Monte Carlo

1. Introduction and Parton Showers

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Course Plan

**Improve understanding of physics at the LHC**

Complementary to the “textbook” picture of particle physics, since event generators is close to how things work “in real life”.

**Lecture 1**  
Introduction and generator survey  
Parton showers: final and initial

**Lecture 2**  
Combining matrix elements and parton showers  
Multiparton interactions and other soft physics  
Hadronization  
Conclusions

**Tutorials**  
Use PYTHIA to study aspects of Higgs physics

Learn more:


also “PYTHIA 6.4 Physics and Manual”, JHEP05 (2006) 026
A tour to Monte Carlo

... because Einstein was wrong: God does throw dice!
Quantum mechanics: amplitudes \(\rightarrow\) probabilities
Anything that possibly can happen, will! (but more or less often)

Event generators: trace evolution of event structure.
Random numbers \(\approx\) quantum mechanical choices.
The structure of an event – 1

Warning: schematic only, everything simplified, nothing to scale, . . .

Incoming beams: parton densities
Hard subprocess: described by matrix elements
Resonance decays: correlated with hard subprocess
The structure of an event – 4

Initial-state radiation: spacelike parton showers
The structure of an event – 5

Final-state radiation: timelike parton showers
The structure of an event – 6

Multiple parton–parton interactions …
The structure of an event – 7

...with its initial- and final-state radiation
The structure of an event – 8

Beam remnants and other outgoing partons
The structure of an event – 9

Everything is connected by colour confinement strings
Recall! Not to scale: strings are of hadronic widths
The strings fragment to produce primary hadrons
The structure of an event – 11

Many hadrons are unstable and decay further
These are the particles that hit the detector
The Monte Carlo method

Want to generate events in as much detail as Mother Nature

\[ \implies \text{get average } \textit{and} \text{ fluctuations right} \]
\[ \implies \text{make random choices, } \sim \text{ as in nature} \]

\[ \sigma_{\text{final state}} = \sigma_{\text{hard process}} P_{\text{tot,hard process}} \rightarrow \text{final state} \]

(appropriately summed & integrated over non-distinguished final states)

where \( P_{\text{tot}} = P_{\text{res}} P_{\text{ISR}} P_{\text{FSR}} P_{\text{MPI}} P_{\text{remnants}} P_{\text{hadronization}} P_{\text{decays}} \)

with \( P_i = \prod_j P_{ij} = \prod_j \prod_k P_{ijk} = \ldots \) in its turn

\[ \implies \text{divide and conquer} \]

an event with \( n \) particles involves \( \mathcal{O}(10^n) \) random choices,

(flavour, mass, momentum, spin, production vertex, lifetime, \ldots)

LHC: \( \sim 100 \) charged and \( \sim 200 \) neutral (+ intermediate stages)

\[ \implies \text{several thousand choices} \]

(of \( \mathcal{O}(100) \) different kinds)
Event Generator Position

```
Machine ⇒ events
LHC

Detector, Data Acquisition
ATLAS, CMS, LHC-B, ALICE

Event Reconstruction
CMSSW, ATHENA

Physics Analysis
ROOT, FastJet

conclusions, articles, talks, ...
```

“real life”

produce events

“virtual reality”

observe & store events

what is knowable?

compare real and simulated data

“quick and dirty”

Rivet

Event Generator
PYTHIA, HERWIG

Detector Simulation
Geant4, LCG
Why generators?

- Allow theoretical and experimental studies of complex multiparticle physics
- Large flexibility in physical quantities that can be addressed
- Vehicle of ideology to disseminate ideas from theorists to experimentalists

Can be used to

- predict event rates and topologies
  ⇒ can estimate feasibility
- simulate possible backgrounds
  ⇒ can devise analysis strategies
- study detector requirements
  ⇒ can optimize detector/trigger design
- study detector imperfections
  ⇒ can evaluate acceptance corrections
HERWIG, PYTHIA and SHERPA offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

PYTHIA (successor to JETSET, begun in 1978):
- originated in hadronization studies: the Lund string
- leading in development of MPI for MB/UE
- pragmatic attitude to showers & matching

HERWIG (successor to EARWIG, begun in 1984):
- originated in coherent-shower studies (angular ordering)
- cluster hadronization & underlying event pragmatic add-on
- large process library with spin correlations in decays

