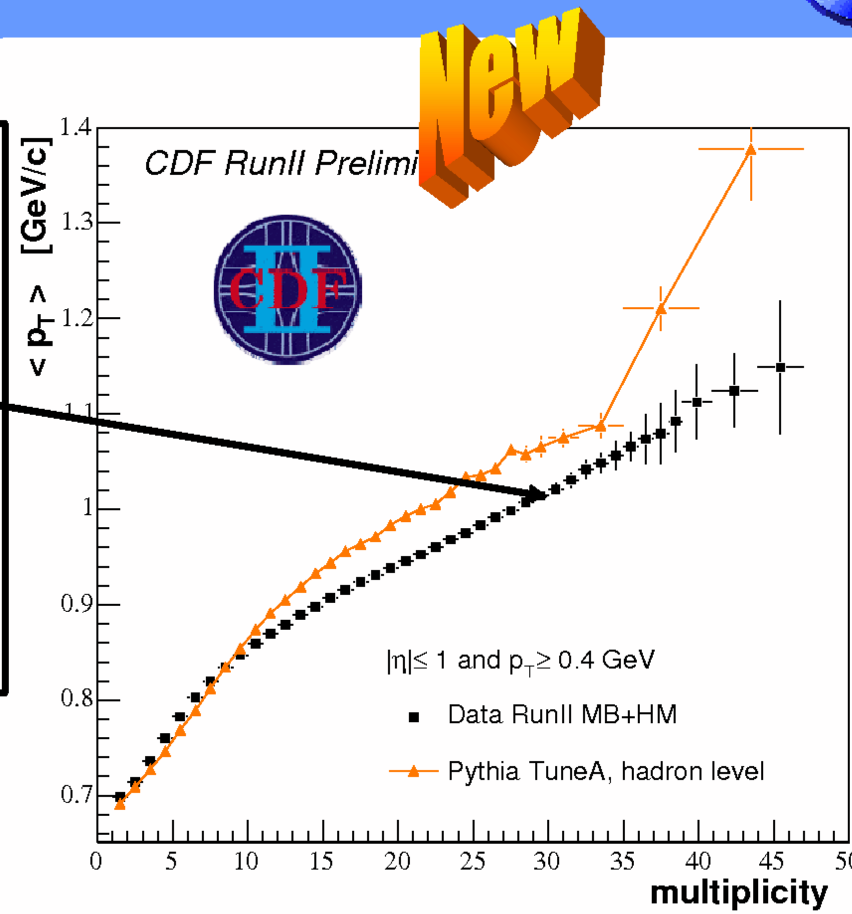




- Look at the $<p_T>$ of particles in the "transverse" region ($p_T > 0.5$ GeV/c, $|\eta| < 1$) versus the number of particles in the "transverse" region: $<p_T>$ vs Nchg.
- Shows $<p_T>$ versus Nchg in the "transverse" region ($p_T > 0.5$ GeV/c, $|\eta| < 1$) for "Leading Jet" and "Back-to-Back" events with $30 < E_T$(jet#1) < 70 GeV compared with "min-bias" collisions.

Min-Bias Correlations





- **Data at 1.96 TeV on the average p_T of charged particles versus the number of charged particles (p_T > 0.4 GeV/c, |η| < 1) for "min-bias" collisions at CDF Run 2. The data are corrected to the particle level and are compared with PYTHIA Tune A at the particle level (*i.e.* generator level).**

Multiple Interactions Outlook

Issues requiring further thought and study:

- Multi-parton PDF's $f_{a_1 a_2 a_3 \cdots}(x_1, Q_1^2, x_2, Q_2^2, x_3, Q_3^2, \ldots)$
- Close-packing in initial state, especially small x
- Impact-parameter picture and (x, b) correlations
  e.g. large-x partons more central!, valence quarks more central?
- Details of colour-screening mechanism
- Rescattering: one parton scattering several times
- Intertwining: one parton splits in two that scatter separately
- Colour sharing: two FS–IS dipoles become one FS–FS one
- Colour reconnection: required for $\langle p_{\perp} \rangle (n_{\mathrm{charged}})$
- Collective effects (e.g. QGP, cf. Hadronization above)
- Relation to diffraction: eikonalization, multi-gap topologies, ...

Action items:

- Vigorous experimental program at LHC
- Study energy dependence: RHIC (pp) $\longrightarrow$ Tevatron $\longrightarrow$ LHC
- Develop new frameworks and refine existing ones

Much work ahead!