Monte Carlo
1. Introduction and Parton Showers

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TRIUMF, Vancouver, Canada, 4 July 2013

Course Plan

Improve understanding of physics at the LHC

Complementary to the "textbook" picture of particle physics, since event generators is close to how things work "in real life".

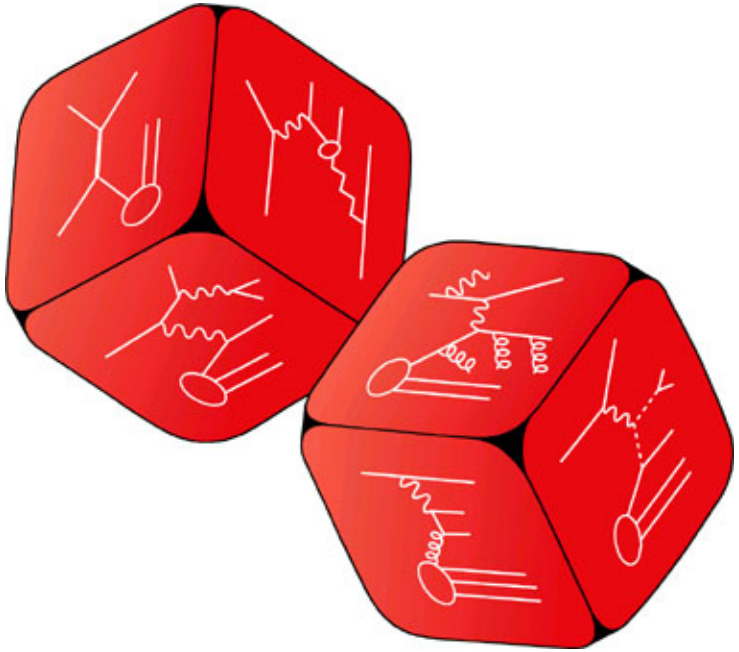
Lecture 1	Introduction and generator survey
	Parton showers: final and initial
Lecture 2	Combining matrix elements and parton showers
	Multiparton interactions and other soft physics
	Hadronization
	Conclusions
Tutorials	Use PYTHIA to study aspects of Higgs physics

Learn more:

A. Buckley et al., "General-purpose event generators for LHC physics", Phys. Rep. 504 (2011) 145 [arXiv:1101.2599[hep-ph]];

also "PYTHIA 6.4 Physics and Manual", JHEP05 (2006) 026

A tour to Monte Carlo



. . . because Einstein was wrong: God does throw dice!
Quantum mechanics: amplitudes $\Longrightarrow$ probabilities
Anything that possibly can happen, will! (but more or less often)

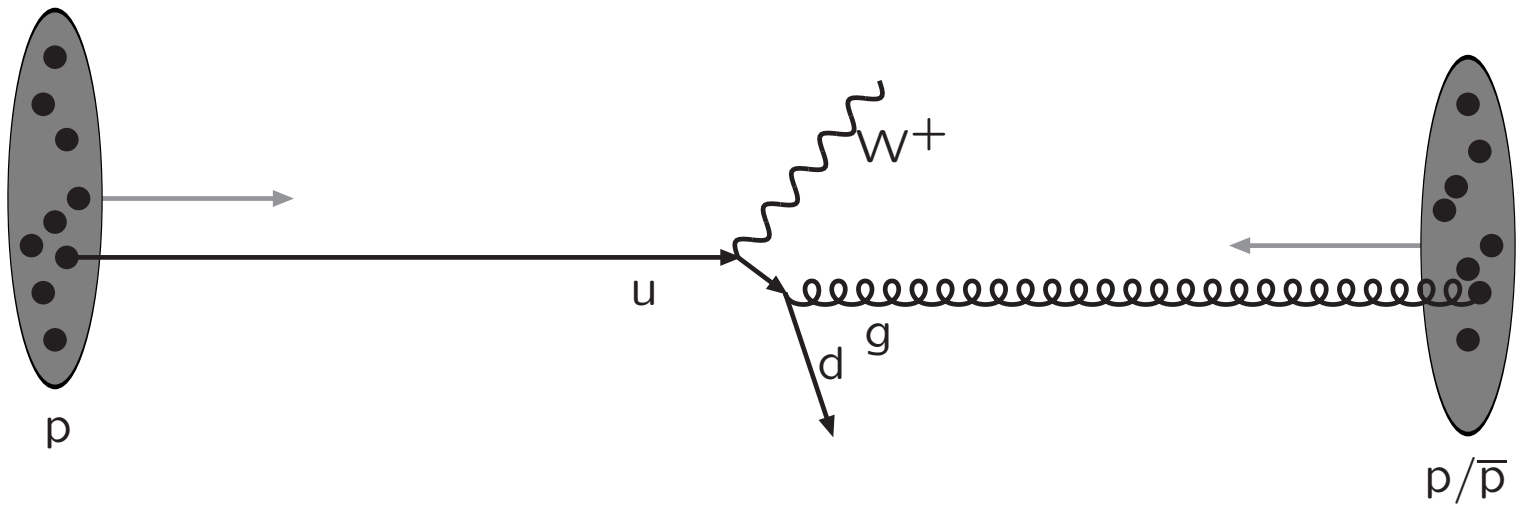
Event generators: trace evolution of event structure.
Random numbers $\approx$ quantum mechanical choices.

The structure of an event – 1

Warning: schematic only, everything simplified, nothing to scale, . . .

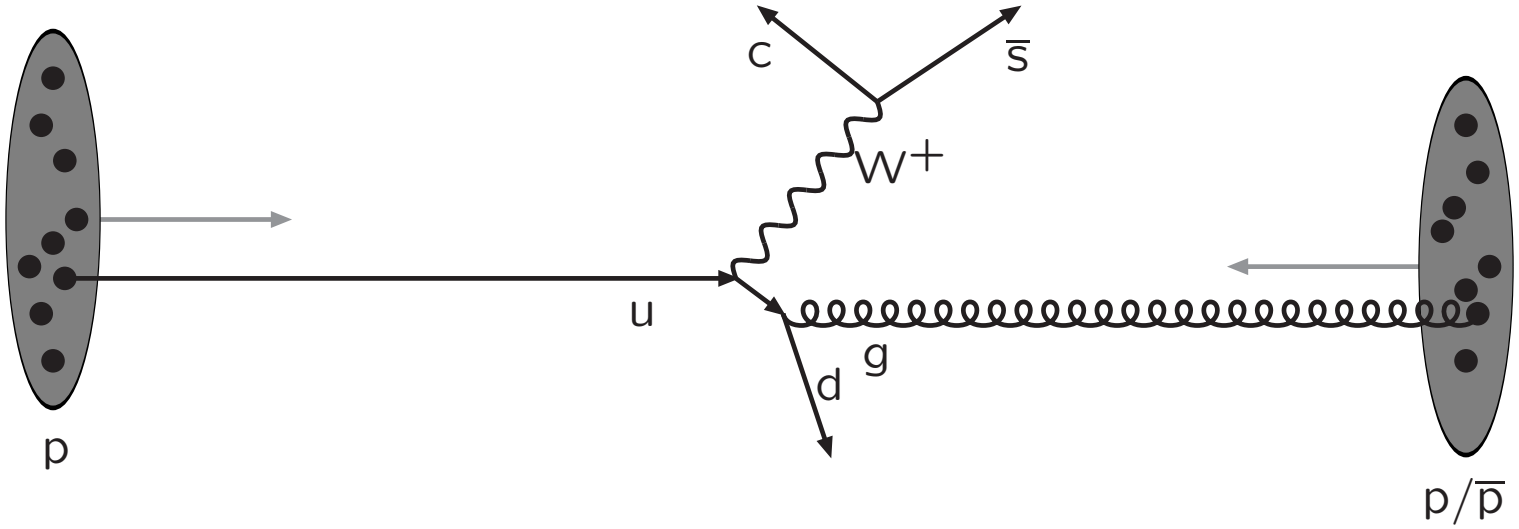
Incoming beams: parton densities

The structure of an event – 2



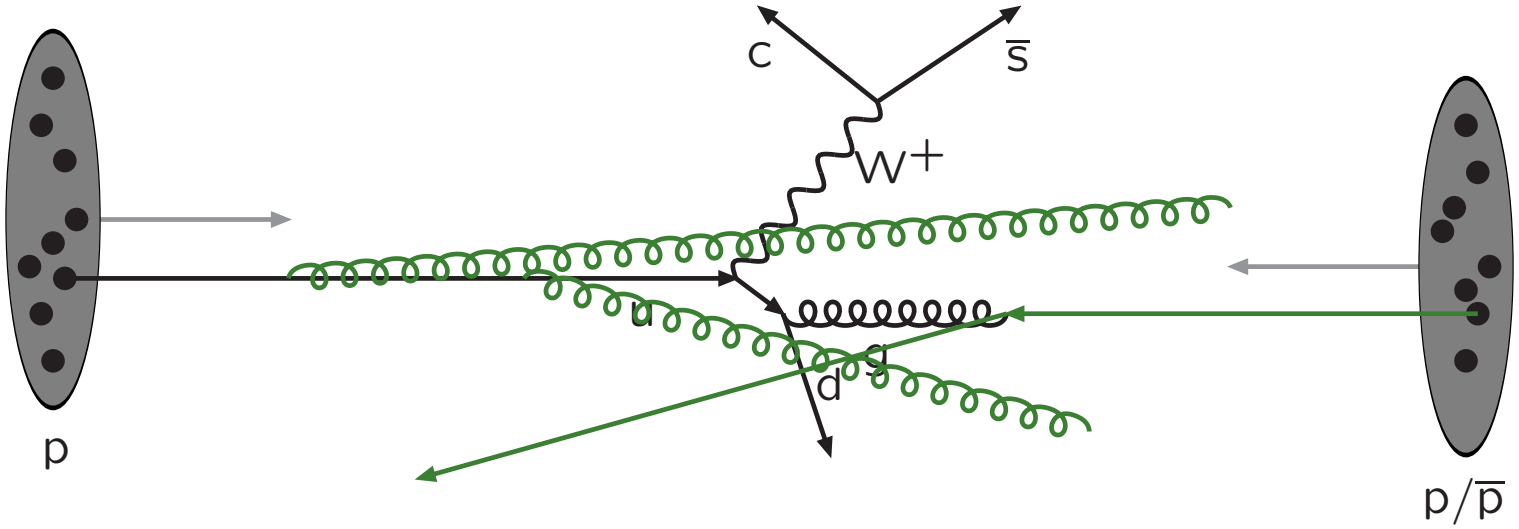
Hard subprocess: described by matrix elements

The structure of an event – 3



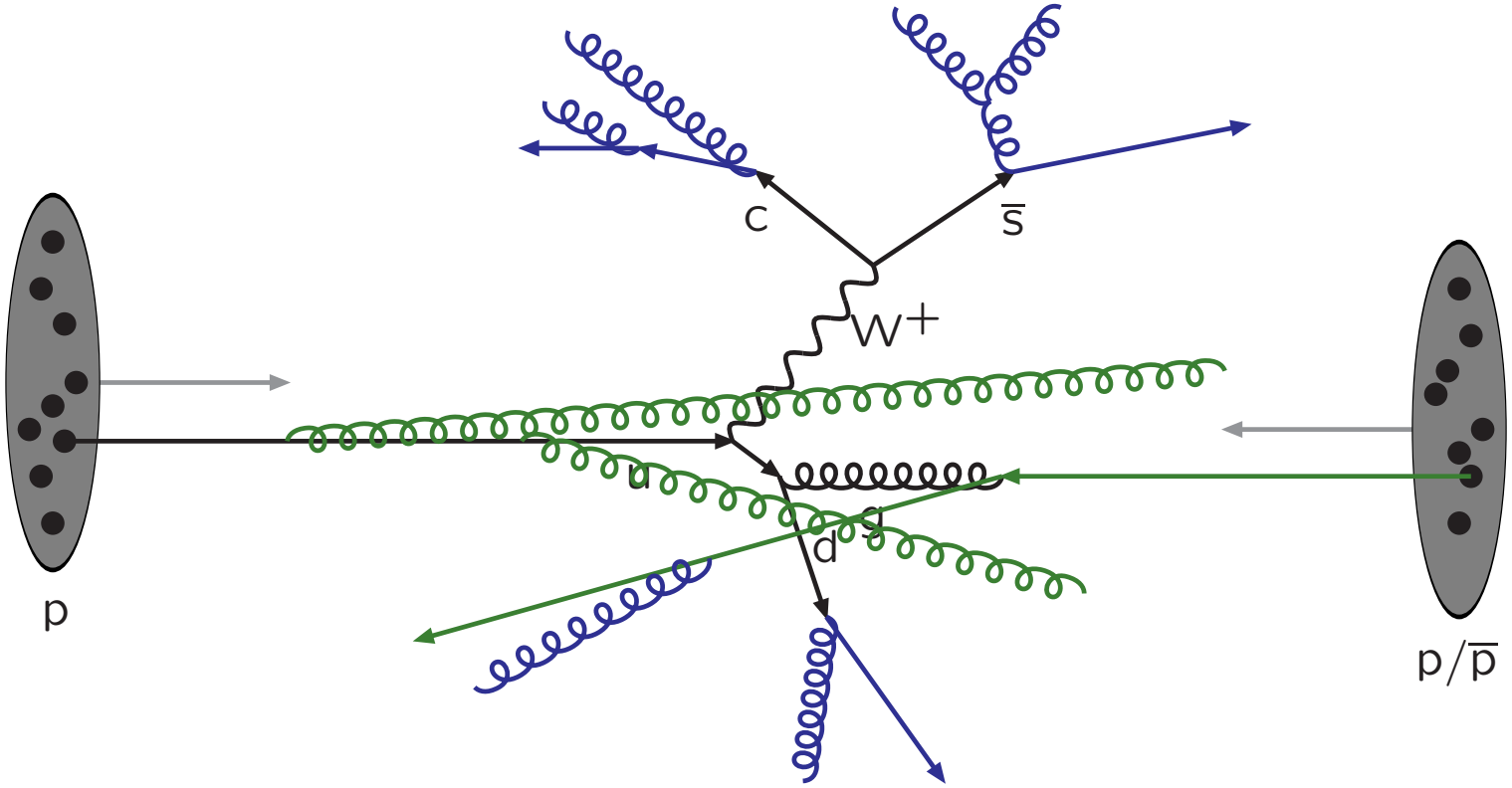
Resonance decays: correlated with hard subprocess