SHERPA (APACIC++/AMEGIC++, begun in 2000):
- own matrix-element calculator/generator
- extensive machinery for CKKW ME/PS matching
- hadronization & min-bias physics under development

PYTHIA and HERWIG originally in Fortran, but now all in C++. 
MCnet projects:
- PYTHIA (+ VINCIA)
- HERWIG
- SHERPA
- MadGraph
- Ariadne (+ DIPSY)
- Cedar (Rivet/Professor)

Activities include
- summer schools
- short-term studentships
- graduate students
- postdocs
- meetings (open/closed)

3-6 month fully funded studentships for current PhD students at one of the MCnet nodes. An excellent opportunity to really understand and improve the Monte Carlos you use!

Application rounds every 3 months.

for details go to: www.montecarlonet.org
Other Relevant Software

Some examples (with apologies for many omissions):

- **Other event/shower generators**: PhoJet, Ariadne, Dipsy, Cascade, Vincia
- **Matrix-element generators**: MadGraph/MadEvent, CompHep, CalcHep, Helac, Whizard, Sherpa, GoSam, aMC@NLO
- **Matrix element libraries**: AlpGen, POWHEG BOX, MCFM, NLOjet++, VBFNLO, BlackHat, Rocket
- **Special BSM scenarios**: Prospino, Charybdis, TrueNoir
- **Mass spectra and decays**: SOFTSUSY, SPHENO, HDelay, SDecay
- **Feynman rule generators**: FeynRules
- **PDF libraries**: LHAPDF
- **Resummed ($p_T$) spectra**: ResBos
- **Approximate loops**: LoopSim
- **Jet finders**: anti-$k_T$ and FastJet
- **Analysis packages**: Rivet, Professor, MCPLLOTS
- **Detector simulation**: GEANT, Delphes
- **Constraints (from cosmology etc)**: DarkSUSY, MicrOmegas
- **Standards**: PDF identity codes, LHA, LHEF, SLHA, Binoth LHA, HepMC

**Can be meaningfully combined and used for LHC physics!**
Putting it together

Standardized interfaces essential!

...but wide range of possible processes, some with special quirks.
Multijets – the need for showers

6 Jet Event in 7 TeV Collisions

An event with 6 jets taken on April 4th, 2010. The jets have calibrated transverse momenta between 30 GeV and 70 GeV and are well separated in the detector.

Basic $2 \rightarrow 2$ process dressed up by bremsstrahlung!?
Order-by-order calculations: challenges more math than physics.

### Availability of exact calculations (hadron colliders)

- Fixed order matrix elements ("parton level") are exact to a given perturbative order.
  (and often quite a pain!)
- Important to understand limitations:
  Only tree-level fully automated, 1-loop-level ongoing.

(courtesy Frank Krauss)
Order-by-order calculations: challenges more math than physics.

- **LO**: solved for all practical applications.
- **NLO**: in process of being automatized.
- **NNLO**: the current calculational frontier.
- Another bottleneck: efficient phase space sampling.

\[ gg \rightarrow H^0 \] illustrates problems:

- Need high-precision calculations
- to search for BSM physics,
- but limited by poorly-understood slow convergence.

Perturbative calculations reliable for well separated jets, but . . .
Divergences

Emission rate \( q \rightarrow qg \) diverges when
- collinear: opening angle \( \theta_{qg} \rightarrow 0 \)
- soft: gluon energy \( E_g \rightarrow 0 \)

Almost identical to \( e \rightarrow e\gamma \)
but QCD is non-Abelian so additionally
- \( g \rightarrow gg \) similarly divergent
- \( \alpha_s(Q^2) \) diverges for \( Q^2 \rightarrow 0 \)
  (actually for \( Q^2 \rightarrow \Lambda_{QCD}^2 \))

Big probability for one emission \( \implies \) also big for several.

With ME’s need to calculate to high order and with many loops
\( \implies \) extremely demanding technically (not solved!), and involving big cancellations between positive and negative contributions.

Alternative approach: parton showers
The Parton-Shower Approach

\[ 2 \rightarrow n = (2 \rightarrow 2) \oplus \text{ISR} \oplus \text{FSR} \]

FSR = Final-State Radiation = timelike shower
\[ Q_i^2 \sim m^2 > 0 \text{ decreasing} \]

ISR = Initial-State Radiation = spacelike showers
\[ Q_i^2 \sim -m^2 > 0 \text{ increasing} \]
Why “time” like and “space” like?

