

Renormalization group running of ChPT LECs

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There is some confusion around how the running of Chiral perturbation theory low-energy constants beyond one loop happens. The correct expressions have been derived in [1, 2].

Below follows a slightly different but equivalent derivation. The main confusion follows from the fact that the scale dependence for a dimension d not quite 4 is relevant for the derivation of the running, but in physical quantities only the values at $d = 4$ are relevant.

I give here the derivation using the two-flavour notation, but it is the same for any flavour, or other field theories if treated in the same way.

The definition of renormalization at NLO and NNLO order with the bare LECs at NLO denoted by l_i and at NNLO by c_i . F is the LO decay constant and does not depend on μ . The dimensions of F, l_i, c_i respectively are $(d-2)/2, d-4, d-6$.

$$\Lambda = \frac{1}{16\pi^2(d-4)}, \quad (1)$$

$$l_i = (c\mu)^{d-4} (l_i^r + \gamma_i \Lambda), \quad (2)$$

$$c_i = \frac{1}{F^2} (c\mu)^{2(d-4)} \left(c_i^r - \gamma_i^{(2)} \Lambda^2 - \gamma_i^{(1)} \Lambda - \gamma_i^{(L)} \Lambda \right). \quad (3)$$

The $\gamma_i, \gamma_i^{(2)}, \gamma_i^{(1)}$ are pure numbers.

$$\gamma_i^{(L)} = \sum_j \gamma_{ij}^{(L)} l_j^r, \quad (4)$$

with the $\gamma_{ij}^{(L)}$ pure numbers.

The renormalization definitions (2) and (3) constitute a choice of renormalization scheme, one could have added terms explicitly depending on $d-4$ inside the brackets on the right hand side. The ChPT scheme has put those terms to zero. c is usually set to

$$\ln c = -\frac{1}{2} (\ln 4\pi + \Gamma'(1) + 1) \quad (5)$$

in ChPT. The usual $\overline{MS}$ in QCD drops the 1.

From (2) and the fact that the bare constants do not depend on μ we get

$$0 = \mu \frac{dl_i}{d\mu} = (c\mu)^{d-4} \left(\mu \frac{dl_i^r}{d\mu} + (d-4)l_i^r + \frac{\gamma_i}{16\pi^2} \right), \quad (6)$$

$$\mu \frac{dl_i^r}{d\mu} = -(d-4)l_i^r - \frac{\gamma_i}{16\pi^2}. \quad (7)$$

The second equation follows from the third and note the extra term present when $d \neq 4$. We now use (7) and (4) to obtain

$$\mu \frac{d\gamma_i^{(L)}}{d\mu} = -(d-4)\gamma_i^{(L)} - \sum_j \gamma_{ij}^{(L)} \frac{\gamma_j}{16\pi^2}. \quad (8)$$

In the same way from $\mu(dc_i/d\mu) = 0$ and using (8) we obtain

$$0 = \mu \frac{dc_i^r}{d\mu} + 2(d-4)c_i^r - \frac{2\gamma_i^{(2)}}{16\pi^2}\Lambda - \frac{2\gamma_i^{(1)}}{16\pi^2} - \frac{\gamma_i^{(L)}}{16\pi^2} + \sum_j \gamma_{ij}^{(L)} \frac{\gamma_j}{16\pi^2}\Lambda. \quad (9)$$

The term with $\gamma_i^{(L)}$ does not have the factor of 2 due to the first term in (8). The absence of nonlocal divergences forces the coefficient of Λ to be zero or

$$2\gamma_i^{(2)} = \sum_j \gamma_{ij}^{(L)} \gamma_j \quad (10)$$

This is often called the Weinberg consistency condition and first derived in [3]. The running of the c_i^r then becomes

$$\mu \frac{dc_i^r}{d\mu} = -2(d-4)c_i^r + \frac{2\gamma_i^{(1)}}{16\pi^2} + \frac{\gamma_i^{(L)}}{16\pi^2}. \quad (11)$$

For physical quantities, after the renormalization definitions (2) and (3) are used, all divergences cancel and the limit $d \rightarrow 4$ can be taken. Thus in the end only the numerical values of F , l_i^r and c_i^r at $d = 4$ are relevant and the first term in (7) and (11) can be dropped. They must however be taken into account when deriving the μ -dependence of the renormalized couplings.

References

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