The structure of an event – 4



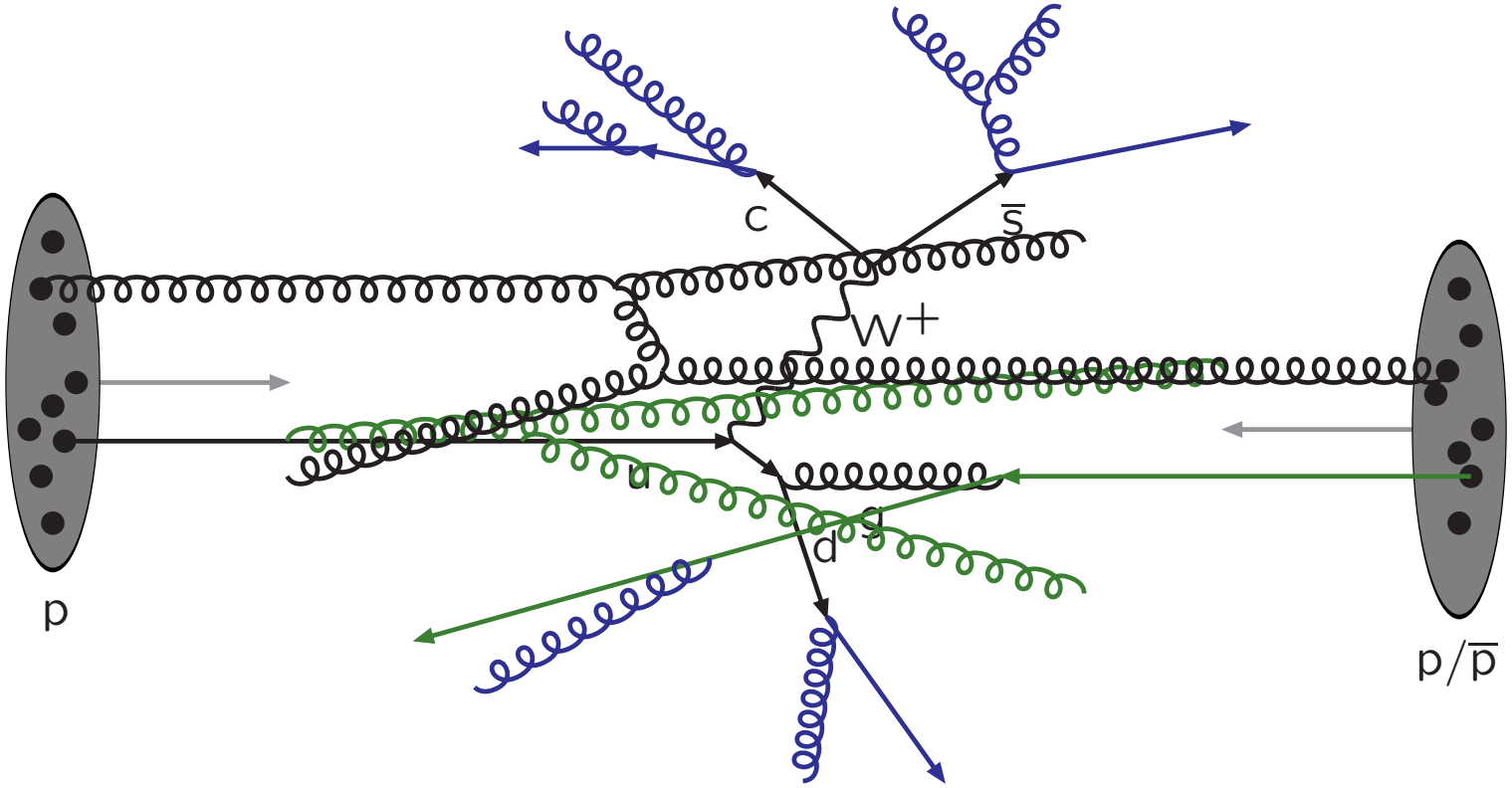
Initial-state radiation: spacelike parton showers

The structure of an event – 5



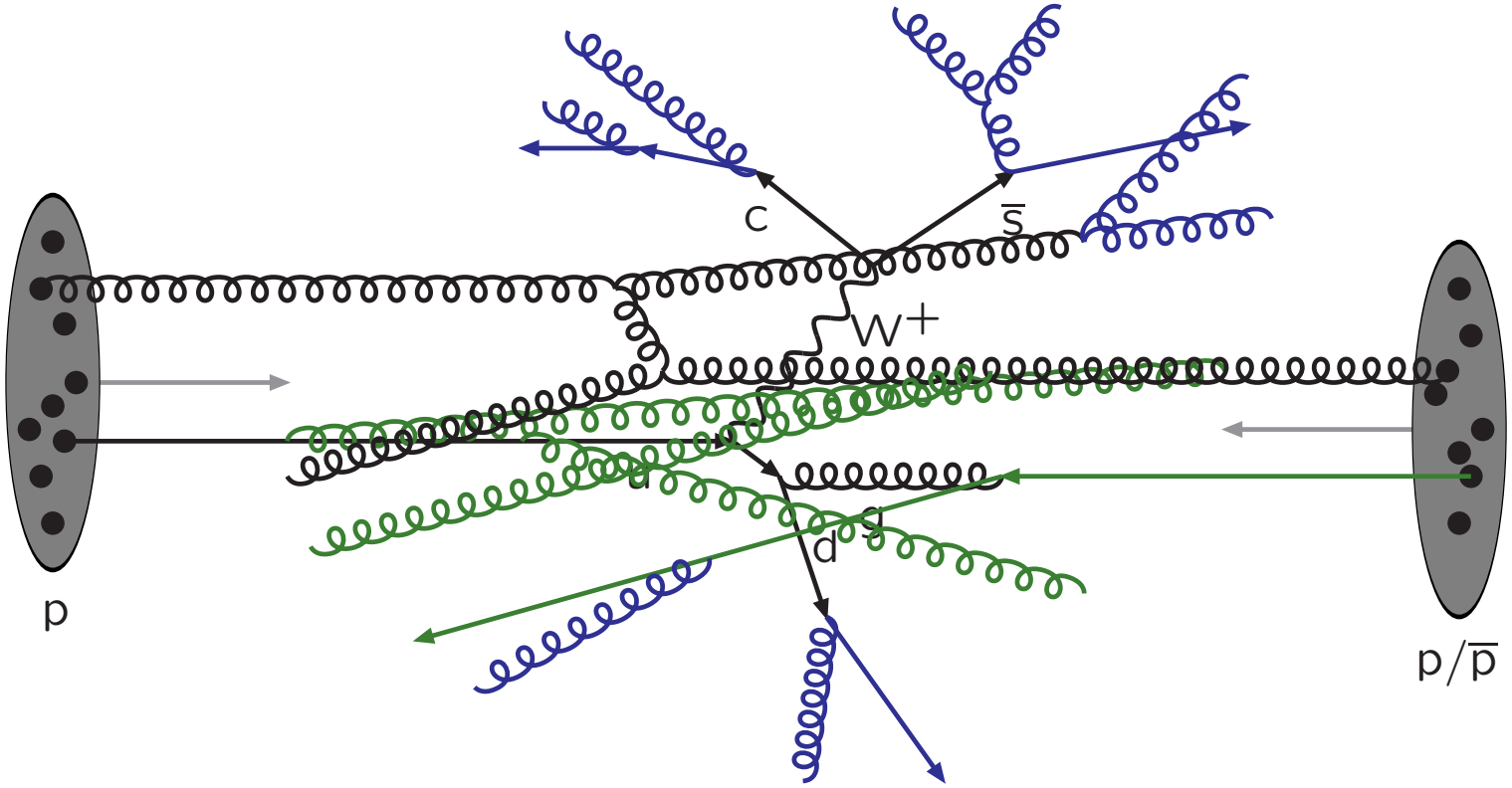
Final-state radiation: timelike parton showers

The structure of an event – 6



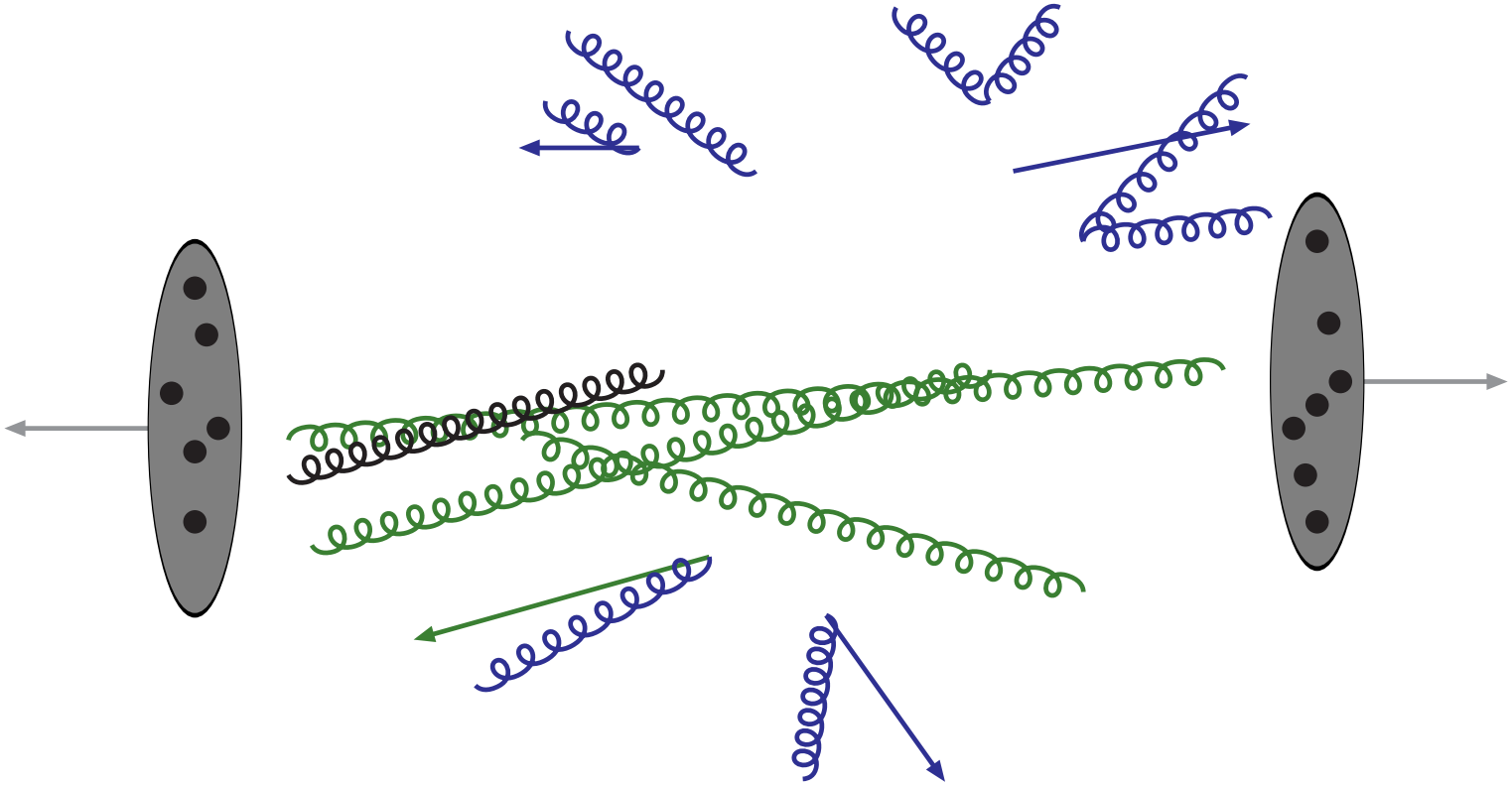
Multiple parton–parton interactions . . .

The structure of an event – 7



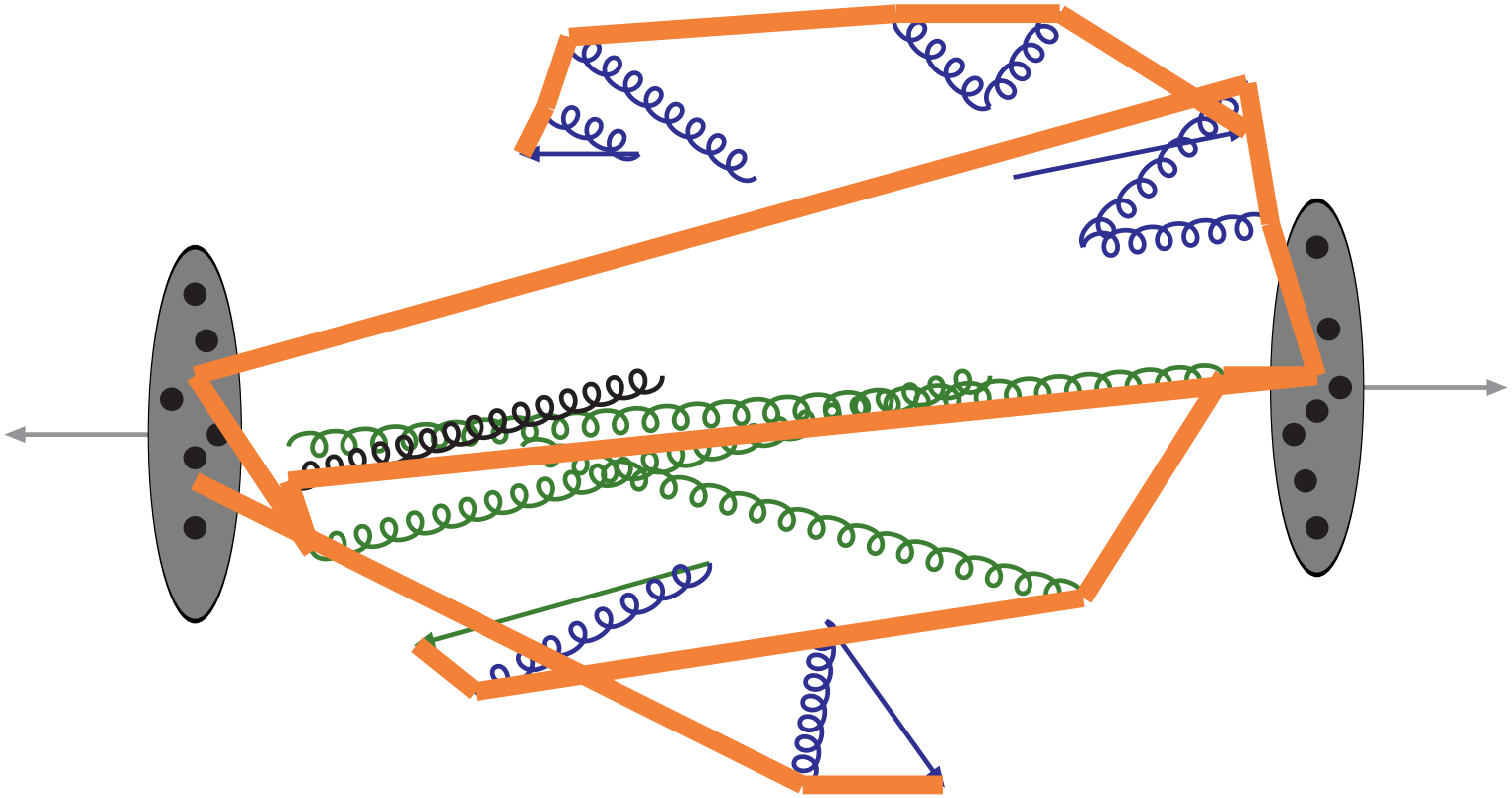
. . . with its initial- and final-state radiation