Consider four-momentum conservation in a branching $a \to b \, c$

$$p_{\perp a} = 0 \implies p_{\perp c} = -p_{\perp b}$$

$$p_+ = E + p_L \implies p_{+a} = p_{+b} + p_{+c}$$

$$p_- = E - p_L \implies p_{-a} = p_{-b} + p_{-c}$$

Define $p_{+b} = z \, p_{+a}$, $p_{+c} = (1 - z) \, p_{+a}$

Use $p_+ p_- = E^2 - p_L^2 = m^2 + p_{\perp}^2$

$$\frac{m_{a}^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_{b}^2 + p_{\perp b}^2}{z \, p_{+a}} + \frac{m_{c}^2 + p_{\perp c}^2}{(1 - z) \, p_{+a}}$$

$$\implies m_{a}^2 = \frac{m_{b}^2 + p_{\perp}^2}{z} + \frac{m_{c}^2 + p_{\perp}^2}{1 - z} = \frac{m_{b}^2}{z} + \frac{m_{c}^2}{1 - z} + \frac{p_{\perp}^2}{z(1 - z)}$$

Final-state shower: $m_b = m_c = 0 \implies m_{a}^2 = \frac{p_{\perp}^2}{z(1 - z)} > 0 \implies $timelike

Initial-state shower: $m_a = m_c = 0 \implies m_{b}^2 = -\frac{p_{\perp}^2}{1 - z} < 0 \implies $spacelike
Shower evolution is viewed as a probabilistic process, which occurs with unit total probability: *the cross section is not directly affected*

However, more complicated than that

- PDF evolution \( \approx \) showers \( \Rightarrow \) enters in convoluted cross section, e.g. for \( 2 \to 2 \) processes

\[
\sigma = \int \int \int dx_1 \, dx_2 \, d\hat{t} \, f_i(x_1, Q^2) \, f_j(x_2, Q^2) \frac{d\hat{\sigma}_{ij}}{d\hat{t}}
\]

- Shower affects event shape
  - E.g. start from 2-jet event with \( p_{\perp 1} = p_{\perp 2} = 100 \text{ GeV} \).
  - ISR gives third jet, plus recoil to existing two, so
    \( p_{\perp 1} = 110 \text{ GeV}, \, p_{\perp 2} = 90 \text{ GeV}, \, p_{\perp 1} = 20 \text{ GeV} \):
      - inclusive \( p_{\perp \text{jet}} \) spectrum goes up
      - hardest \( p_{\perp \text{jet}} \) spectrum goes up
      - two-jets with both jets above some \( p_{\perp \text{min}} \) comes down
      - three-jet rate goes up
Doublecounting

A $2 \rightarrow n$ graph can be “simplified” to $2 \rightarrow 2$ in different ways:

$g \rightarrow q\bar{q} \oplus qg \rightarrow qg$

$g \rightarrow gg \oplus gg \rightarrow q\bar{q}$

Do not doublecount: $2 \rightarrow 2 = $ most virtual = shortest distance

Conflict: theory derivations assume virtualities strongly ordered; interesting physics often in regions where this is not true!
Final-state radiation

Standard process $e^+e^- \rightarrow q\bar{q}g$ by two Feynman diagrams:

\[
x_i = \frac{2E_i}{E_{cm}}
\]
\[
x_1 + x_2 + x_3 = 2
\]

\[
\frac{d\sigma_{ME}}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} \, dx_1 \, dx_2
\]

Convenient (but arbitrary) subdivision to “split” radiation:

\[
\frac{1}{(1-x_1)(1-x_2)} \frac{(1-x_1) + (1-x_2)}{x_3} = \frac{1}{(1-x_2)x_3} + \frac{1}{(1-x_1)x_3}
\]
Rewrite for $x_2 \to 1$, i.e. $q\!-\!g$ collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\text{cm}}^2} = \frac{Q^2}{E_{\text{cm}}^2} \Rightarrow dx_2 = \frac{dQ^2}{E_{\text{cm}}^2}$$

define $z$ as fraction $q$ retains in branching $q \to qg$

$$x_1 \approx z \Rightarrow dx_1 \approx dz$$

$$x_3 \approx 1 - z$$

$$\Rightarrow d\mathcal{P} = \frac{d\sigma}{\sigma_0} = \frac{\alpha_s}{2\pi} \frac{dx_2}{(1 - x_2)} \frac{4}{3} \frac{x_2^2 + x_1^2}{(1 - x_1)} dx_1 \approx \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} \frac{4}{3} \frac{1 + z^2}{1 - z} dz$$