The structure of an event – 8



Beam remnants and other outgoing partons

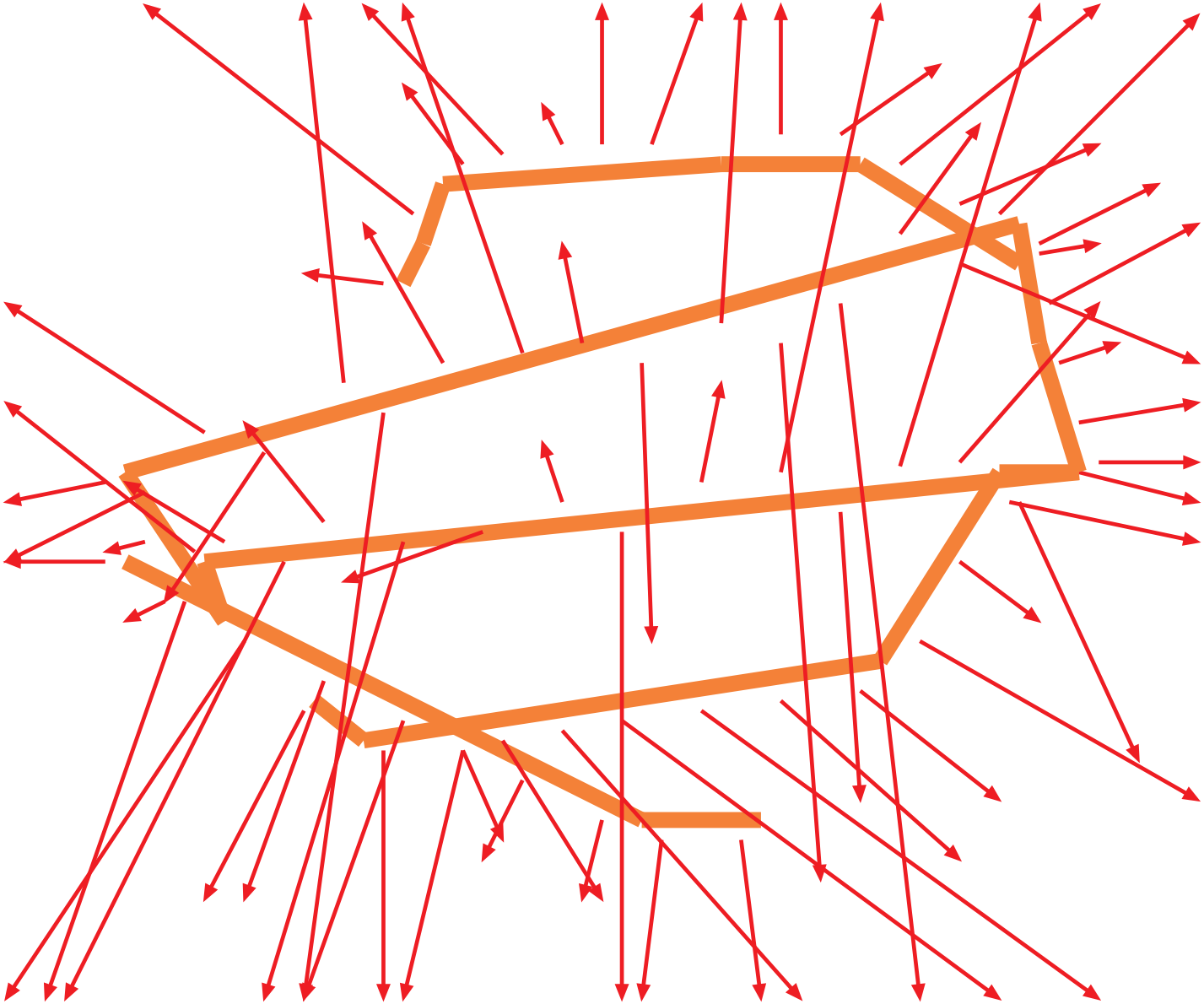
The structure of an event – 9



Everything is connected by colour confinement strings

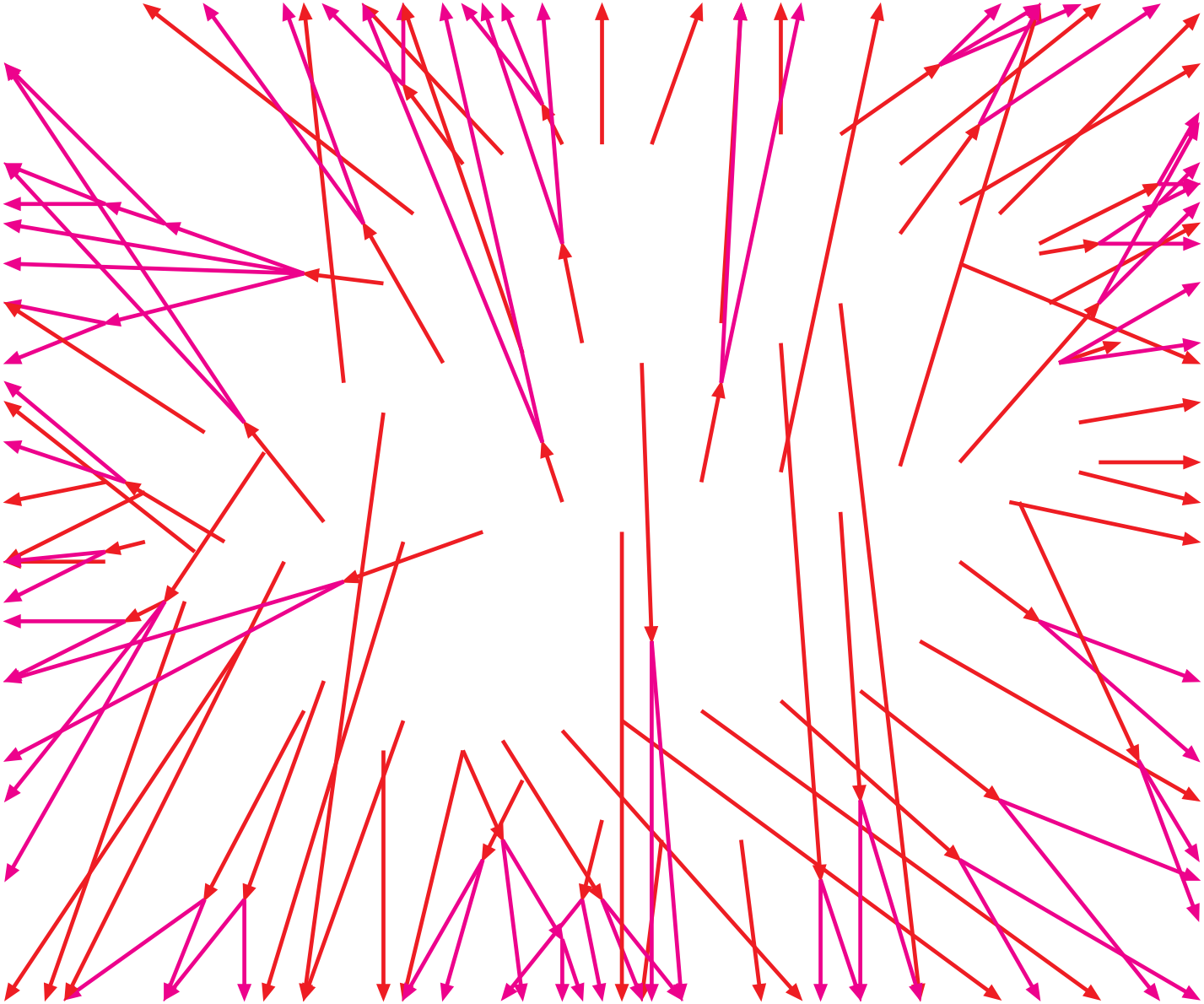
Recall! Not to scale: strings are of hadronic widths

The structure of an event – 10



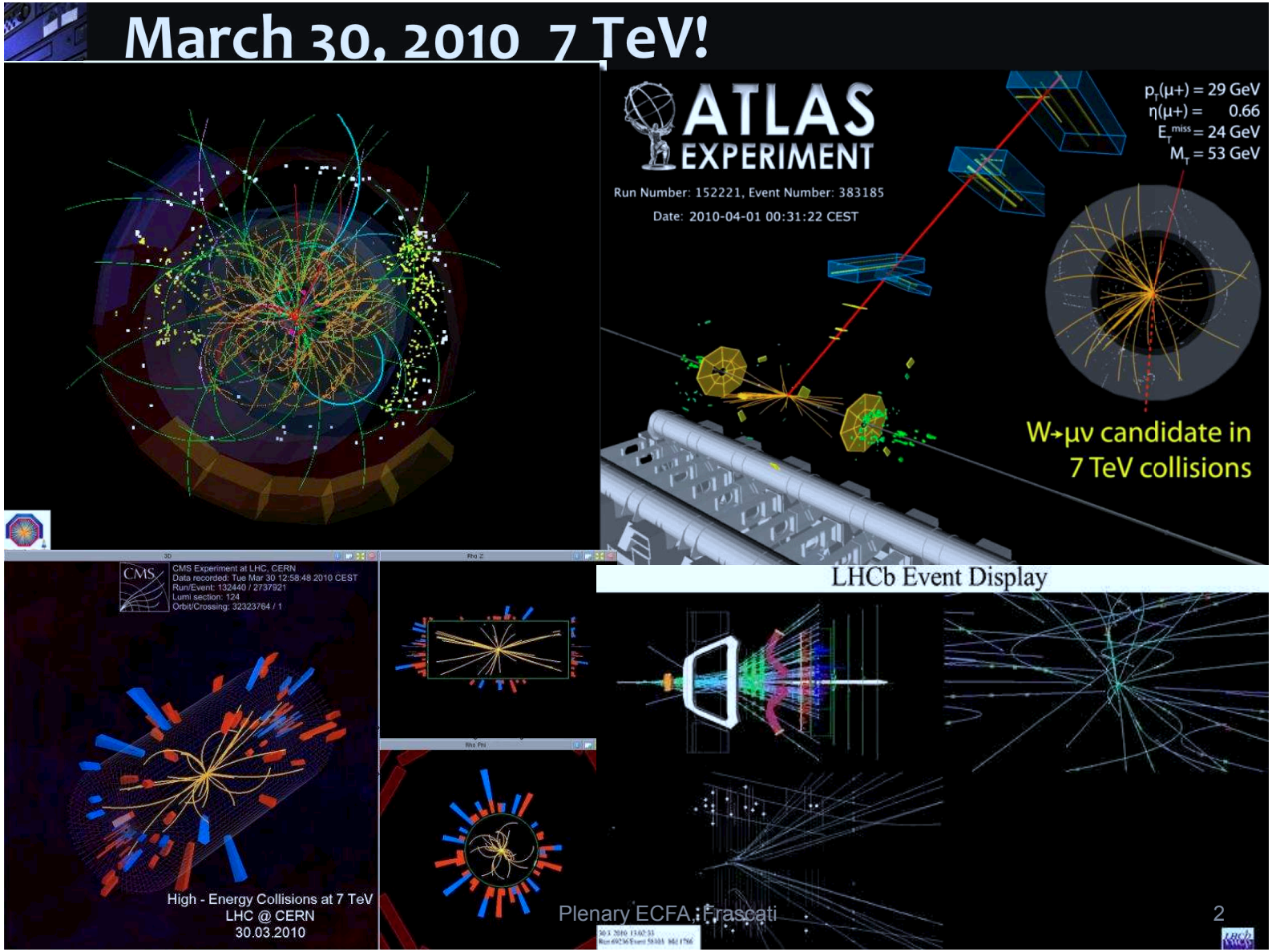
The strings fragment to produce primary hadrons

The structure of an event – 11



Many hadrons are unstable and decay further

The structure of an event – 12



These are the particles that hit the detector

The Monte Carlo method

Want to generate events in as much detail as Mother Nature
$\Longrightarrow$ get average *and* fluctutations right
$\Longrightarrow$ make random choices, $\sim$ as in nature

$$\sigma_{\text{final state}} = \sigma_{\text{hard process}}\, \mathcal{P}_{\text{tot,hard process}\to\text{final state}}$$

(appropriately summed & integrated over non-distinguished final states)

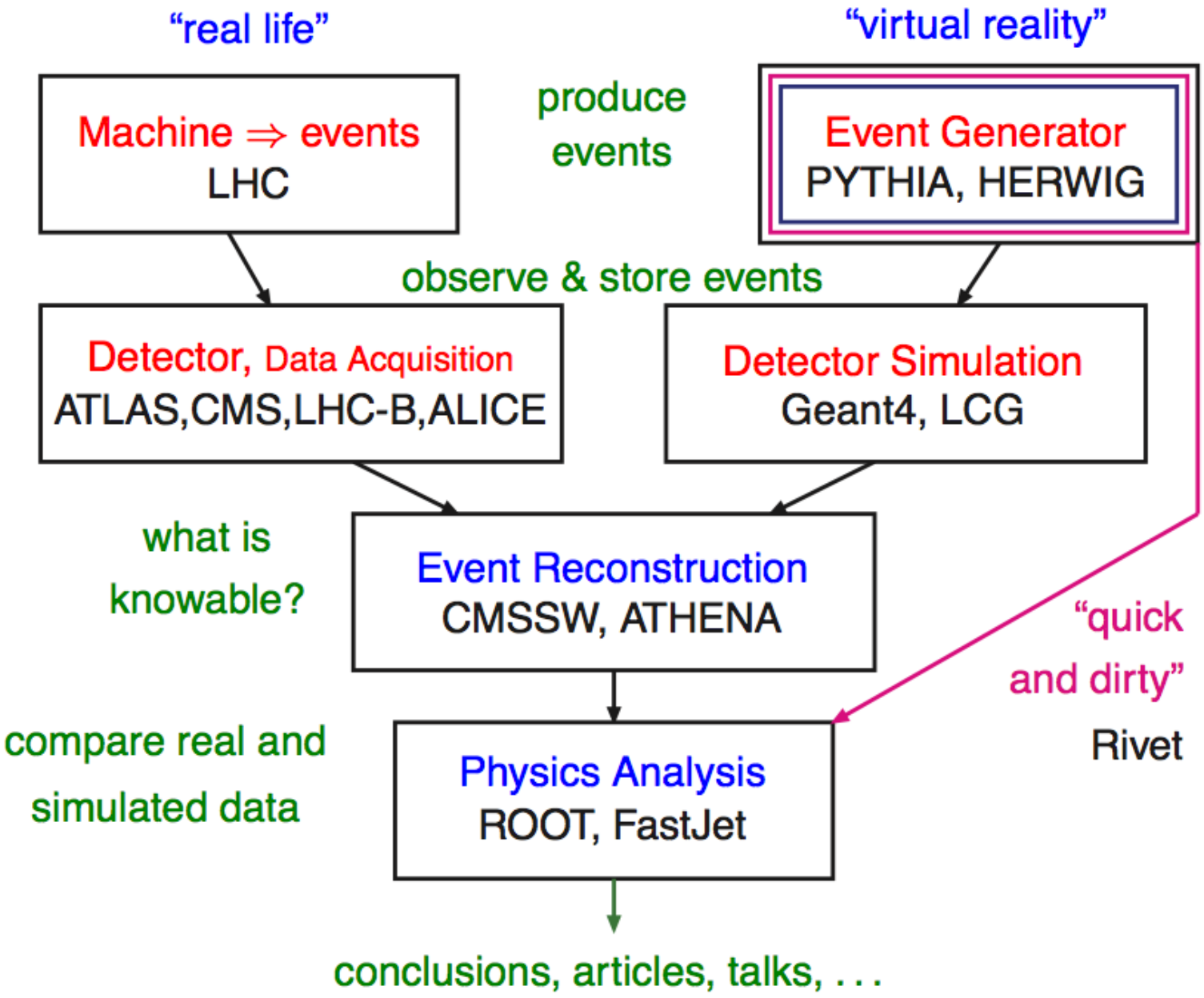
where $\mathcal{P}_{\text{tot}} = \mathcal{P}_{\text{res}}\, \mathcal{P}_{\text{ISR}}\, \mathcal{P}_{\text{FSR}}\, \mathcal{P}_{\text{MPI}} \mathcal{P}_{\text{remnants}}\, \mathcal{P}_{\text{hadronization}}\, \mathcal{P}_{\text{decays}}$

with $\mathcal{P}_i = \prod_j \mathcal{P}_{ij} = \prod_j \prod_k \mathcal{P}_{ijk} = \ldots$ in its turn

$\Longrightarrow$ **divide and conquer**

an event with n particles involves $\mathcal{O}(10n)$ random choices,
(flavour, mass, momentum, spin, production vertex, lifetime, ...)
LHC: $\sim$ 100 charged and $\sim$ 200 neutral (+ intermediate stages)
$\Longrightarrow$ several thousand choices
(of $\mathcal{O}(100)$ different kinds)

Event Generator Position



Why generators?

- Allow theoretical and experimental studies of *complex* multiparticle physics
- Large flexibility in physical quantities that can be addressed
- Vehicle of ideology to disseminate ideas from theorists to experimentalists

Can be used to

- predict event rates and topologies
  $\Rightarrow$ can estimate feasibility
- simulate possible backgrounds
  $\Rightarrow$ can devise analysis strategies
- study detector requirements
  $\Rightarrow$ can optimize detector/trigger design
- study detector imperfections
  $\Rightarrow$ can evaluate acceptance corrections

The workhorses: what are the differences?

HERWIG, PYTHIA and SHERPA offer convenient frameworks for LHC physics studies, but with slightly different emphasis:

PYTHIA (successor to JETSET, begun in 1978):

- originated in hadronization studies: the Lund string
- leading in development of MPI for MB/UE
- pragmatic attitude to showers & matching

HERWIG (successor to EARWIG, begun in 1984):

- originated in coherent-shower studies (angular ordering)
- cluster hadronization & underlying event pragmatic add-on
- large process library with spin correlations in decays

SHERPA (APACIC++/AMEGIC++, begun in 2000):

- own matrix-element calculator/generator
- extensive machinery for CKKW ME/PS matching
- hadronization & min-bias physics under development

PYTHIA and HERWIG originally in Fortran, but now all in C++.

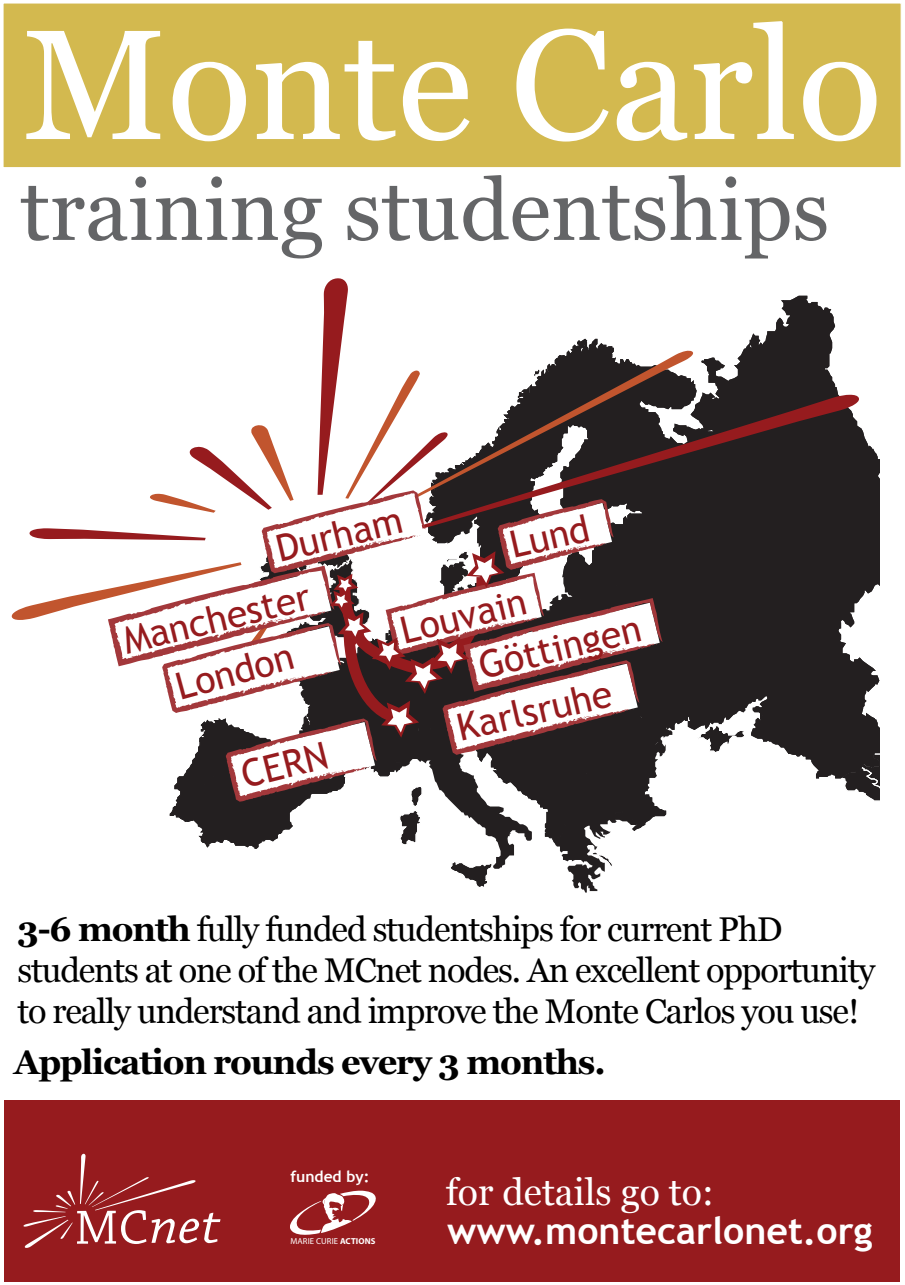
MCnet

MCnet projects:

- PYTHIA (+ VINCIA)
- HERWIG
- SHERPA
- MadGraph
- Ariadne (+ DIPSY)
- Cedar (Rivet/Professor)

Activities include

- summer schools
- short-term studentships
- graduate students
- postdocs
- meetings (open/closed)



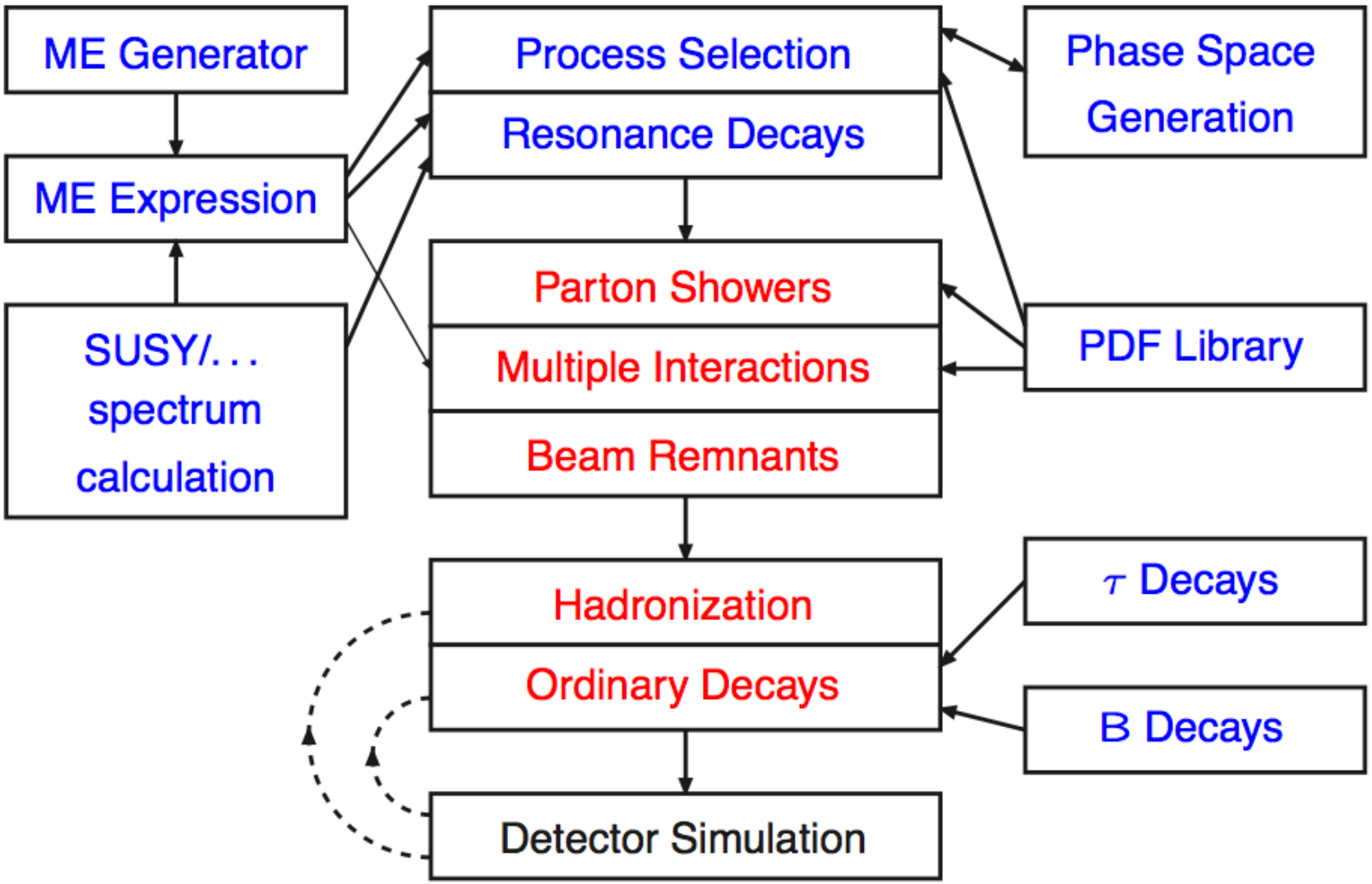
Other Relevant Software

Some examples (with apologies for many omissions):

- Other event/shower generators: PhoJet, Ariadne, Dipsy, Cascade, Vincia
- Matrix-element generators: MadGraph/MadEvent, CompHep, CalcHep, Helac, Whizard, Sherpa, GoSam, aMC@NLO
- Matrix element libraries: AlpGen, POWHEG BOX, MCFM, NLOjet++, VBFNLO, BlackHat, Rocket
- Special BSM scenarios: Prospino, Charybdis, TrueNoir
- Mass spectra and decays: SOFTSUSY, SPHENO, HDecay, SDecay
- Feynman rule generators: FeynRules
- PDF libraries: LHAPDF
- Resummed ($p_\perp$) spectra: ResBos
- Approximate loops: LoopSim
- Jet finders: anti-$k_\perp$ and FastJet
- Analysis packages: Rivet, Professor, MCPLOTS
- Detector simulation: GEANT, Delphes
- Constraints (from cosmology etc): DarkSUSY, MicrOmegas
- Standards: PDF identity codes, LHA, LHEF, SLHA, Binoth LHA, HepMC

Can be meaningfully combined and used for LHC physics!

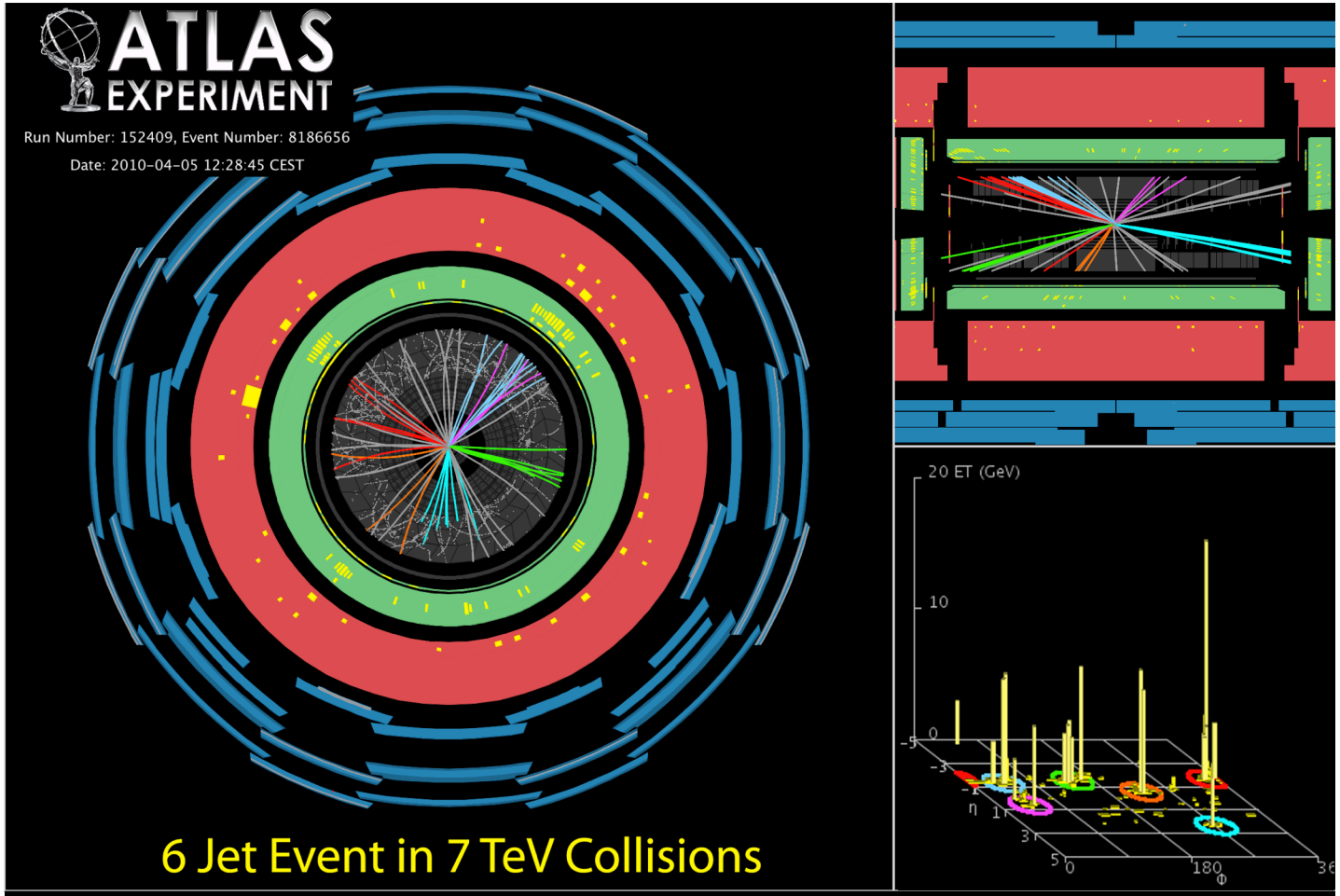
Putting it together



Standardized interfaces essential!

...but wide range of possible processes, some with special quirks.

Multijets – the need for showers



An event with 6 jets taken on April 4th, 2010. The jets have calibrated transverse momenta between 30 GeV and 70 GeV and are well separated in the detector. 12

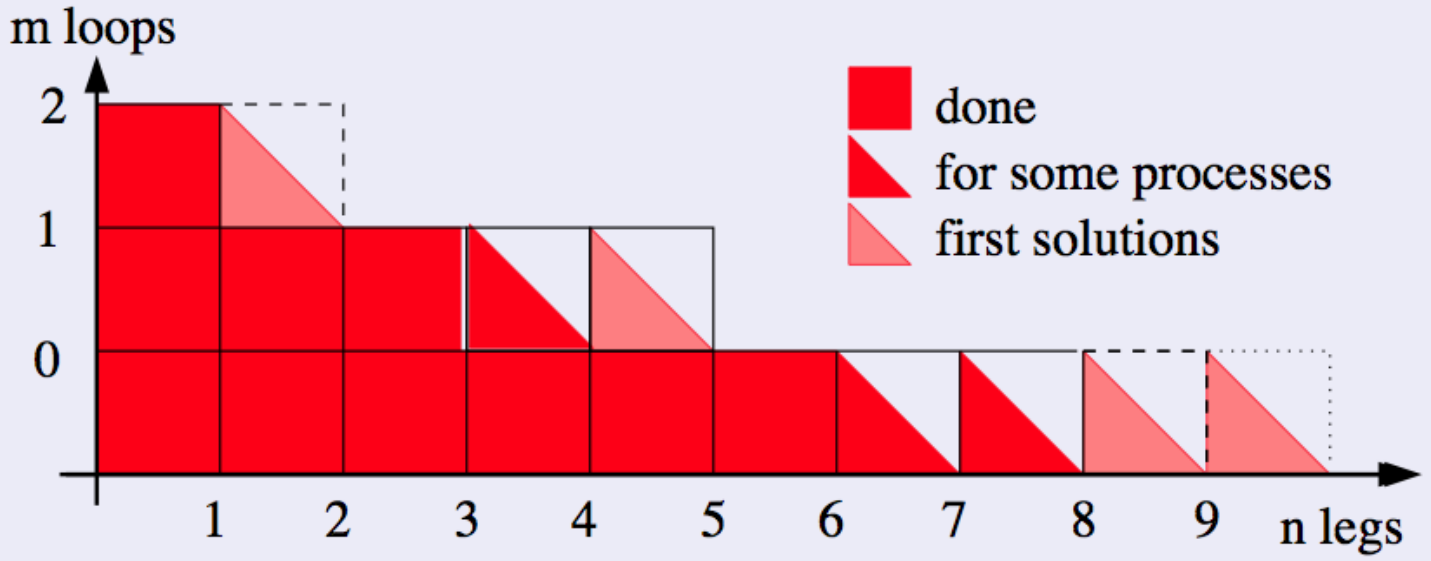
Basic $2 \to 2$ process dressed up by bremsstrahlung!?

Perturbative QCD

Order-by-order calculations: challenges more math than physics.

Availability of exact calculations (hadron colliders)

- Fixed order matrix elements ("parton level") are exact to a given perturbative order. (and often quite a pain!)
- Important to understand limitations:
  Only tree-level fully automated, 1-loop-level ongoing.