In limit $x_1 \to 1$ same result, but for $\bar{q} \to \bar{q}g$.

$$dQ^2/Q^2 = dm^2/m^2: \text{“mass (or collinear) singularity”}$$

$$dz/(1 - z) = d\omega/\omega \text{ “soft singularity”}$$
The DGLAP equations

Generalizes to

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

\[
\begin{align*}
\frac{d}{dz} \mathcal{P}_{a\to bc} &= \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) \; dz \\
P_{q\to qg} &= \frac{4}{3} \frac{1 + z^2}{1 - z} \\
P_{g\to gg} &= 3 \left(1 - z(1 - z)\right)^2 \frac{1}{z(1 - z)} \\
P_{g\to q\bar{q}} &= \frac{n_f}{2} \left(z^2 + (1 - z)^2\right) \quad (n_f = \text{no. of quark flavours})
\end{align*}
\]

Universality: any matrix element reduces to DGLAP in collinear limit.

e.g. \[
\frac{d\sigma(H^0 \to q\bar{q}g)}{d\sigma(H^0 \to q\bar{q})} = \frac{d\sigma(Z^0 \to q\bar{q}g)}{d\sigma(Z^0 \to q\bar{q})} \quad \text{in collinear limit}
\]
The iterative structure

Generalizes to many consecutive emissions if strongly ordered,
$Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots \ (\approx \text{time-ordered}).$

To cover “all” of phase space use DGLAP in whole region
$Q_1^2 > Q_2^2 > Q_3^2 \ldots .

Iteration gives
final-state
parton showers:

Need soft/collinear cuts to stay away from nonperturbative physics.
Details model-dependent, but around 1 GeV scale.
The ordering variable

In the evolution with

\[ d\mathcal{P}_{a\rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\rightarrow bc}(z) \, dz \]

\( Q^2 \) orders the emissions (memory).

If \( Q^2 = m^2 \) is one possible evolution variable then \( Q'^2 = f(z)Q^2 \) is also allowed, since

\[
\left| \frac{d(Q'^2, z)}{d(Q^2, z)} \right| = \left| \begin{array}{cc} \frac{\partial Q'^2}{\partial Q^2} & \frac{\partial Q'^2}{\partial z} \\ \frac{\partial Q^2}{\partial z} & \frac{\partial Q^2}{\partial z} \end{array} \right| = \left| \begin{array}{cc} f(z) & f'(z)Q^2 \\ 0 & 1 \end{array} \right| = f(z)
\]

\[ \Rightarrow d\mathcal{P}_{a\rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{f(z) dQ^2}{f(z)Q^2} P_{a\rightarrow bc}(z) \, dz = \frac{\alpha_s}{2\pi} \frac{dQ'^2}{Q'^2} P_{a\rightarrow bc}(z) \, dz 
\]

- \( Q'^2 = E_a^2 \theta_{a\rightarrow bc}^2 \approx m^2/(z(1-z)); \) angular-ordered shower
- \( Q'^2 = p_\perp^2 \approx m^2z(1-z); \) transverse-momentum-ordered
Time evolution, conservation of total probability:
\[ \mathcal{P}(\text{no emission}) = 1 - \mathcal{P}(\text{emission}). \]

Multiplicativeness, with \( T_i = (i/n)T, \ 0 \leq i \leq n): \]
\[
\mathcal{P}_{\text{no}}(0 \leq t < T) = \lim_{n \to \infty} \prod_{i=0}^{n-1} \mathcal{P}_{\text{no}}(T_i \leq t < T_{i+1})
\]
\[
= \lim_{n \to \infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\text{em}}(T_i \leq t < T_{i+1}))
\]
\[
= \exp \left( - \lim_{n \to \infty} \sum_{i=0}^{n-1} \mathcal{P}_{\text{em}}(T_i \leq t < T_{i+1}) \right)
\]
\[
= \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{em}}(t)}{dt} dt \right)
\]
\[
\implies d\mathcal{P}_{\text{first}}(T) = d\mathcal{P}_{\text{em}}(T) \exp \left( - \int_0^T \frac{d\mathcal{P}_{\text{em}}(t)}{dt} dt \right)
\]
The Sudakov form factor – 2