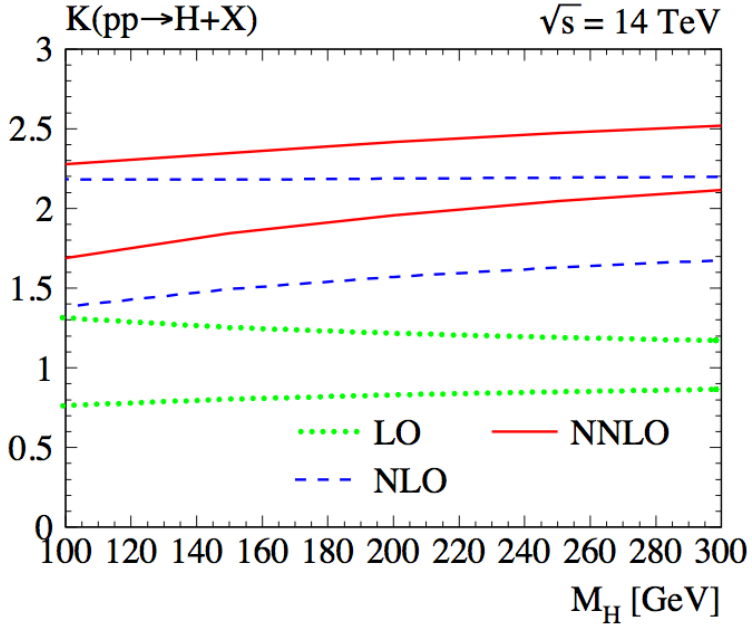


(courtesy Frank Krauss)

Perturbative QCD

Order-by-order calculations: challenges more math than physics.

- LO: solved for all practical applications.
- NLO: in process of being automatized.
- NNLO: the current calculational frontier.
- Another bottleneck: efficient phase space sampling.



$\mathrm{gg} \to \mathrm{H}^0$ illustrates problems:

- Need high-precision calculations
- to search for BSM physics,
- but limited by poorly-understood slow convergence.

Perturbative calculations reliable for well separated jets, but ...

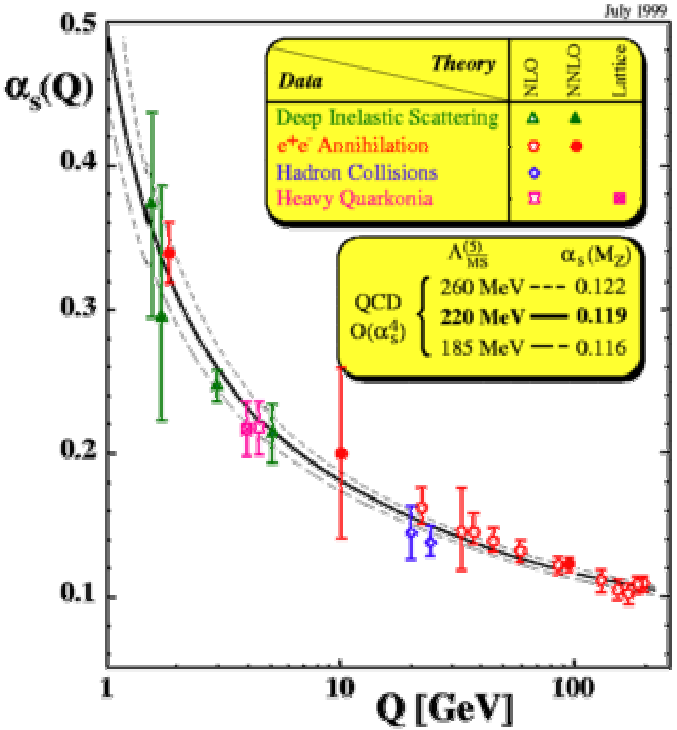
Divergences

Emission rate $\mathrm{q} \to \mathrm{qg}$ diverges when

- collinear: opening angle $\theta_{\mathrm{qg}} \to 0$
- soft: gluon energy $E_{\mathrm{g}} \to 0$

Almost identical to $\mathrm{e} \to \mathrm{e}\gamma$
but QCD is non-Abelian so additionally

- $\mathrm{g} \to \mathrm{gg}$ similarly divergent
- $\alpha_s(Q^2)$ diverges for $Q^2 \to 0$
  (actually for $Q^2 \to \Lambda^2_{\mathrm{QCD}}$)



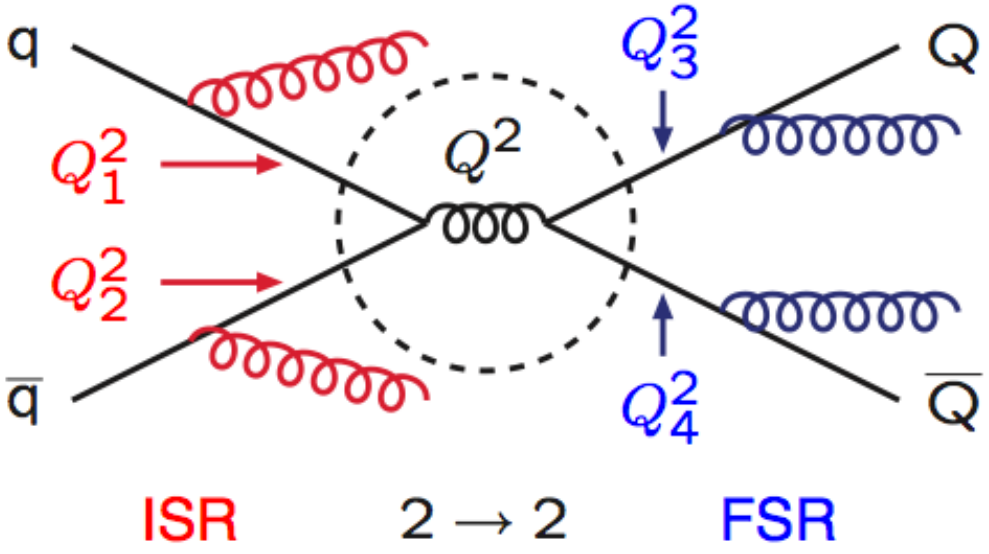
Big probability for one emission $\Longrightarrow$ also big for several.

With ME's need to calculate to high order **and** with many loops $\Longrightarrow$ extremely demanding technically (not solved!), and involving big cancellations between positive and negative contributions.

Alternative approach: **parton showers**

The Parton-Shower Approach

$$2 \to n \quad = \quad (2 \to 2) \oplus \text{ISR} \oplus \text{FSR}$$

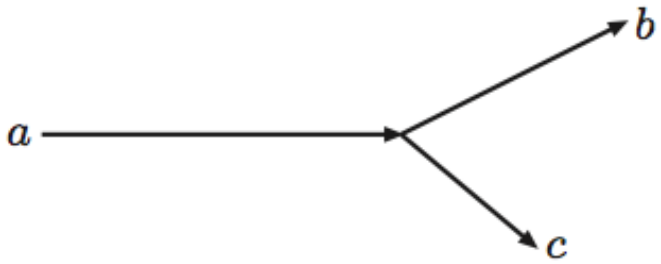


FSR = Final-State Radiation = timelike shower
$Q_i^2 \sim m^2 > 0$ decreasing
ISR = Initial-State Radiation = spacelike showers
$Q_i^2 \sim -m^2 > 0$ increasing

Why "time" like and "space" like?

Consider four-momentum conservation in a branching $a \to b\,c$

$$\mathbf{p}_{\perp a} = 0 \quad \Rightarrow \quad \mathbf{p}_{\perp c} = -\mathbf{p}_{\perp b}$$
$$p_+ = E + p_{\mathrm{L}} \quad \Rightarrow \quad p_{+a} = p_{+b} + p_{+c}$$
$$p_- = E - p_{\mathrm{L}} \quad \Rightarrow \quad p_{-a} = p_{-b} + p_{-c}$$



Define $p_{+b} = z\,p_{+a}, \quad p_{+c} = (1-z)\,p_{+a}$

Use $p_+ p_- = E^2 - p_{\mathrm{L}}^2 = m^2 + p_\perp^2$

$$\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{z\,p_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1-z)\,p_{+a}}$$

$$\Rightarrow \quad m_a^2 = \frac{m_b^2 + p_\perp^2}{z} + \frac{m_c^2 + p_\perp^2}{1-z} = \frac{m_b^2}{z} + \frac{m_c^2}{1-z} + \frac{p_\perp^2}{z(1-z)}$$

Final-state shower: $m_b = m_c = 0 \Rightarrow m_a^2 = \frac{p_\perp^2}{z(1-z)} > 0 \Rightarrow$ timelike

Initial-state shower: $m_a = m_c = 0 \Rightarrow m_b^2 = -\frac{p_\perp^2}{1-z} < 0 \Rightarrow$ spacelike

Showers and cross sections

Shower evolution is viewed as a probabilistic process, which occurs with unit total probability:
the cross section is not directly affected

However, more complicated than that

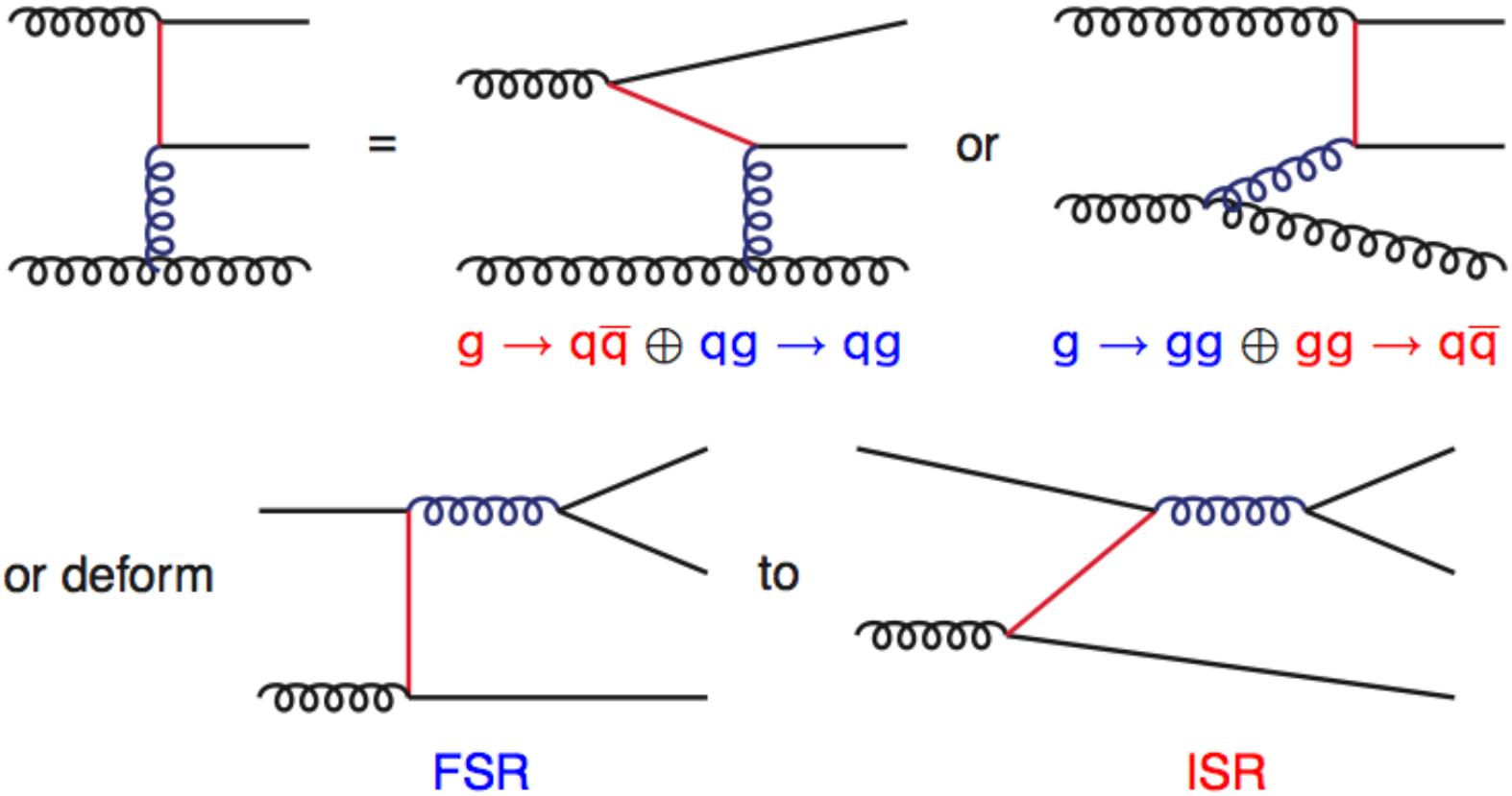
- PDF evolution $\approx$ showers $\Rightarrow$ enters in convoluted cross section, e.g. for $2 \to 2$ processes

$$\sigma = \iiint \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}\hat{t}\; f_i(x_1, Q^2)\, f_j(x_2, Q^2)\, \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}\hat{t}}$$

- Shower affects event shape
  E.g. start from 2-jet event with $\mathrm{p}_{\perp 1} = \mathrm{p}_{\perp 2} = 100$ GeV.
  ISR gives third jet, plus recoil to existing two, so
  $\mathrm{p}_{\perp 1} = 110$ GeV, $\mathrm{p}_{\perp 2} = 90$ GeV, $\mathrm{p}_{\perp 1} = 20$ GeV:
  - inclusive $\mathrm{p}_{\perp \mathrm{jet}}$ spectrum goes up
  - hardest $\mathrm{p}_{\perp \mathrm{jet}}$ spectrum goes up
  - two-jets with both jets above some $p_{\perp \mathrm{min}}$ comes down
  - three-jet rate goes up

Doublecounting

A $2 \to n$ graph can be "simplified" to $2 \to 2$ in different ways:

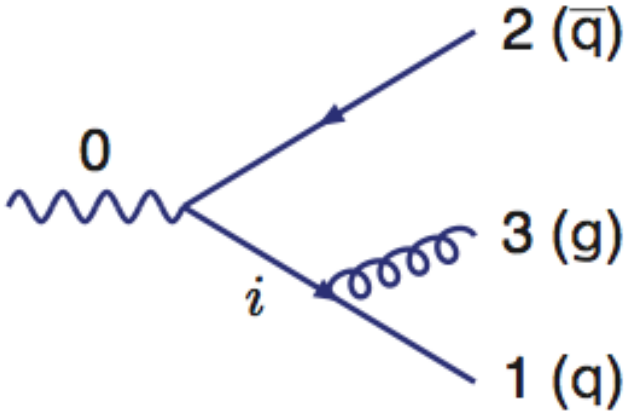


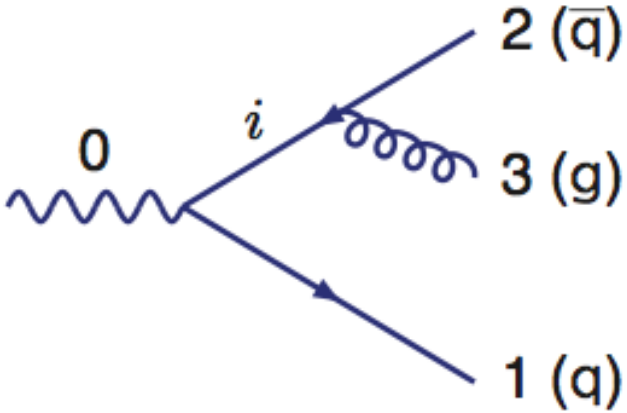
Do not doublecount: $2 \to 2$ = most virtual = shortest distance

Conflict: theory derivations assume virtualities strongly ordered; interesting physics often in regions where this is not true!

Final-state radiation

Standard process $\boxed{\mathrm{e^+e^- \to q\bar{q}g}}$ by two Feynman diagrams:





$$x_i = \frac{2E_i}{E_{\mathrm{cm}}}$$

$$x_1 + x_2 + x_3 = 2$$

$$\frac{\mathrm{d}\sigma_{\mathrm{ME}}}{\sigma_0} = \frac{\alpha_{\mathrm{s}}}{2\pi}\,\frac{4}{3}\,\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)}\,\mathrm{d}x_1\,\mathrm{d}x_2$$

Convenient (but arbitrary) subdivision to "split" radiation:

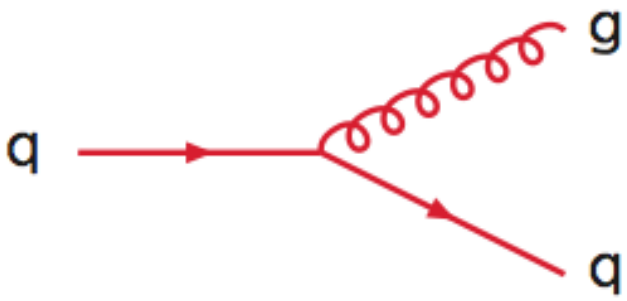
$$\frac{1}{(1-x_1)(1-x_2)}\,\frac{(1-x_1)+(1-x_2)}{x_3} = \frac{1}{(1-x_2)x_3} + \frac{1}{(1-x_1)x_3}$$

From matrix elements to parton showers

Rewrite for $x_2 \to 1$, i.e. q–g collinear limit:

$$1 - x_2 = \frac{m_{13}^2}{E_{\mathrm{cm}}^2} = \frac{Q^2}{E_{\mathrm{cm}}^2} \Rightarrow \mathrm{d}x_2 = \frac{\mathrm{d}Q^2}{E_{\mathrm{cm}}^2}$$

define z as fraction q retains in branching $\mathrm{q} \to \mathrm{qg}$



$$\begin{aligned} x_1 &\approx z \quad \Rightarrow \quad \mathrm{d}x_1 \approx \mathrm{d}z \\ x_3 &\approx 1 - z \end{aligned}$$

$$\Rightarrow \mathrm{d}\mathcal{P} = \frac{\mathrm{d}\sigma}{\sigma_0} = \frac{\alpha_\mathrm{s}}{2\pi} \frac{\mathrm{d}x_2}{(1-x_2)} \frac{4}{3} \frac{x_2^2 + x_1^2}{(1-x_1)} \mathrm{d}x_1 \approx \frac{\alpha_\mathrm{s}}{2\pi} \frac{\mathrm{d}Q^2}{Q^2} \frac{4}{3} \frac{1+z^2}{1-z} \mathrm{d}z$$

In limit $x_1 \to 1$ same result, but for $\overline{\mathrm{q}} \to \overline{\mathrm{q}}\mathrm{g}$.

$\mathrm{d}Q^2/Q^2 = \mathrm{d}m^2/m^2$: "mass (or collinear) singularity"

$\mathrm{d}z/(1-z) = \mathrm{d}\omega/\omega$ "soft singularity"

The DGLAP equations

Generalizes to

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

$$
\begin{aligned}
\mathrm{d}\mathcal{P}_{a\to bc} &= \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \\
P_{\mathrm{q}\to\mathrm{qg}} &= \frac{4}{3}\,\frac{1+z^2}{1-z} \\
P_{\mathrm{g}\to\mathrm{gg}} &= 3\,\frac{(1-z(1-z))^2}{z(1-z)} \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}} &= \frac{n_f}{2}\,(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})
\end{aligned}
$$

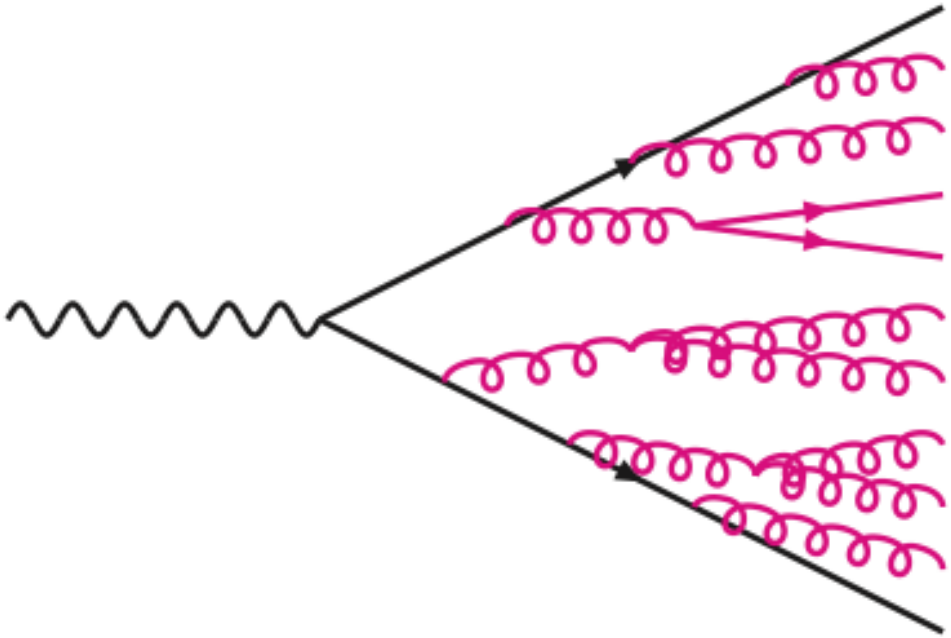
Universality: any matrix element reduces to DGLAP in collinear limit.

$$
\text{e.g.}\quad \frac{\mathrm{d}\sigma(\mathrm{H}^0\to\mathrm{q\bar{q}g})}{\mathrm{d}\sigma(\mathrm{H}^0\to\mathrm{q\bar{q}})} = \frac{\mathrm{d}\sigma(\mathrm{Z}^0\to\mathrm{q\bar{q}g})}{\mathrm{d}\sigma(\mathrm{Z}^0\to\mathrm{q\bar{q}})} \quad \text{in collinear limit}
$$

The iterative structure

Generalizes to many consecutive emissions if strongly ordered, $Q_1^2 \gg Q_2^2 \gg Q_3^2 \ldots$ ($\approx$ time-ordered).
To cover "all" of phase space use DGLAP in whole region $Q_1^2 > Q_2^2 > Q_3^2 \ldots$.

Iteration gives final-state parton showers:



Need soft/collinear cuts to stay away from nonperturbative physics.
Details model-dependent, but around 1 GeV scale.

The ordering variable

In the evolution with

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z$$

Q^2 orders the emissions (memory).

If $Q^2 = m^2$ is one possible evolution variable
then $Q'^2 = f(z)Q^2$ is also allowed, since

$$\left|\frac{\mathrm{d}(Q'^2, z)}{\mathrm{d}(Q^2, z)}\right| = \begin{vmatrix} \frac{\partial Q'^2}{\partial Q^2} & \frac{\partial Q'^2}{\partial z} \\ \frac{\partial z}{\partial Q^2} & \frac{\partial z}{\partial z} \end{vmatrix} = \begin{vmatrix} f(z) & f'(z)Q^2 \\ 0 & 1 \end{vmatrix} = f(z)$$

$$\Rightarrow \mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{f(z)\mathrm{d}Q^2}{f(z)Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{\mathrm{d}Q'^2}{Q'^2}\,P_{a\to bc}(z)\,\mathrm{d}z$$

- $Q'^2 = E_a^2\theta_{a\to bc}^2 \approx m^2/(z(1-z))$; angular-ordered shower
- $Q'^2 = p_\perp^2 \approx m^2 z(1-z)$; transverse-momentum-ordered

The Sudakov form factor – 1

Time evolution, conservation of total probability:
$\mathcal{P}(\text{no emission}) = 1 - \mathcal{P}(\text{emission})$.

Multiplicativeness, with $T_i = (i/n)T$, $0 \leq i \leq n$:

$$\begin{aligned}
\mathcal{P}_{\rm no}(0 \leq t < T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\rm no}(T_i \leq t < T_{i+1}) \\
&= \lim_{n\to\infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\rm em}(T_i \leq t < T_{i+1})) \\
&= \exp\left(- \lim_{n\to\infty} \sum_{i=0}^{n-1} \mathcal{P}_{\rm em}(T_i \leq t < T_{i+1}) \right) \\
&= \exp\left(- \int_0^T \frac{{\rm d}\mathcal{P}_{\rm em}(t)}{{\rm d}t} {\rm d}t \right) \\
\Longrightarrow \quad {\rm d}\mathcal{P}_{\rm first}(T) &= {\rm d}\mathcal{P}_{\rm em}(T) \exp\left(- \int_0^T \frac{{\rm d}\mathcal{P}_{\rm em}(t)}{{\rm d}t} {\rm d}t \right)
\end{aligned}$$

The Sudakov form factor – 2

Expanded, with $Q \sim 1/t$ (Heisenberg)

$$\begin{aligned} \mathrm{d}\mathcal{P}_{a\to bc} &= \frac{\alpha_\mathrm{s}}{2\pi}\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \\ &\times \exp\left(-\sum_{b,c}\int_{Q^2}^{Q^2_\mathrm{max}}\frac{\mathrm{d}Q'^2}{Q'^2}\int\frac{\alpha_\mathrm{s}}{2\pi}\,P_{a\to bc}(z')\,\mathrm{d}z'\right) \end{aligned}$$

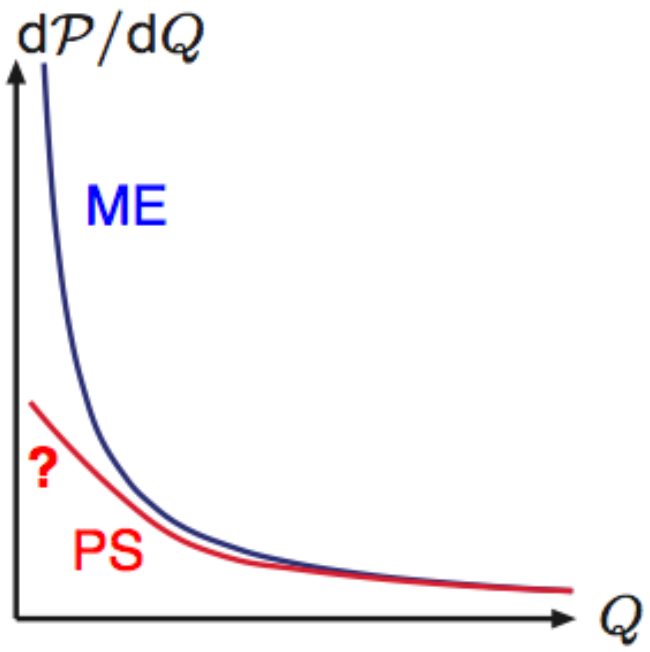
where the exponent is (one definition of) the Sudakov form factor

A given parton can only branch once, i.e. if it did not already do so

Note that $\sum_{b,c}\int\int\ \mathrm{d}\mathcal{P}_{a\to bc} \equiv 1 \Rightarrow$ *convenient for Monte Carlo* ($\equiv 1$ if extended over whole phase space, else possibly nothing happens before you reach $Q_0 \approx 1$ GeV).

The Sudakov form factor – 3

Sudakov regulates singularity for *first* emission . . .



. . . but in limit of *repeated soft* emissions $\mathrm{q} \to \mathrm{qg}$ (but no $\mathrm{g} \to \mathrm{gg}$) one obtains the same inclusive Q emission spectrum as for ME,

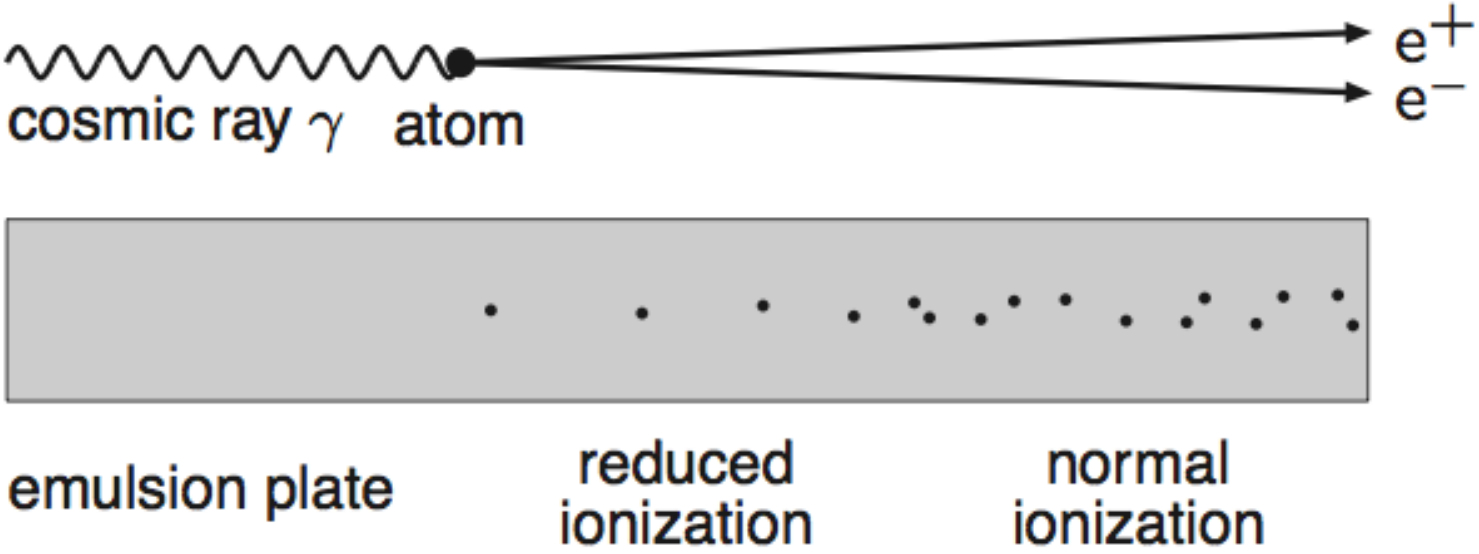
i.e. divergent ME spectrum
$\Longleftrightarrow$ infinite number of PS emissions

More complicated in reality:

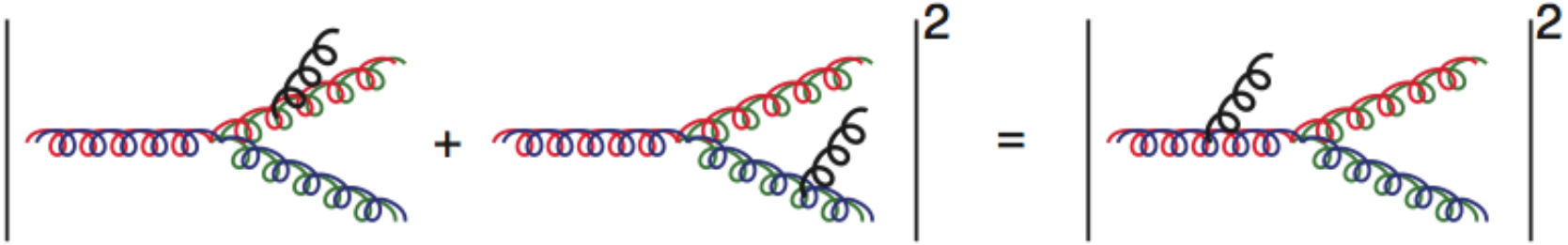
- energy-momentum conservation effects big since α_s big, so hard emissions frequent
- $\mathrm{g} \to \mathrm{gg}$ branchings leads to accelerated multiplication of partons

Coherence

QED: Chudakov effect (mid-fifties)