Expanded, with $Q \sim 1/t$ (Heisenberg)

$$d\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\to bc}(z) \, dz$$

$$\times \exp \left( - \sum_{b,c} \int_{Q^2}^{Q_{\text{max}}^2} \frac{dQ'^2}{Q'^2} \int \frac{\alpha_s}{2\pi} P_{a\to bc}(z') \, dz' \right)$$

where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c} \int \int d\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow \text{convenient for Monte Carlo}$

($\equiv 1$ if extended over whole phase space, else possibly nothing happens before you reach $Q_0 \approx 1$ GeV).
The Sudakov form factor – 3

Sudakov regulates singularity for *first* emission . . .

. . . but in limit of *repeated soft* emissions $q \to qg$ (but no $g \to gg$) one obtains the same inclusive $Q$ emission spectrum as for ME, i.e. divergent ME spectrum $\iff$ infinite number of PS emissions

More complicated in reality:

- energy-momentum conservation effects big since $\alpha_s$ big, so hard emissions frequent
- $g \to gg$ branchings leads to accelerated multiplication of partons
Coherence

QED: Chudakov effect (mid-fifties)

QCD: colour coherence for soft gluon emission

solved by • requiring emission angles to be decreasing
or • requiring transverse momenta to be decreasing
Common Showering Algorithms

Standard shower language with $a \rightarrow bc$ successive branchings:

HERWIG: $Q^2 \approx E^2(1 - \cos \theta) \approx E^2\theta^2/2$

old PYTHIA: $Q^2 = m^2$ (+ brute-force coherence)

Newer ARIADNE picture of dipole emission $ab \rightarrow cde$:

is the basis for most current-day algorithms (HERWIG excepted)
Hadrons are composite, with time-dependent structure:

\[ f_i(x, Q^2) = \text{number density of partons } i \]

at momentum fraction \( x \) and probing scale \( Q^2 \).

Linguistics (example):

\[ F_2(x, Q^2) = \sum_i e_i^2 \, x f_i(x, Q^2) \]

structure function \quad parton distributions
PDF evolution

Initial conditions at small $Q_0^2$ unknown: nonperturbative.
Resolution dependence perturbative, by DGLAP:

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

\[
\frac{df_b(x, Q^2)}{d(\ln Q^2)} = \sum_a \int_x^1 \frac{dz}{z} f_a(y, Q^2) \frac{\alpha_s}{2\pi} P_{a \rightarrow bc} \left( z = \frac{x}{y} \right)
\]

DGLAP already introduced for (final-state) showers:

\[
dP_{a \rightarrow bc} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a \rightarrow bc}(z) \, dz
\]

Same equation, but different context:

- $dP_{a \rightarrow bc}$ is probability for the individual parton to branch; while
- $df_b(x, Q^2)$ describes how the ensemble of partons evolve by the branchings of individual partons as above.
• Parton cascades in $p$ are continuously born and recombined.
• Structure at $Q$ is resolved at a time $t \sim 1/Q$ before collision.
• A hard scattering at $Q^2$ probes fluctuations up to that scale.
• A hard scattering inhibits full recombination of the cascade.

• Convenient reinterpretation:

\[
\begin{align*}
m^2 &= 0 & m^2 &= 0 & Q^2 &= -m^2 > 0 \\
m^2 &< 0 & m^2 &> 0 & \text{and increasing}
\end{align*}
\]
Event generation could be addressed by **forwards evolution**: pick a complete partonic set at low $Q_0$ and evolve, consider collisions at different $Q^2$ and pick by $\sigma$ of those.

**Inefficient:**

1. have to evolve and check for *all* potential collisions, but 99.9...% inert
2. impossible (or at least very complicated) to steer the production, e.g. of a narrow resonance (Higgs)

**Backwards evolution** is viable and ~equivalent alternative: start at hard interaction and trace what happened “before”
Monte Carlo approach, based on *conditional probability*: recast

\[
\frac{df_b(x, Q^2)}{dt} = \sum_a \int_x^1 \frac{dz}{z} f_a(x', Q^2) \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z)
\]

with \( t = \ln\left(\frac{Q^2}{\Lambda^2}\right) \) and \( z = \frac{x}{x'} \) to

\[
d\mathcal{P}_b = \frac{df_b}{f_b} = |dt| \sum_a \int dz \frac{x'f_a(x', t)}{xf_b(x, t)} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z)
\]

then solve for decreasing \( t \), i.e. backwards in time, starting at high \( Q^2 \) and moving towards lower, with Sudakov form factor \( \exp\left(-\int d\mathcal{P}_b\right) \).