QCD: colour coherence for **soft** gluon emission



solved by
- requiring emission angles to be decreasing

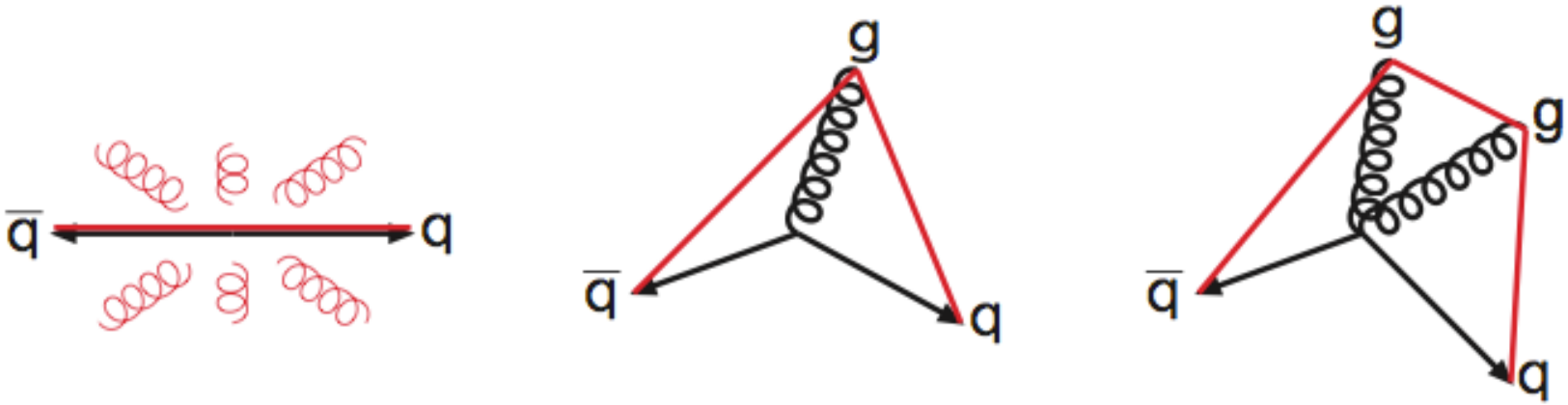
or
- requiring transverse momenta to be decreasing

Common Showering Algorithms

Standard shower language with $a \to bc$ successive branchings:

HERWIG: $Q^2 \approx E^2(1 - \cos\theta) \approx E^2\theta^2/2$
old PYTHIA: $Q^2 = m^2$ (+ brute-force coherence)

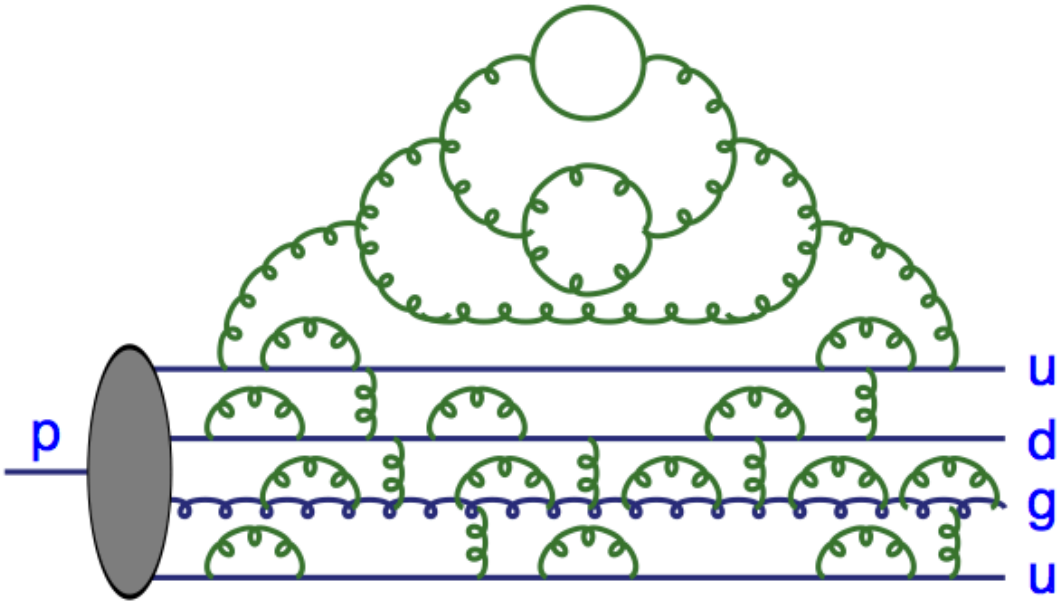
Newer ARIADNE picture of dipole emission $ab \to cde$:



is the basis for most current-day algorithms (HERWIG excepted)

Parton Distribution Functions

Hadrons are composite, with time-dependent structure:



$f_i(x, Q^2)$ = number density of partons i
at momentum fraction x and probing scale Q^2.
Linguistics (example):

$$F_2(x, Q^2) \quad = \quad \sum_i e_i^2 \, x f_i(x, Q^2)$$

structure function parton distributions

PDF evolution

Initial conditions at small Q_0^2 unknown: nonperturbative.

Resolution dependence perturbative, by DGLAP:

DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}(\ln Q^2)} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z}\, f_a(y,Q^2)\, \frac{\alpha_\mathrm{s}}{2\pi}\, P_{a\to bc}\left(z=\frac{x}{y}\right)$$

DGLAP already introduced for (final-state) showers:

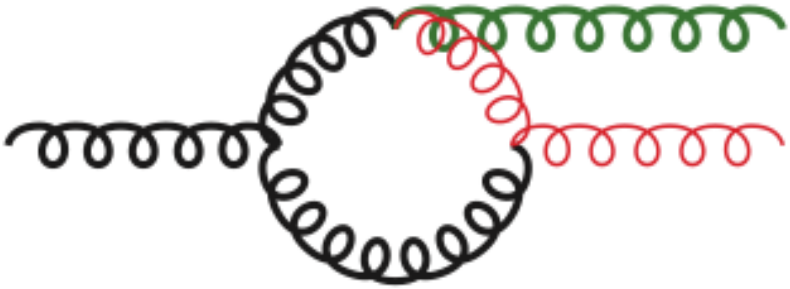
$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\, P_{a\to bc}(z)\,\mathrm{d}z$$

Same equation, but different context:

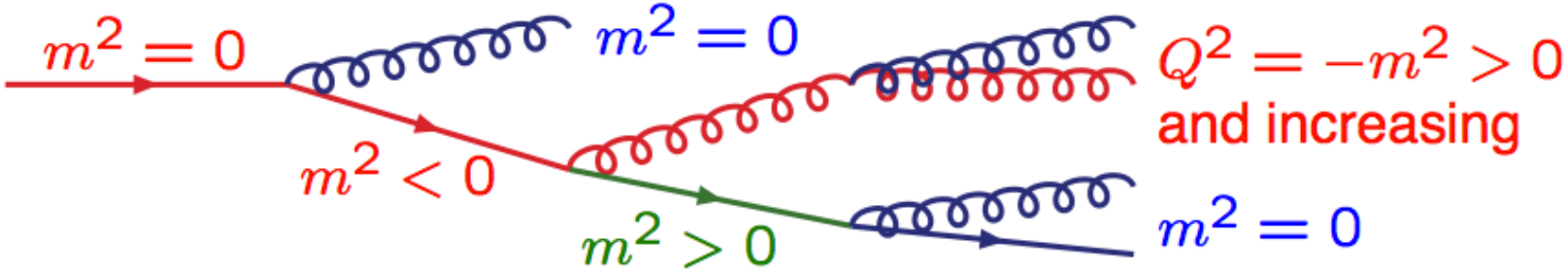
- $\mathrm{d}\mathcal{P}_{a\to bc}$ is probability for the individual parton to branch; while
- $\mathrm{d}f_b(x,Q^2)$ describes how the ensemble of partons evolve by the branchings of individual partons as above.

Initial-State Shower Basics

- Parton cascades in p are continuously born and recombined.
- Structure at Q is resolved at a time $t \sim 1/Q$ *before* collision.
- A hard scattering at Q^2 probes fluctuations up to that scale.
- A hard scattering inhibits full recombination of the cascade.



- Convenient reinterpretation:



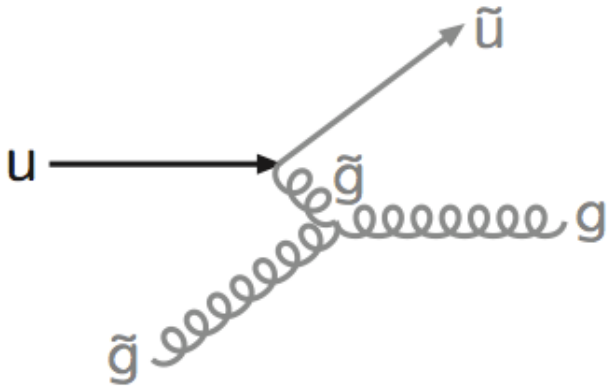
Forwards vs. backwards evolution

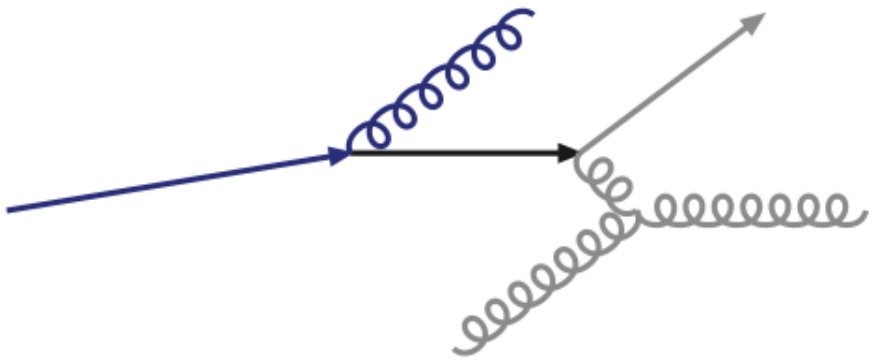
Event generation could be addressed by **forwards evolution**:
pick a complete partonic set at low Q_0 and evolve,
consider collisions at different Q^2 and pick by σ of those.

Inefficient:

1. have to evolve and check for *all* potential collisions, but 99.9. . . % inert
2. impossible (or at least very complicated) to steer the production, e.g. of a narrow resonance (Higgs)

Backwards evolution is viable and ∼equivalent alternative:
start at hard interaction and trace what happened "before"





Backwards evolution master formula

Monte Carlo approach, based on *conditional probability*: recast

$$\frac{\mathrm{d}f_b(x,Q^2)}{\mathrm{d}t} = \sum_a \int_x^1 \frac{\mathrm{d}z}{z}\, f_a(x',Q^2)\, \frac{\alpha_\mathrm{s}}{2\pi}\, P_{a\to bc}(z)$$

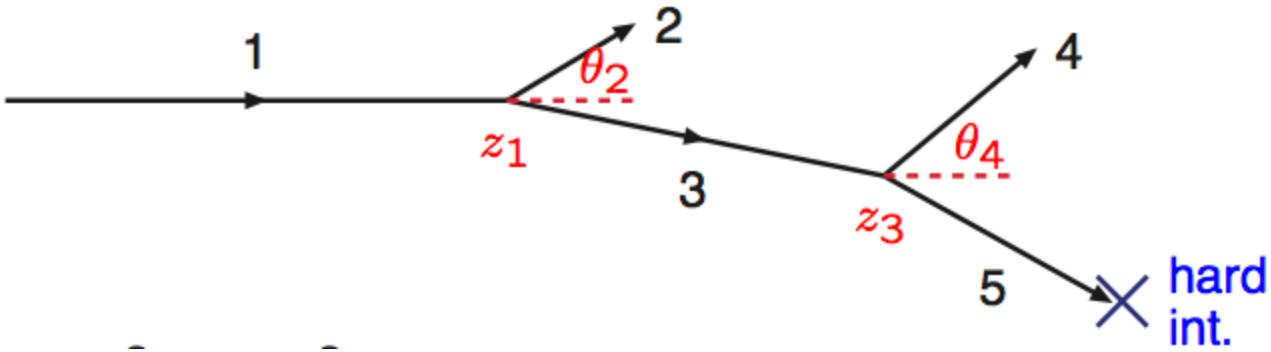
with $t = \ln(Q^2/\Lambda^2)$ and $z = x/x'$ to

$$\mathrm{d}\mathcal{P}_b = \frac{\mathrm{d}f_b}{f_b} = |\mathrm{d}t| \sum_a \int \mathrm{d}z\, \frac{x' f_a(x',t)}{x f_b(x,t)}\, \frac{\alpha_\mathrm{s}}{2\pi}\, P_{a\to bc}(z)$$

then solve for *de*creasing t, i.e. backwards in time,
starting at high Q^2 and moving towards lower,
with Sudakov form factor $\exp(-\int \mathrm{d}\mathcal{P}_b)$

Webber: can be recast by noting that total change of PDF at x is difference between gain by branchings from higher x and loss by branchings to lower x.

Coherence in spacelike showers

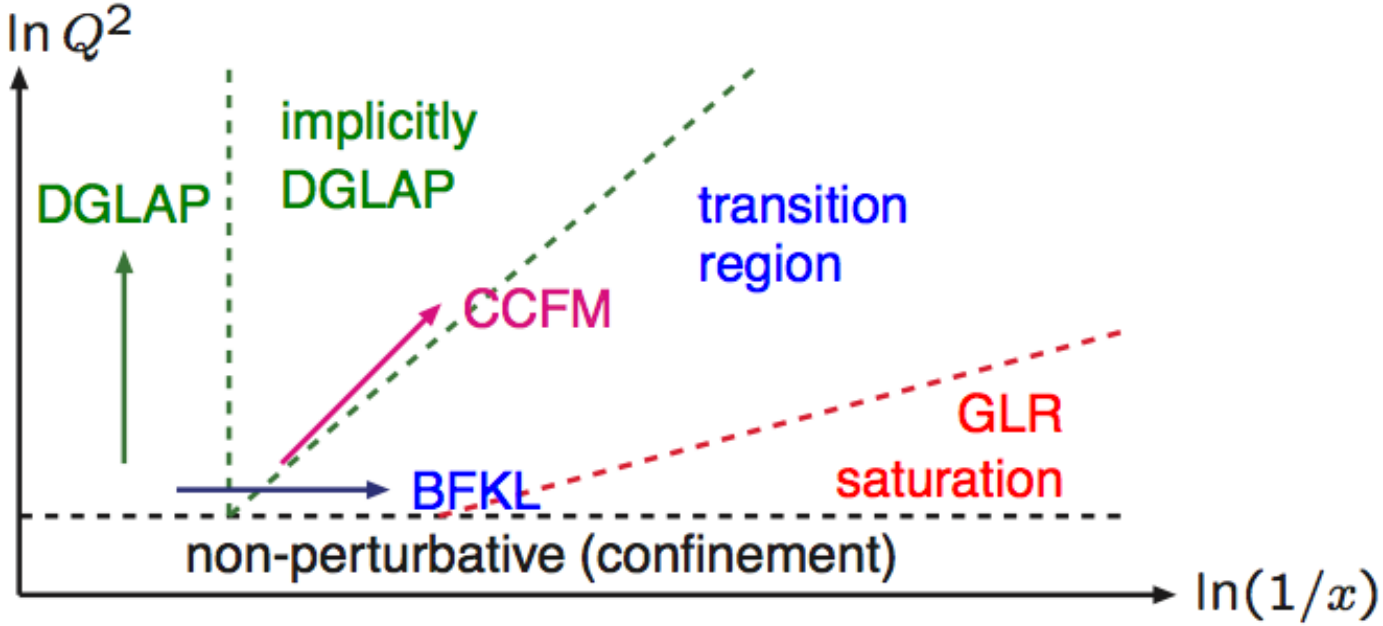


$z_1 = E_3/E_1$
$z_3 = E_5/E_3$
$\theta_2 = \theta_{12}$
$\theta_4 = \theta_{14}$!!

with $Q^2 = -m^2 =$ spacelike virtuality

- kinematics only:
  $Q_3^2 > z_1 Q_1^2,\ Q_5^2 > z_3 Q_3^2,\ \ldots$
  i.e. Q_i^2 need not even be ordered
- coherence of leading collinear singularities:
  $Q_5^2 > Q_3^2 > Q_1^2$, i.e. Q^2 ordered
- coherence of leading soft singularities (more messy):
  $E_3\theta_4 > E_1\theta_2$, i.e. $z_1\theta_4 > \theta_2$
  - $z \ll 1$: $E_1\theta_2 \approx p_{\perp 2}^2 \approx Q_3^2,\ E_3\theta_4 \approx p_{\perp 4}^2 \approx Q_5^2$
    i.e. reduces to Q^2 ordering as above
  - $z \approx 1$: $\theta_4 > \theta_2$, i.e. angular ordering of soft gluons
    $\Longrightarrow$ reduced phase space

Evolution procedures

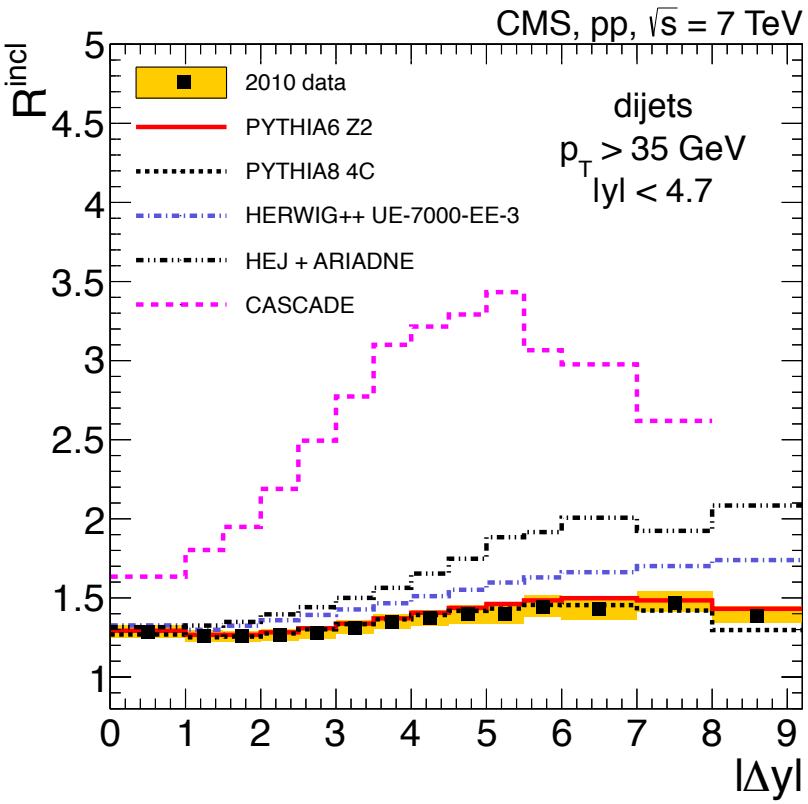


DGLAP: Dokshitzer–Gribov–Lipatov–Altarelli–Parisi
evolution towards larger Q^2 and (implicitly) towards smaller x
BFKL: Balitsky–Fadin–Kuraev–Lipatov
evolution towards smaller x (with small, unordered Q^2)
CCFM: Ciafaloni–Catani–Fiorani–Marchesini
interpolation of DGLAP and BFKL
GLR: Gribov–Levin–Ryskin
nonlinear equation in dense-packing (saturation) region,
where partons recombine, not only branch

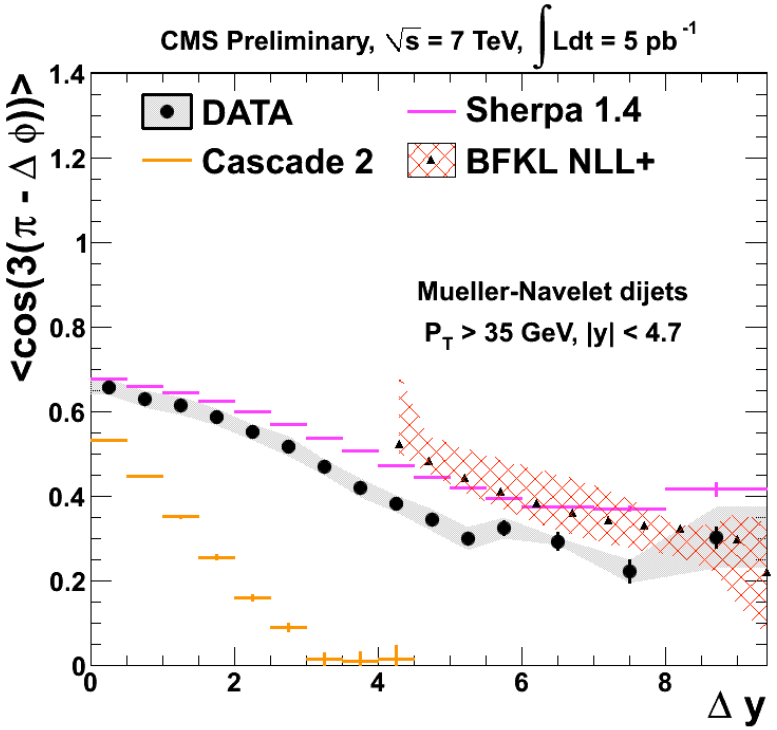
Did we reach BFKL regime?

Study events with ≥ 2 jets as a function of their y separation.

Ratio of the inclusive to exclusive dijet cross sections:



Azimuthal decorrelation:



No strong indications for BFKL/CCFM behaviour onset so far!

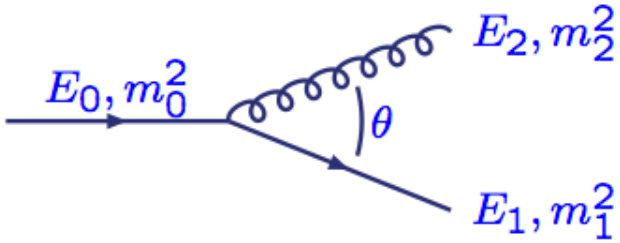
Initial- vs. final-state showers

Both controlled by same evolution equations

$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_\mathrm{s}}{2\pi}\,\frac{\mathrm{d}Q^2}{Q^2}\,P_{a\to bc}(z)\,\mathrm{d}z \;\cdot\; (\mathrm{Sudakov})$$

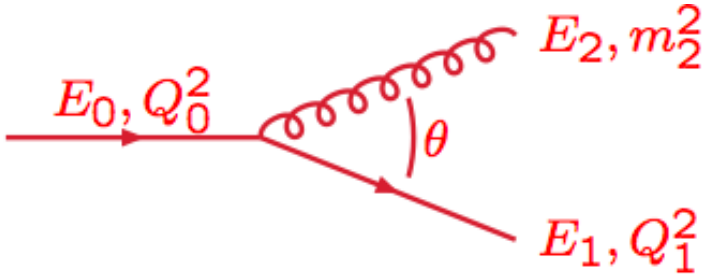
but

Final-state showers:
Q^2 timelike ($\sim m^2$)



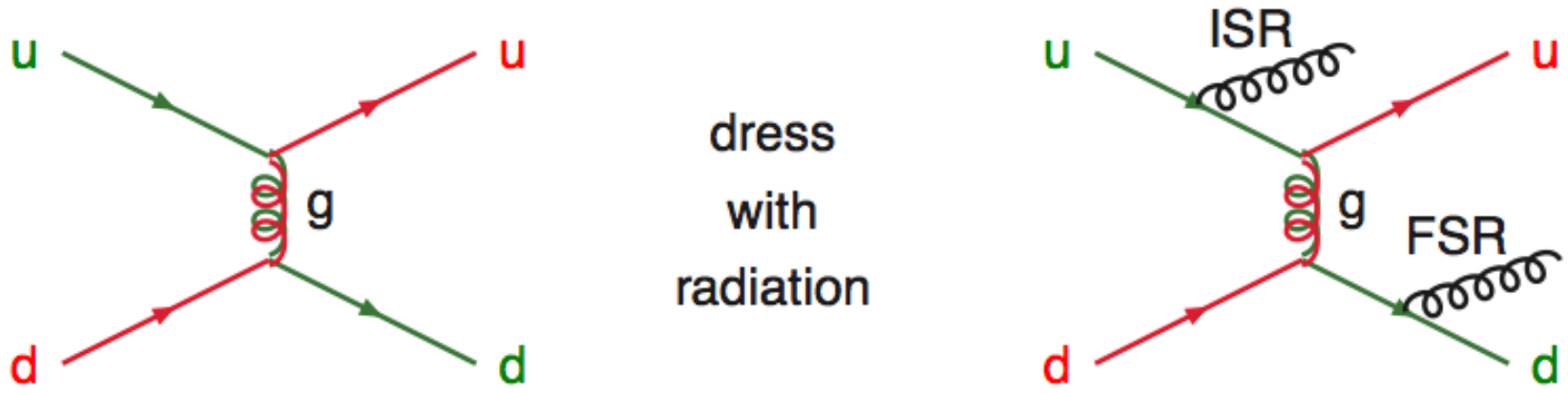
decreasing E, m^2, θ
both daughters $m^2 \geq 0$
physics relatively simple
$\Rightarrow$ "minor" variations:
Q^2, shower vs. dipole, ...

Initial-state showers:
Q^2 spacelike ($\approx -m^2$)

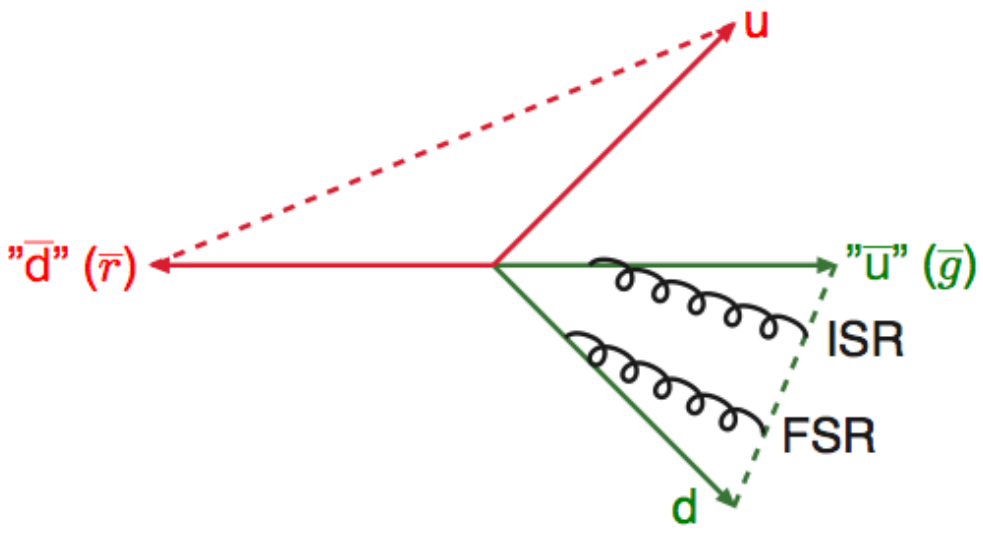


decreasing E, increasing Q^2, θ
one daughter $m^2 \geq 0$, one $m^2 < 0$
physics more complicated
$\Rightarrow$ more formalisms:
DGLAP, BFKL, CCFM, GLR, ...

Combining FSR with ISR



Separate processing of ISR and FSR misses interference
($\sim$ colour dipoles)



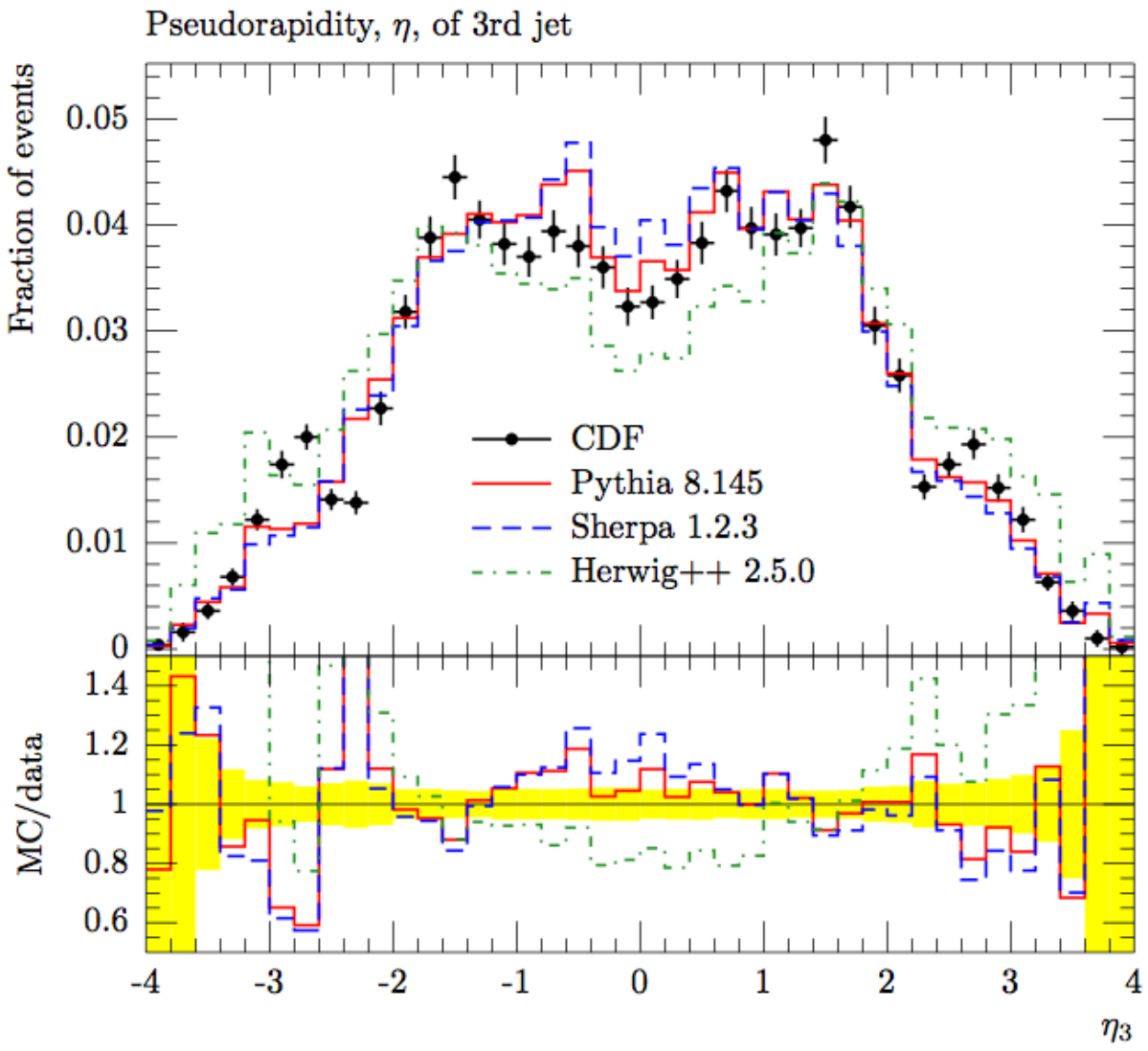
ISR+FSR add coherently in regions of colour flow and destructively else

in "normal" shower by azimuthal anisotropies

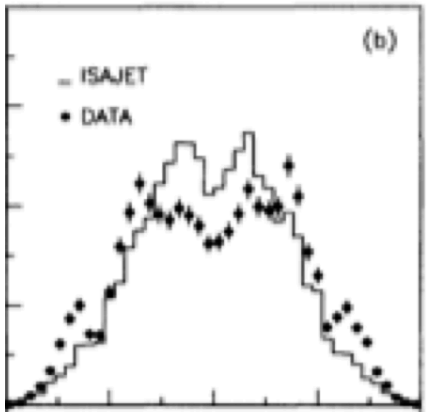
automatic in dipole (by proper boosts)

Coherence tests

Current-day generators for pseudorapidity of third jet:



and past incoherent:



Summary and Outlook

- A multitude of physics mechanisms at play in pp collisions.
- Event generators separate problem into manageable chunks.
- Random numbers $\approx$ quantum mechanical choices.
- Often need to combine several software packages.
- Matrix element calculations at core of process selection.
- Parton shower offers convenient alternative to HO ME's.
- Unitarity by Sudakov form factor.

Tutorial today: begin using PYTHIA; Higgs production as example.

Tomorrow:

- Combining matrix elements and parton showers.
- Multiparton interactions and other soft physics.
- Hadronization.
- Conclusions.