Webber: can be recast by noting that total change of PDF at \( x \) is difference between gain by branchings from higher \( x \) and loss by branchings to lower \( x \).
Coherence in spacelike showers

with $Q^2 = -m^2 = \text{spacelike virtuality}$

- **kinematics only:**
  \[ Q_3^2 > z_1 Q_1^2, \quad Q_5^2 > z_3 Q_3^2, \ldots \]
  i.e. $Q_i^2$ need not even be ordered

- **coherence of leading collinear singularities:**
  \[ Q_5^2 > Q_3^2 > Q_1^2, \] i.e. $Q^2$ ordered

- **coherence of leading soft singularities (more messy):**
  \[ E_3 \theta_4 > E_1 \theta_2, \] i.e. $z_1 \theta_4 > \theta_2$
  \[ z \ll 1: \quad E_1 \theta_2 \approx p_{\perp 2}^2 \approx Q_3^2, \quad E_3 \theta_4 \approx p_{\perp 4}^2 \approx Q_5^2 \]
  i.e. reduces to $Q^2$ ordering as above
  \[ z \approx 1: \quad \theta_4 > \theta_2, \] i.e. angular ordering of soft gluons
  $\implies$ reduced phase space
Evolution procedures

DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi
evolution towards larger $Q^2$ and (implicitly) towards smaller $x$
BFKL: Balitsky–Fadin–Kuraev–Lipatov
evolution towards smaller $x$ (with small, unordered $Q^2$)
CCFM: Ciafaloni–Catani–Fiorani–Marchesini
interpolation of DGLAP and BFKL
GLR: Gribov–Levin–Ryskin
nonlinear equation in dense-packing (saturation) region, where partons recombine, not only branch
Did we reach BFKL regime?

Study events with $\geq 2$ jets as a function of their $y$ separation.

Ratio of the inclusive to exclusive dijet cross sections:

Azimuthal decorrelation:

No strong indications for BFKL/CCFM behaviour onset so far!
Initial- vs. final-state showers

Both controlled by same evolution equations

\[
\frac{d\mathcal{P}_{a\rightarrow bc}}{dQ^2} = \frac{\alpha_s}{2\pi} \frac{dQ^2}{Q^2} P_{a\rightarrow bc}(z) dz \cdot (\text{Sudakov})
\]

but

Final-state showers:
- \(Q^2\) timelike (\(\sim m^2\))
- Decreasing \(E, m^2, \theta\)
- Both daughters \(m^2 \geq 0\)
- Physics relatively simple
  \[\Rightarrow \text{"minor" variations:}\]
  - \(Q^2\), shower vs. dipole, ...

Initial-state showers:
- \(Q^2\) spacelike (\(\approx -m^2\))
- Decreasing \(E\), increasing \(Q^2, \theta\)
- One daughter \(m^2 \geq 0\), one \(m^2 < 0\)
- Physics more complicated
  \[\Rightarrow \text{more formalisms:}\]
  - DGLAP, BFKL, CCFM, GLR, ...
Combining FSR with ISR

Separate processing of ISR and FSR misses interference (∼ colour dipoles)

ISR + FSR add coherently in regions of colour flow and destructively else in “normal” shower by azimuthal anisotropies automatic in dipole (by proper boosts)
Coherence tests

Current-day generators for pseudorapidity of third jet:

Pseudorapidity, $\eta$, of 3rd jet

and past incoherent:
Summary and Outlook

- A multitude of physics mechanisms at play in pp collisions.
- Event generators separate problem into manageable chunks.
- Random numbers $\approx$ quantum mechanical choices.
- Often need to combine several software packages.
- Matrix element calculations at core of process selection.
- Parton shower offers convenient alternative to HO ME’s.
- Unitarity by Sudakov form factor.

Tutorial today: begin using PYTHIA; Higgs production as example.

Tomorrow:
- Combining matrix elements and parton showers.
- Multiparton interactions and other soft physics.
- Hadronization.
- Conclusions.